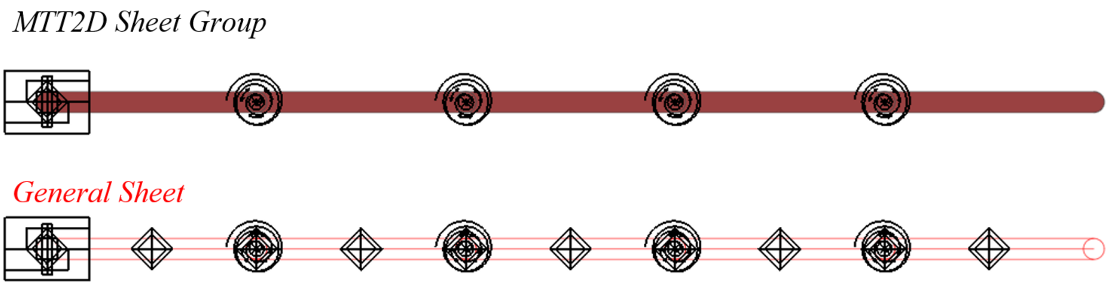
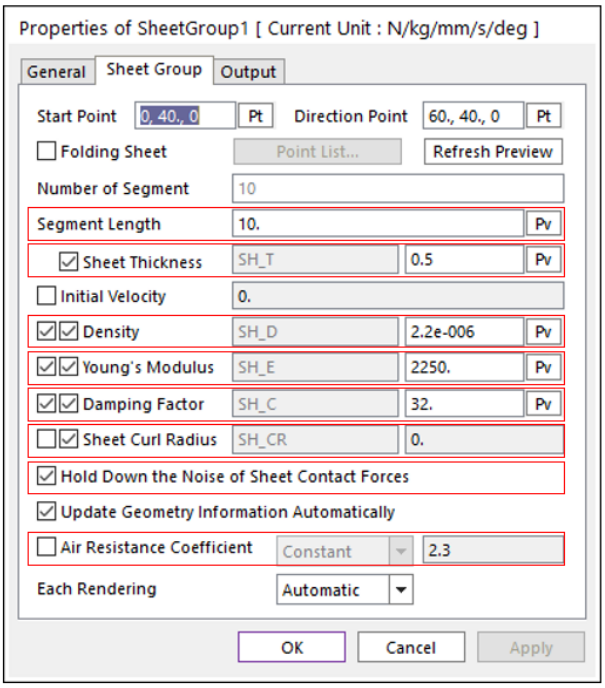
MTT2D Model Validation

Sheet Group

Compare the behavior of MTT2D Sheet with General Sheet properties by defining MTT2D Sheet modeling using RecurDyn General Entities.

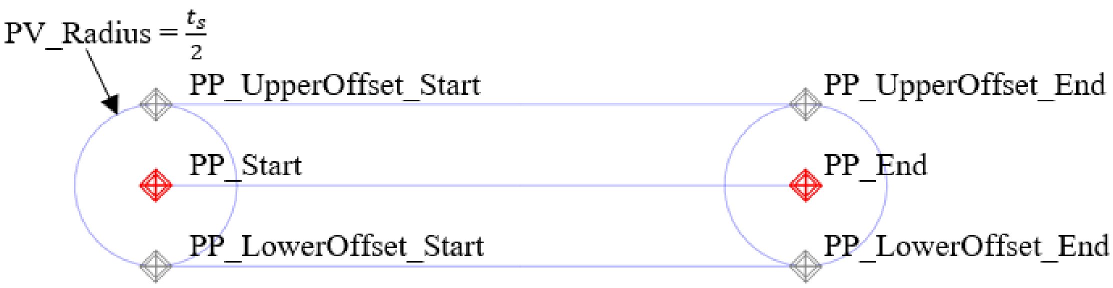




General Sheet Modeling

1) Sheet Segment Body

- Sheet Segment Body is defined by Segment Length, Thickness, and Density.
- Start Point is defined as the origin of Body Local Coordinate, and End Point has x coordinate value of Start Point + Segment Length.
- Segment Body of General Sheet is modeled as Circle and Outline in RecurDyn General Body.
- The parameters of Outline and Circle constituting Sheet are modeled by PPs and PVs that are defined as Segment Length and Sheet Thickness.
- PVs in Body Edit Mode are connected to PVCs of Assembly Mode.



Modeling parameter

Name	Point
PP_Start	0., 0., 0.
PP_End	PV_Length, 0., 0.
PP_UpperOffset_Start	PP_Start.X, PP_Start.Y + PV_Radius, PP_Start.Z
PP_UpperOffset_End	PP_End.X, PP_End.Y + PV_Radius, PP_End.Z
PP_LowerOffset_Start	PP_Start.X, PP_Start.Y – PV_Radius, PP_Start.Z
PP_LowerOffset_End	PP_End.X, PP_End.Y – PV_Radius, PP_End.Z

2) Joint & Force

- Using Revolute Joint and Rotational Spring, we defined the relationships between the General Sheet Segments for the same mechanical behavior as the MTT2D Sheet.
- Origin of Joints and Forces is defined as PP in Assembly Mode through the relative coordinates of Marker set as start point of each Sheet Segment Body.
- The Center of Mass Position is assumed to be located at half the segment length from the start point of each segment on the x axis.

Modeling parameter

Name	Point	Relative to
PP_Sheet1_Start	0, 0, 0	-
PP_Sheet2	Input_SegLength, 0, 0	Sheet1.Marker_Start
PP_Sheet3	Input_SegLength, 0, 0	Sheet2.Marker_Start
PP_Sheet4	Input_SegLength, 0, 0	Sheet3.Marker_Start
PP_Sheet5	Input_SegLength, 0, 0	Sheet4.Marker_Start
PP_CM1	Input_SegLength/2, 0., 0.	Sheet1.Marker_Start
PP_CM2	Input_SegLength/2, 0., 0.	Sheet2.Marker_Start
PP_CM3	Input_SegLength/2, 0., 0.	Sheet3.Marker_Start
PP_CM4	Input_SegLength/2, 0., 0.	Sheet4.Marker_Start
PP_CM5	Input_SegLength/2, 0., 0.	Sheet5.Marker_Start

3) Parametric Values for Sheet Group Modeling

- It is easy to modify the properties of General Sheet by changing the PVs of sheet properties.

Modeling parameter

Name	Point
Input_SegLength	5.
Input_SheetThickness	0.5
Input_Density	2.2e-004
Input_YoungsModulus	2250.
Input_DampingFactor	32.

Input_CurlRadius	0.
Input_Depth	1.

Validation Properties

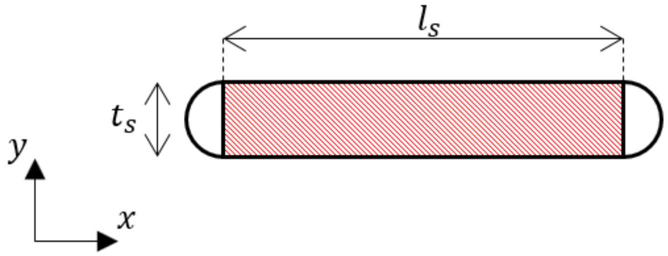
○ Mass & Flexibility

1) Mass & Moment of Inertia

- Mass and Moment of Inertia of Sheet Segment Body are defined as Segment Length, Sheet Thickness, and Density among the input properties.
- The shape of Sheet Segment Body is assumed to be a rectangle, and the Depth is assumed to be 1, which is a unit length.
- Mass(PV_Mass) and I_{zz} (PV_Izz) are defined as follows.

$$m_s = \rho l_s t_s \cdot 1$$

$$I_{zz} = \frac{1}{12}(l_s^{\,2} + t_s^{\,2})$$



2) Flexibility

- Flexibility of Sheet Group is described by connecting each sheet segment with Rotational Spring.
- In this case, the Stiffness and Damping Coefficient (k, c) of Rotational Spring are defined as Young's Modulus (E) and Damping Factor (C_s) among the input properties.
- The torque of Rotational Spring can be calculated by k, c (PV_Rot_K, PV_Rot_C) values derived from the following equations, and the flexibility of the sheet can be described.

$$k = \frac{Et_s^{\ 3}}{12L_s}, \quad c = \frac{C_s t_s^{\ 3}}{12L_s}$$

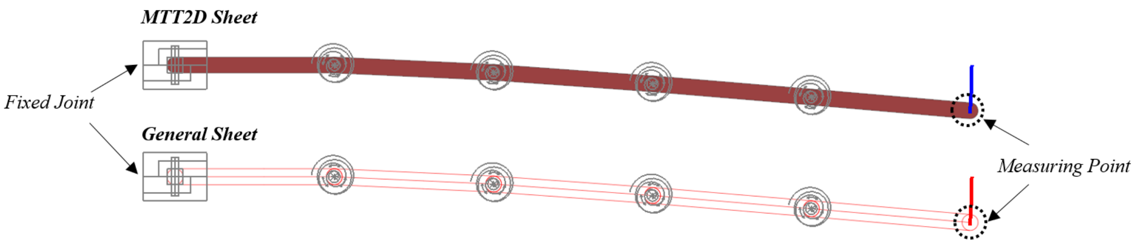
- Fix one end of the sheet like a cantilever in the gravitational field to compare the x, y position at the opposite end when the sheet is bent.

Modeling parameter

Property	Value
Number of Segment	5
Segment Length	5
Sheet Thickness	0.5
Density	2.2e–4
Young's Modulus	2250
Damping Factor	32
Curl Radius	0

Name	Expression
PV_Mass	Input_Density*Input_SegLength*Input_SheetThickness*Input_Depth
PV_Izz	1/12*PV_Mass*(Input_SegLength**2+Input_SheetThickness**2)
PV_Rot_K	(Input_YoungsModulus*Input_SheetThickness**3)/(12*Input_SegLength)
PV_Rot_C	(Input_DampingFactor*Input_SheetThickness**3)/(12*Input_SegLength)

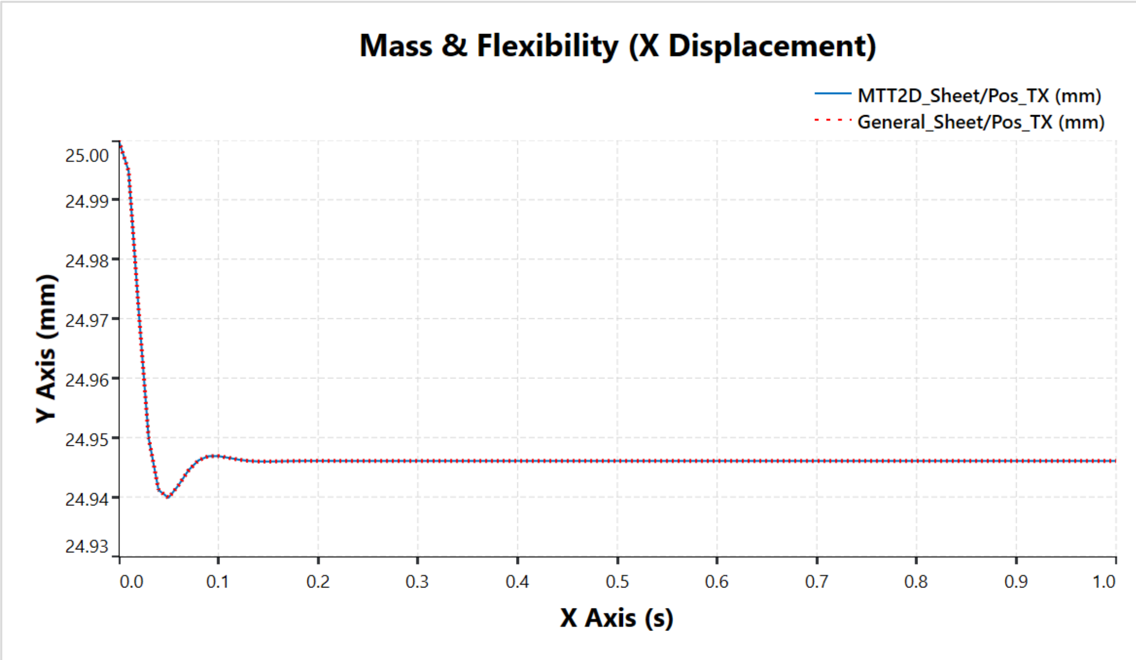
Body modeling and simulation

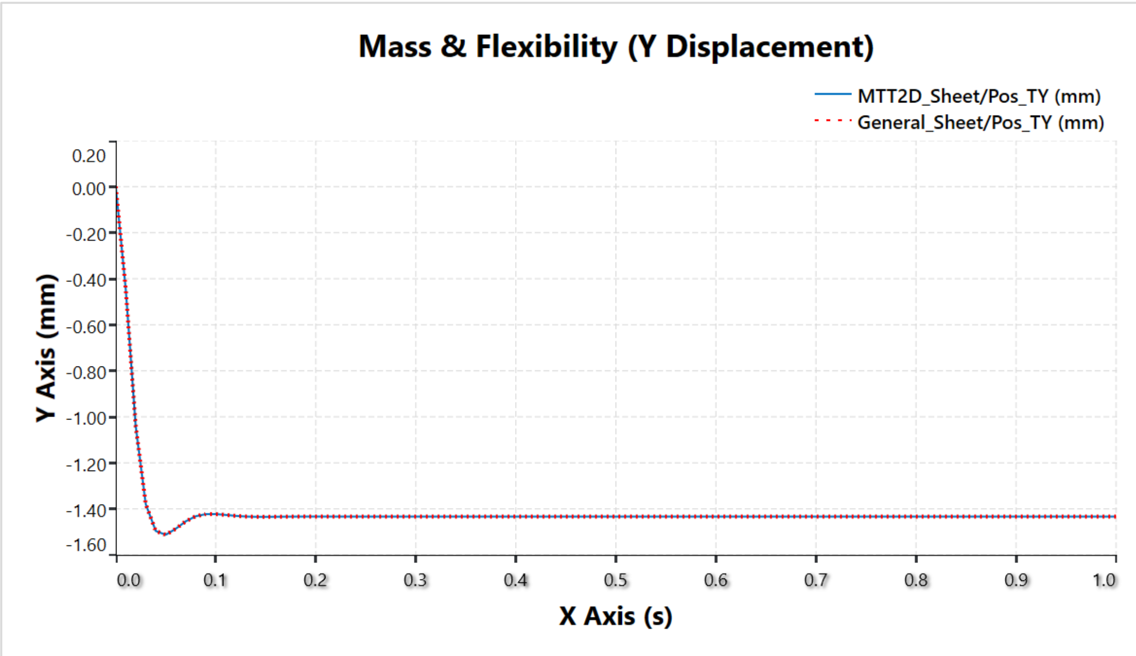


➢ Comparison of results

Plot the results

- Measure the displacement of the end when the analysis result converges.





Object Value	MTT2D Sheet	General Sheet	Error(%)
$\delta_x\ [mm]$	24.95	24.95	0
$\delta_y\ [mm]$	-1.43	-1.43	0

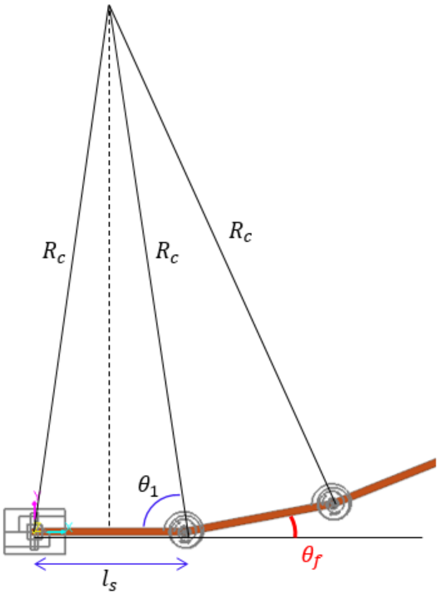
Sheet Curl Radius

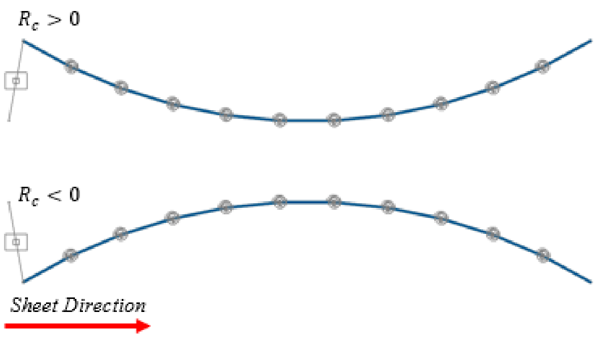
- Sheet Curl Radius is defined by Free Angle of Rotational Spring.
- When the Curl Radius is R_c, the Free Angle (θ_f) of Rotational Spring can be obtained from geometric structure.

$$\theta_1 = \cos^{-1}\left(\frac{l_s/2}{R_c}\right), \quad (-1 < \frac{l_s/2}{R_c} < 1)$$

$$\theta_f = 180 - 2 \times \theta_1 \text{ (deg)}$$

- The bending direction of the sheet depends on the sign of R_c.'
- If the Curl Radius is 0, the θ_1 is not defined, so we implemented "Ex_CheckCosFunc" expression using the IF function so that Free Angle is always 0.
- One end of the sheet was constrained by a Fixed Joint and the y displacement and angular velocity(z-axis) of the last sheet segment were compared when the Curl Radius was 20.





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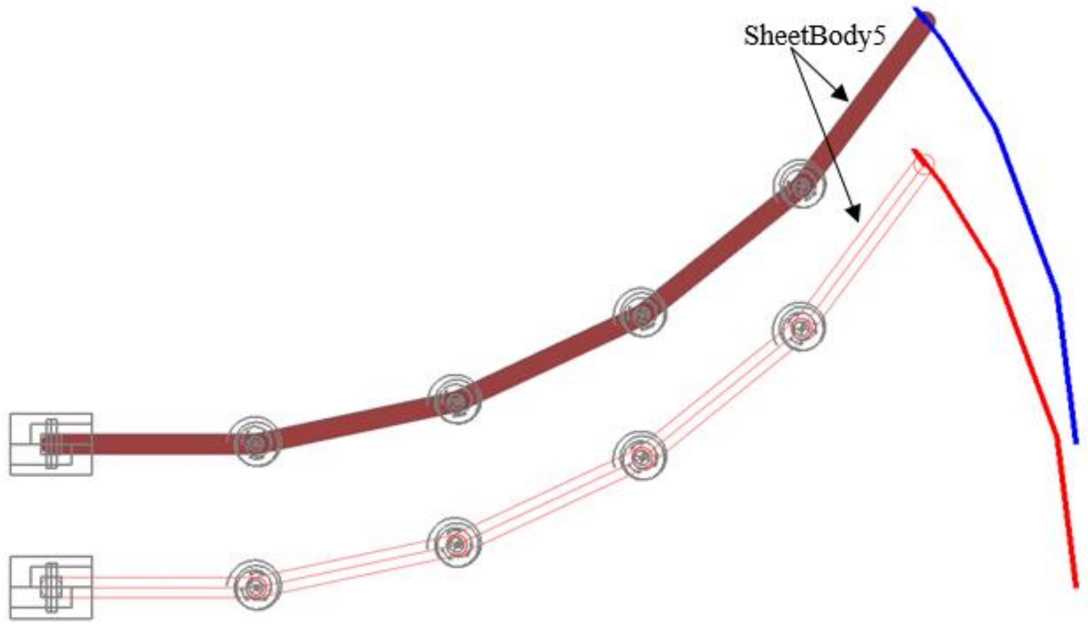
Modeling parameter

Property	Value
Number of Segment	5
Segment Length	5
Sheet Thickness	0.5
Density	2.2e−4
Young's Modulus	2250
Damping Factor	32
Curl Radius	20

Name	Expression
Ex_CheckCosFunc	IF((abs((Input_SegLength)/Input_CurlRadius)-1):((Input_SegLength/2)/Input_CurlRadius),0,0)
PV_CosFunc	Ex_CheckCosFunc
PV_CurlFreeAngle	(180-(acos(PV_CosFunc)*rtod)*2)*dtor

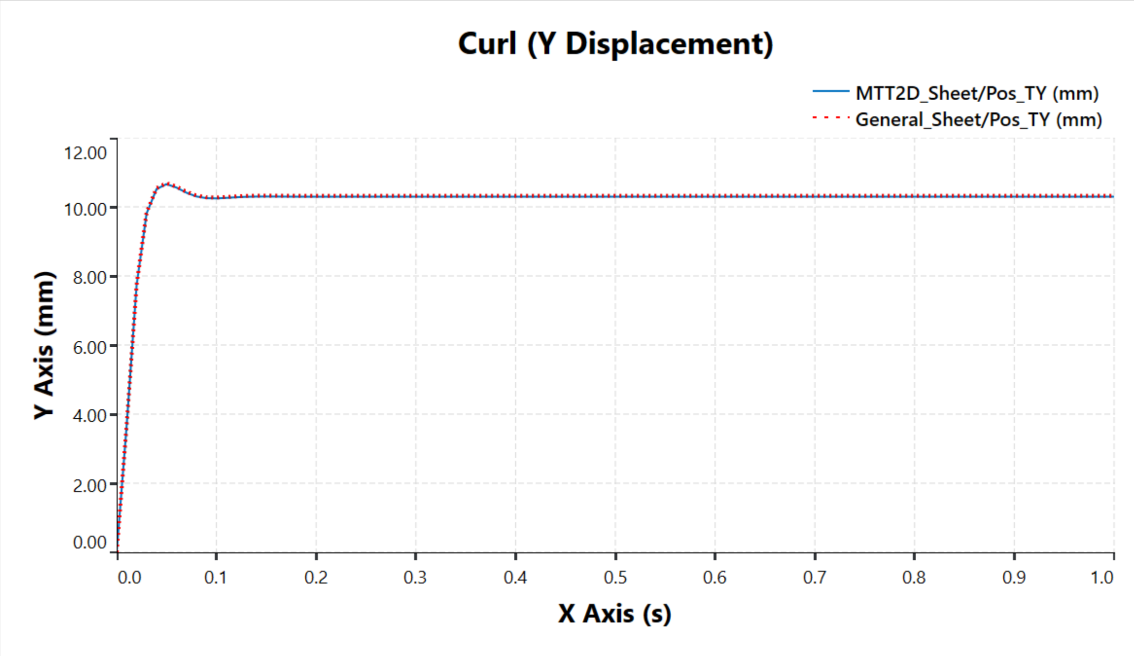
Curl modeling and simulation

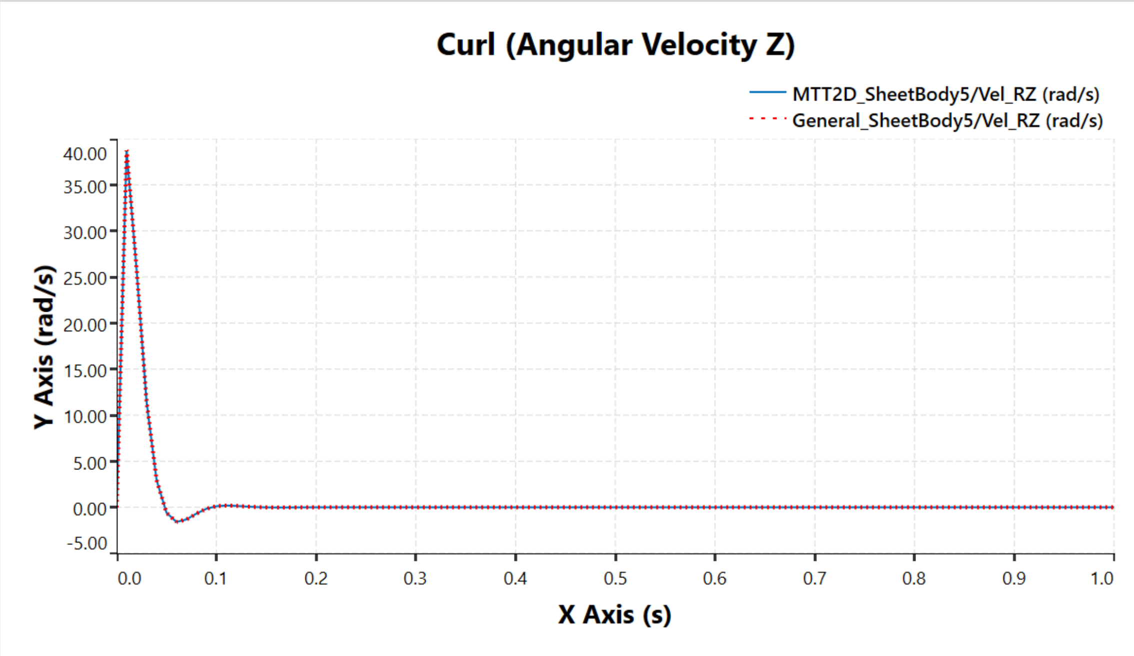


➢ Comparison of results

Plot the results

- y Displacement and Angular Velocity of measuring point.





Object Value	MTT2D Sheet	General Sheet	Error(%)
δ_y $[mm]$	10.29	10.32	0.2
$v_z\, max$ $[rad/s]$	38.78	38.66	0.5

Air Resistance Force

- The air resistance force acting on the sheet segment is defined as follows.

$$F_{air} = \frac{1}{2} C_d \rho_{air} A v^2 \cdot \frac{1}{UCF}$$

Where, C_d is Drag Coefficient (Input), A is area of sheet segment and v is velocity. The air density of ρ_{air} is set to $1.22 \times 10^{-9} (kg/mm^3)$.

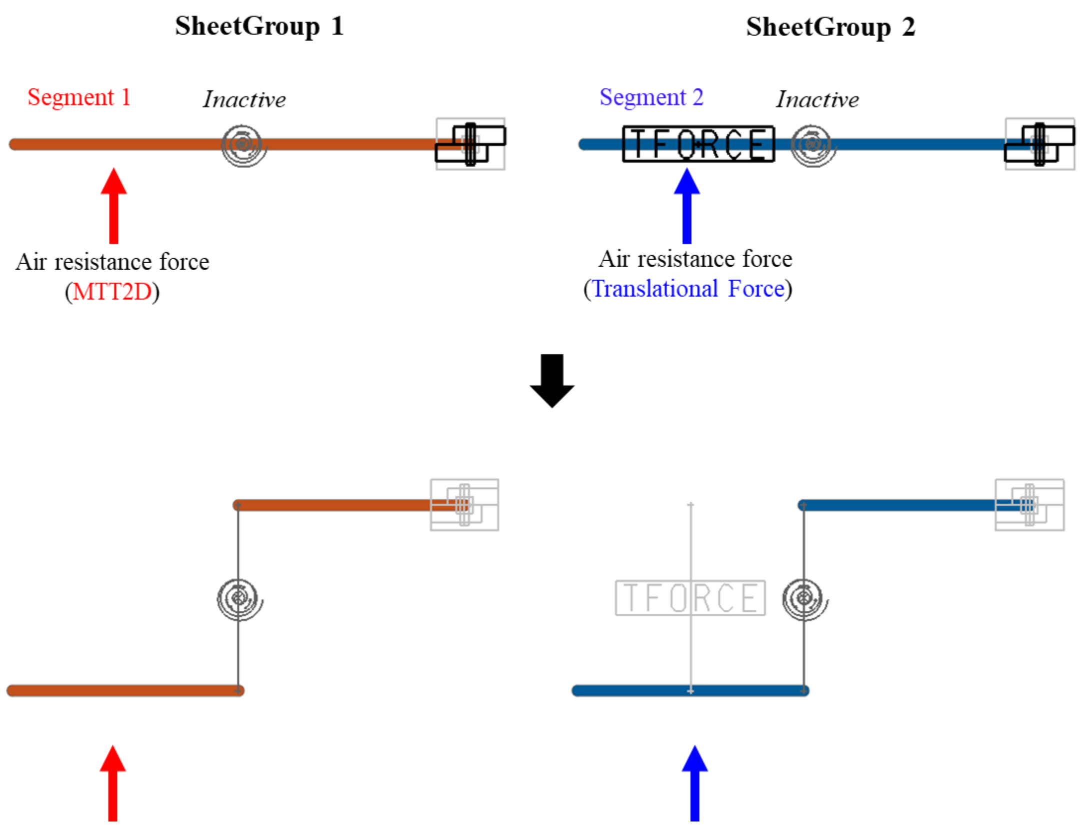
- Default values are used for the Sheet Properties and Drag Coefficient (Input_Cd: 2.3) is added.
- The air resistance force is verified by simulating the free fall of a single sheet segment in when the Revolute Joint and Rotational Spring are inactivated.
- Segment 1 uses the MTT2D's Air Resistance Force function and Segment 2 uses the Translational Force to implement the Air Resistance Force formula.

Modeling parameter

Property	Value
Start Point (Sheet Group1)	0, 40., 0
Direction Point (Sheet Group1)	-40., 40., 0
Start Point (Sheet Group2)	25., 40., 0
Direction Point (Sheet Group2)	5., 40., 0
Number of Segment	2
Segment Length	10
Sheet Thickness	0.5
Density	2.2e−6
Young's Modulus	2250
Damping Factor	32
Drag Coefficient	2.3

Name	Expression
Ex_AirRes_Y	(1/2)*Input_Cd*1.22*1e-9*10*(VY(1,2)*VY(1,2))*PV_UCF
PV_UCF	1.e-003

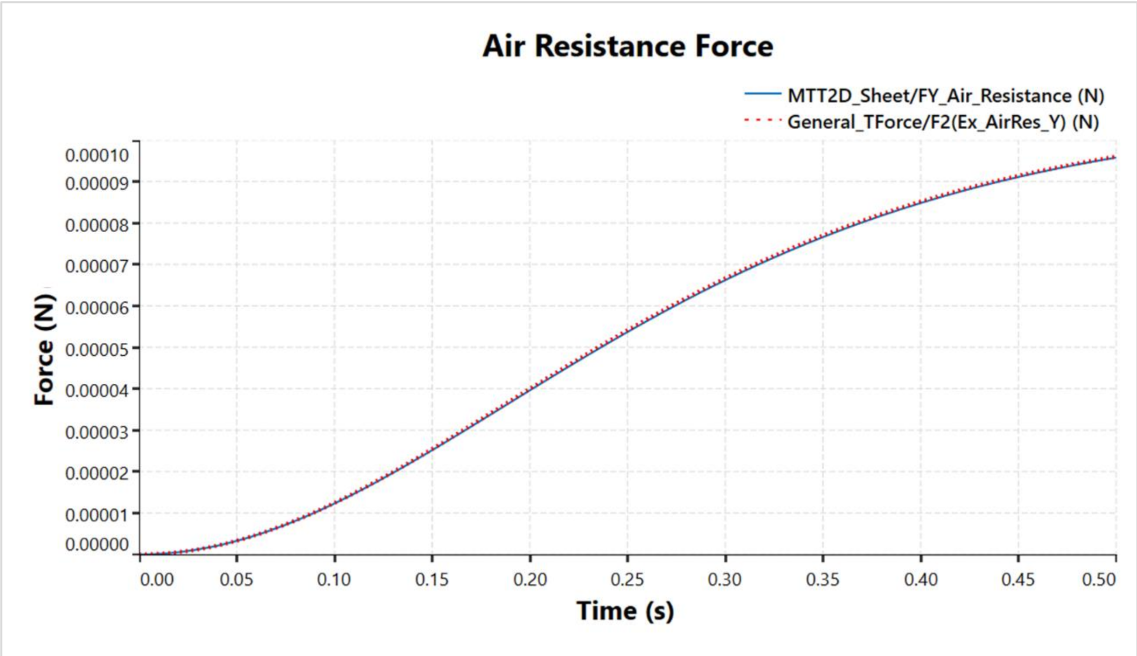
Air Resistance Force modeling and simulation

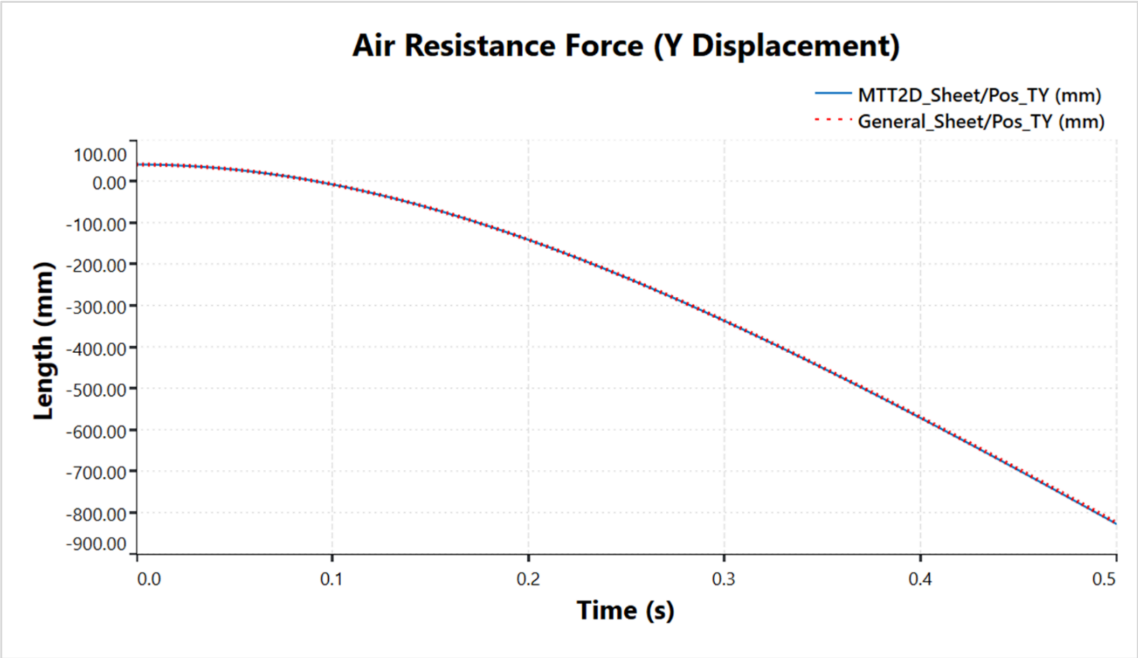


➢ Comparison of results

Plot the results

- Air Resistance Force and y displacement.

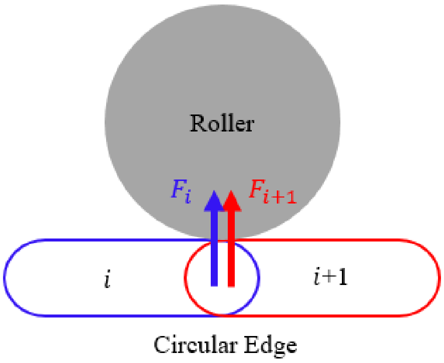


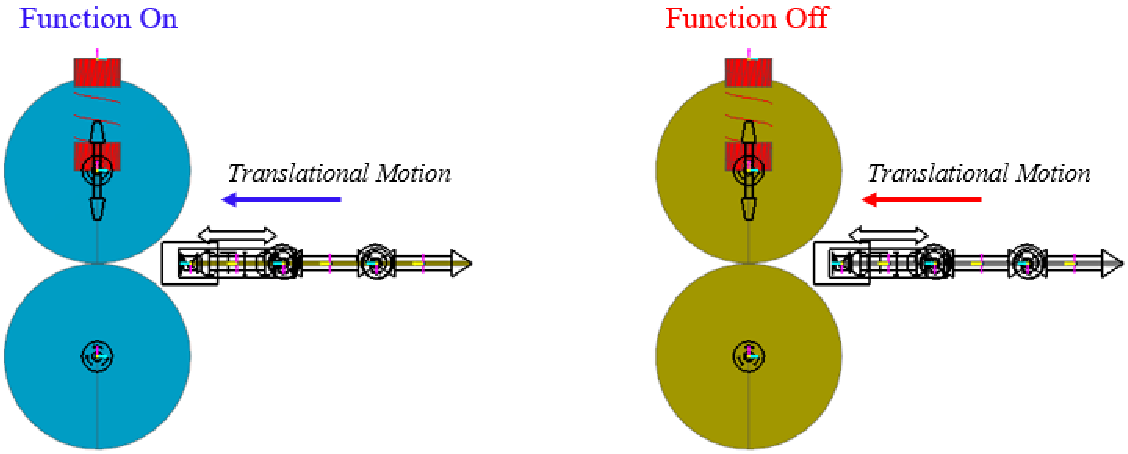


Object Value	MTT2D	TForce	Error(%)
F_{air} [N]	-9.58e-5	-9.60e-5	0.25
δ_y [mm]	-827.78	-825.15	0.31

Hold Down the Noise of Sheet Contact Forces

- Sheet Group is modeled by overlapped Circular Edges of Sheet Segment.
- When a contact occurs between overlapped Circular Edges and Roller, the sum of contact forces ($F_i + F_{i+1}$) due to Segment *i* and Segment *i + 1* acts on a Roller.
- "Hold Down the Noise of Sheet Contact Forces" function can be used to override these contact force nesting phenomena.
- Translational motion is imposed to Sheet to compare the contact forces acting on the Fixed Roller according to On/Off of the function when the sheet is passing through Roller Pair.
- All parameters are default value.





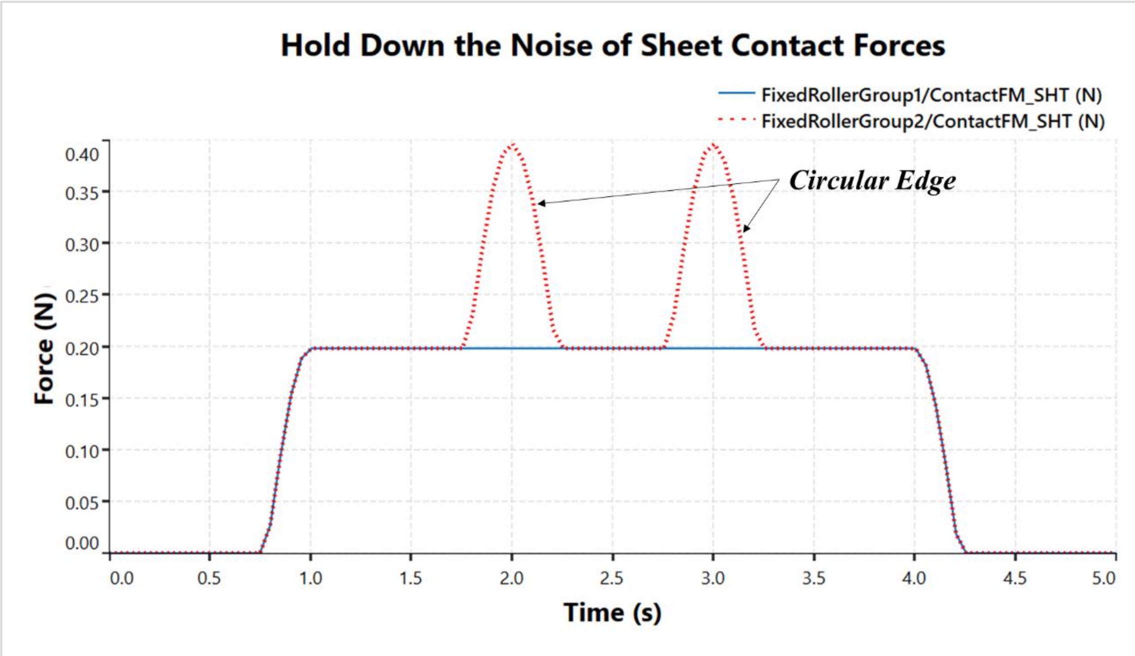
Modeling parameter

Property	Value
Number of Segment	3
Segment Length	10
Sheet Thickness	0.5
Fixed Roller Radius	10
Movable Roller Radius	10

➢ Comparison of results

Plot the results

- Comparison of Contact Force by Function On/Off.



Numerical Results

- When the contact is occurred at the point where the circular edges have overlapped, the value of the contact force is doubled.
- The result below shows the effect of "Hold Down the Noise of Sheet Contact Forces" function.

Object Value	Function On	Function Off
$F_{mag}\ max\ [N]$	0.197	0.395 $(\approx 0.197 \times 2)$

Roller Contact

Contact to Sheet

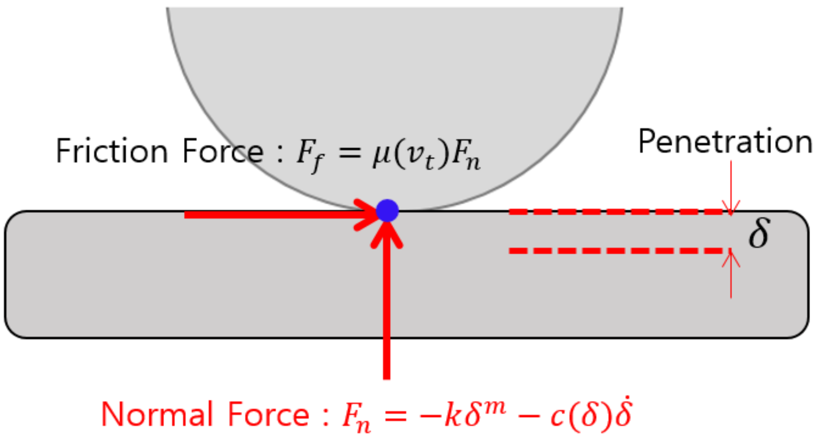
Contact Force

- Contact Normal Force is defined by the following equation. (The value of $c(\delta)$ is defined differently depending on the type used.)

  $f_n = -k\delta^m - c(\delta)\dot{\delta}^n$: Contact Normal Force

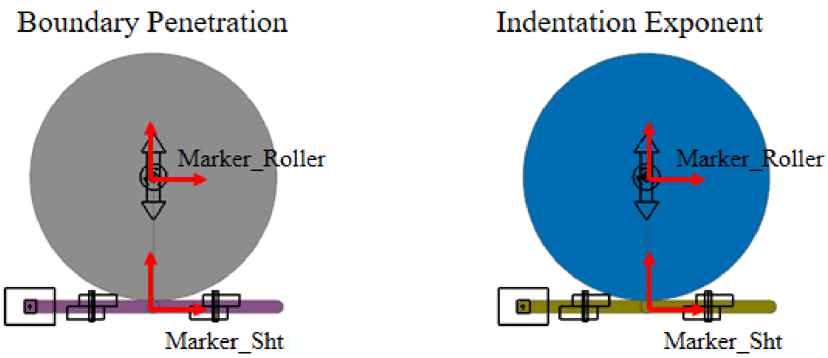
  $c(\delta) = c_{\max}\delta^n$: using the indentation exponent

  $c(\delta) = step(\delta, 0, 0, \delta_b, c_{\max})$: using the boundary penetration



- Verify the normal force of the Fixed Roller - Sheet Contact for two cases using indentation exponent and boundary penetration.
- All the degrees of freedom of the sheet are constrained by the fixed joint at the center of mass, and the revolute joint of roller is inactive so that only Y-axis translational motion is possible.
- MTT2D Contact force and Expression value are compared through expressions using Marker_Roller (1) and Marker_Sht (2) which are defined in Roller and Sheet.

- Results using Boundary Penetration can be found in the left system, and results using Indentation Exponent can be found in the right system.
- Default values are used for all unmarked properties.



Modeling parameter

Property	Value
Roller Center Point (Left)	-10., 0, 0
Roller Radius (Left)	10
Roller Center Point (Right)	30., 0, 0
Roller Radius (Right)	10
Sheet Start Point (Left)	-20., -10.5, 0
Sheet Direction Point (Left)	0, -10.5, 0
Sheet Start Point (Right)	20., -10.5, 0
Sheet Direction Point (Right)	40., -10.5, 0
Sheet Thickness	1
PV_Stiffness	1.2
PV_Exponent	1
PV_MaximumDamping	1.2e-002
PV_BoundaryPenetration	1
PV_IndentationExponent	1

Name	Expression
Ex_Penentration_Boundary	10.50-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel_Boundary	VY(1,2)
Ex_ContactForce_Boundary	-(-PV_Stiffness*VARVAL(1)**PV_Exponent)-(VARVAL(3)*VARVAL(2))

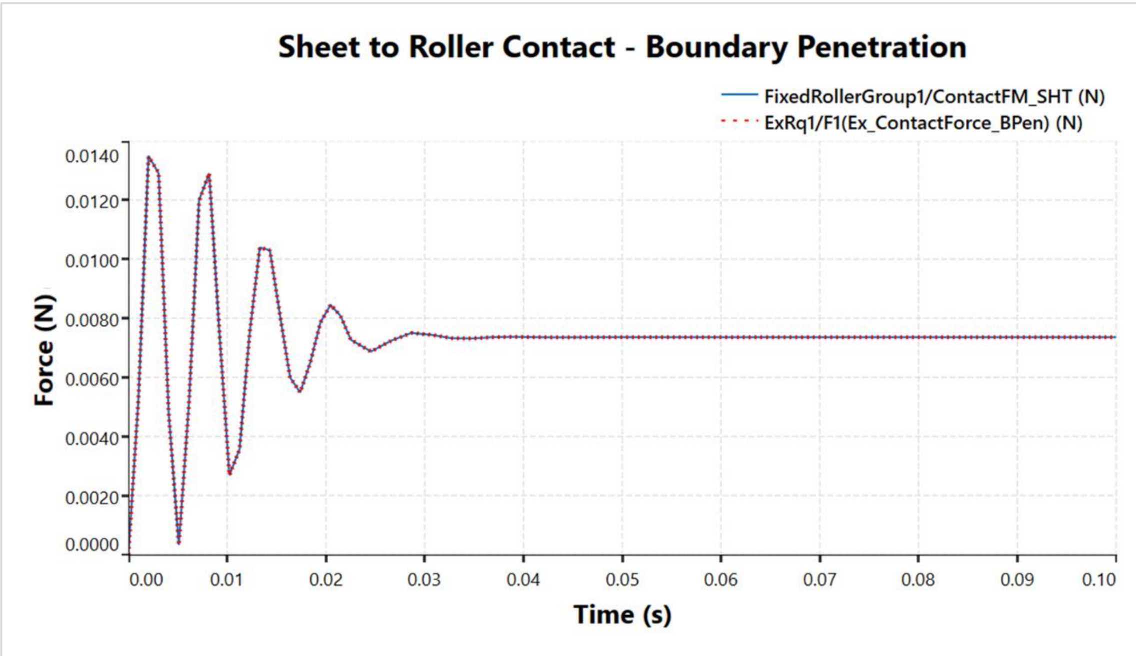
Ex_BoundaryPenetration	step(VARVAL(1),0,0,PV_BoundaryPenetration,PV_MaximumDamping)
Ex_Penentration_Indentation	10.50-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel_Indentation	VY(1,2)
Ex_ContactForce_Indentation	-(-PV_Stiffness*(VARVAL(1)**PV_Exponent)) -(PV_MaximumDamping*(VARVAL(1)**PV_IndentationExponent)*VARVAL(2))*2

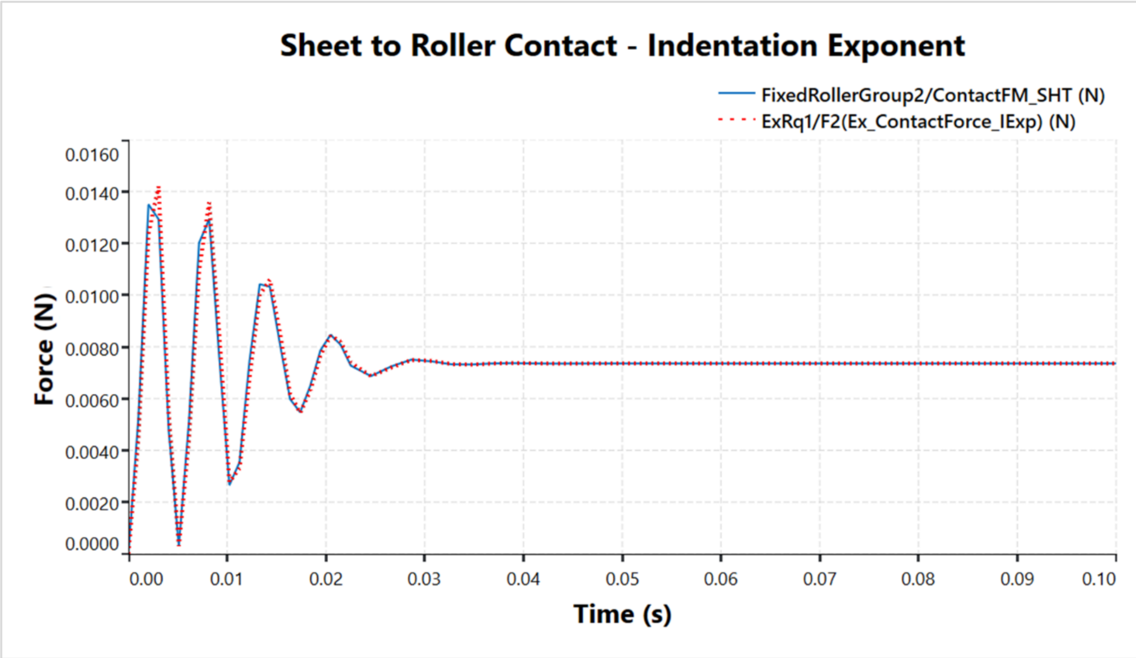
Name	Expression
VE_Penetration_Boundary (1)	Ex_Penentration_ Boundary
VE_PenetrationVel_Boundary (2)	Ex_PenetrationVel_ Boundary
VE_BoundaryPenetration (3)	Ex_BoundaryPenetration
VE_Penetration_Indentation (1)	Ex_Penentration_ Indentation
VE_PenetrationVel_Indentation (2)	Ex_PenetrationVel_ Indentation

➢ Comparison of results

Plot the results

- Sheet to Roller Contact normal force





Object Value	MTT2D	Expression	Error(%)
f_n [N] (Boundary Penetration)	7.35e-3	7.35e-3	0
f_n [N] (Indentation Exponent)	7.35e-3	7.35e-3	0

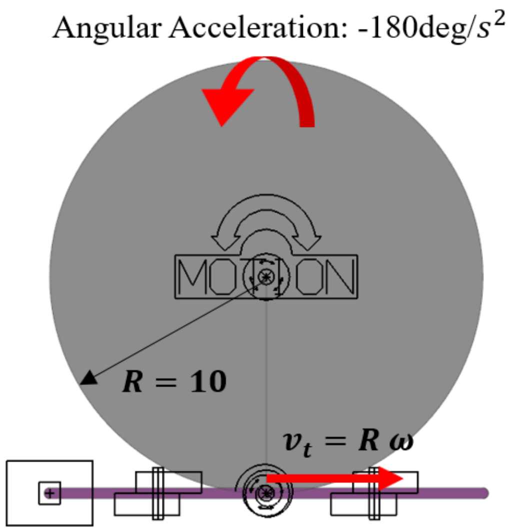
Friction Force

- The Friction coefficient is defined relative to the tangential velocity at the contact point.

$$f_f = \mu(v_t) f_n$$

- Verify the Friction Force when the roller rotates at a constant acceleration with the Sheet fixed.
- The penetration is constant at 0.25 mm.
- Verify Friction Coefficient with Contact Normal Force and Friction Force when v_t is equal to Threshold Velocity.

$$\mu(v_t) = \frac{f_f}{f_n}$$



Modeling parameter

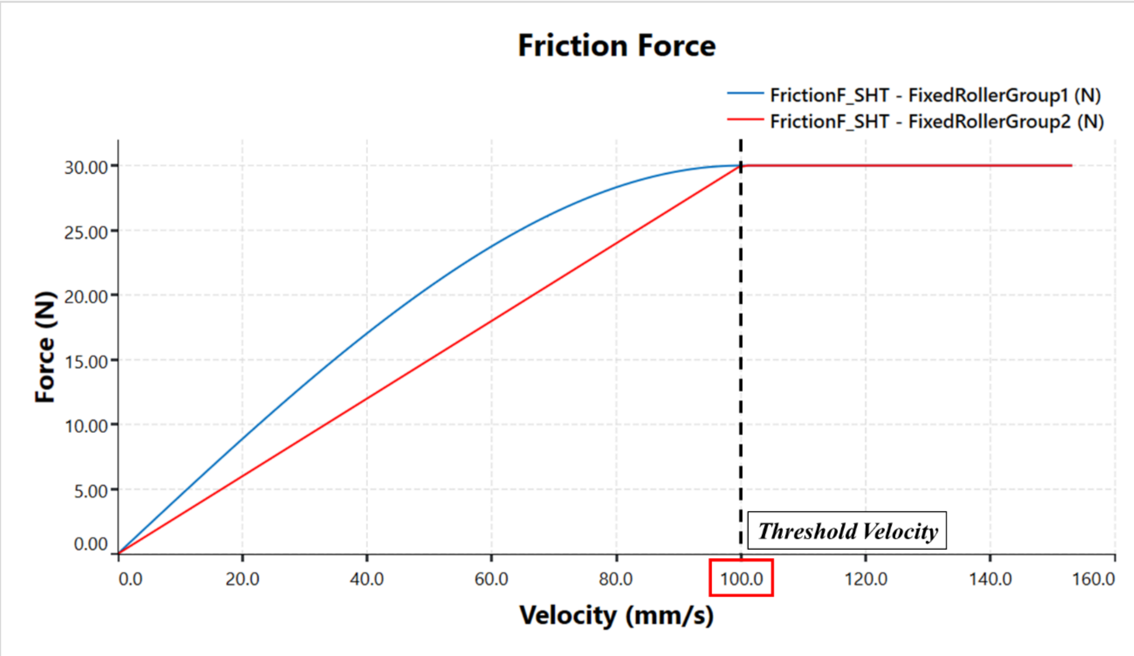
Property	Value
Roller Center Point	-10., 0, 0
Roller Radius	10
Sheet Start Point	-20., -10., 0
Sheet Direction Point	0, -10., 0
Sheet Thickness	0.5
Friction Coefficient	100
Threshold Velocity	100
Penetration	0.25

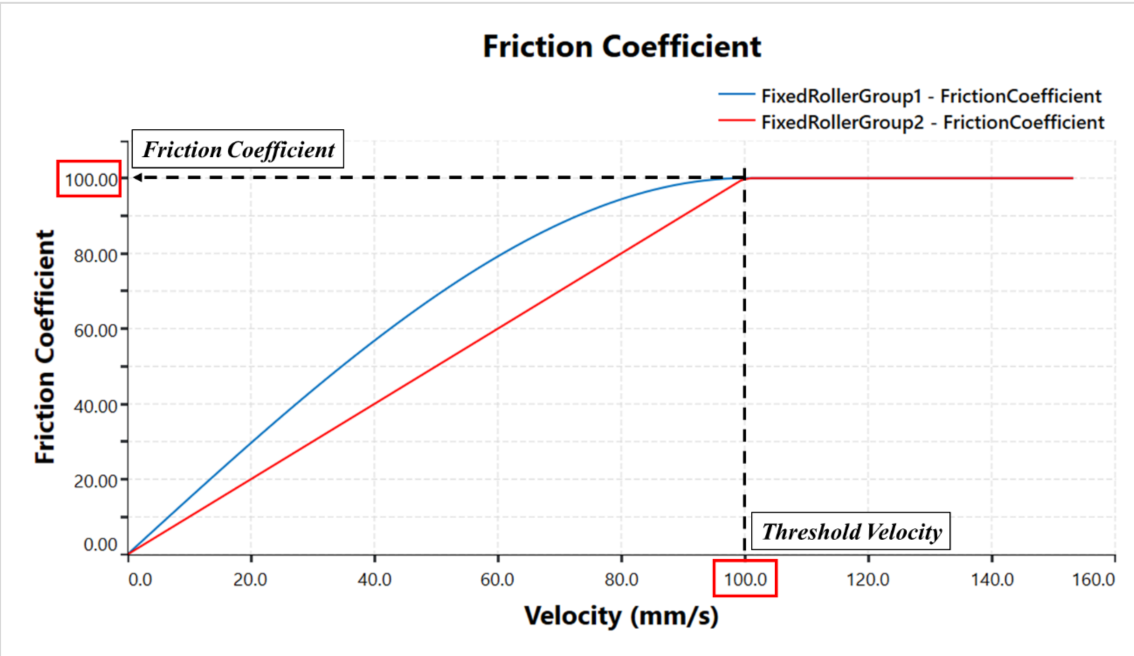
Name	Expression
Ex_AngularAcc	-180D
Ex_Tangential_Velocity	WM(1,2)*(10-0.25)

➢ Comparison of results

Plot the results

- STEP/Linear Type Friction Force and Friction Coefficient





Object Value	STEP	Linear
$f_f\ [N],\ v_t = 100$	30	30
$\mu(v_t),\ v_t = 100$	100	100

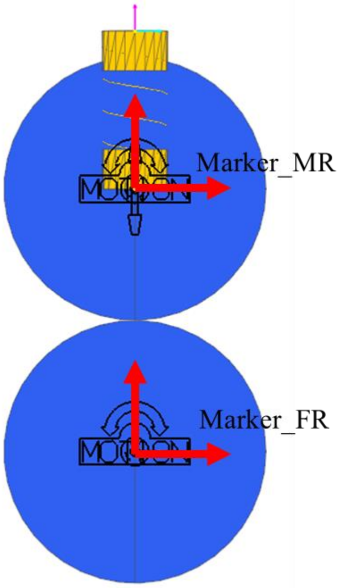
Contact to Roller

Fixed to Movable Roller Contact Normal Force

- Contact Normal Force of Fixed Roller to Movable Roller is defined by the same formula as the previous Sheet Contact.

$$f_n = -k\delta^m - c(\delta)\dot{\delta}^n$$

- Both Roller radii is created as 10, and 0 is input to Revolute Joint Motion to ignore Friction.
- Normal Force Expression is implemented with Penetration and Penetration Velocity using Marker_Roller (1) and Marker_Sht (2).



Modeling parameter

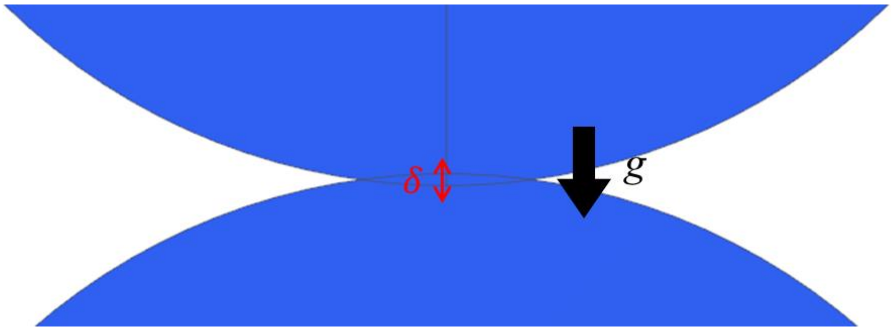
Property	Value
Fixed Roller Radius	10
Fixed Roller Center Point	30., 0, 0
Movable Roller Radius	10
Movable Roller Direction	0, 20., 0
PV_Stiffness	0.8
PV_Exponent	1

PV_MaximumDamping	1.2e-002
PV_BoundaryPenetration	2

Name	Expression
Ex_RevJoint	0
Ex_Penetration	20-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel	VY(1,2)
Ex_BoundaryPenetration	step(VARVAL(1),0,0,PV_BoundaryPenetration,PV_MaximumDamping)
Ex_ContactForce_Boundary	-(-PV_Stiffness*VARVAL(1)**PV_Exponent)+(VARVAL(3)*VARVAL(2))
Ex_ContactForce_Indentation	-(-PV_Stiffness*(VARVAL(1)**PV_Exponent)) -(PV_MaximumDamping*(VARVAL(1)**PV_Indentation Exponent)*VARVAL(2))*2

Name	Expression
VE_Penetration (1)	Ex_Penentration
VE_PenetrationVel (2)	Ex_PenetrationVel
VE_BoundaryPenetration (3)	Ex_BoundaryPenetration

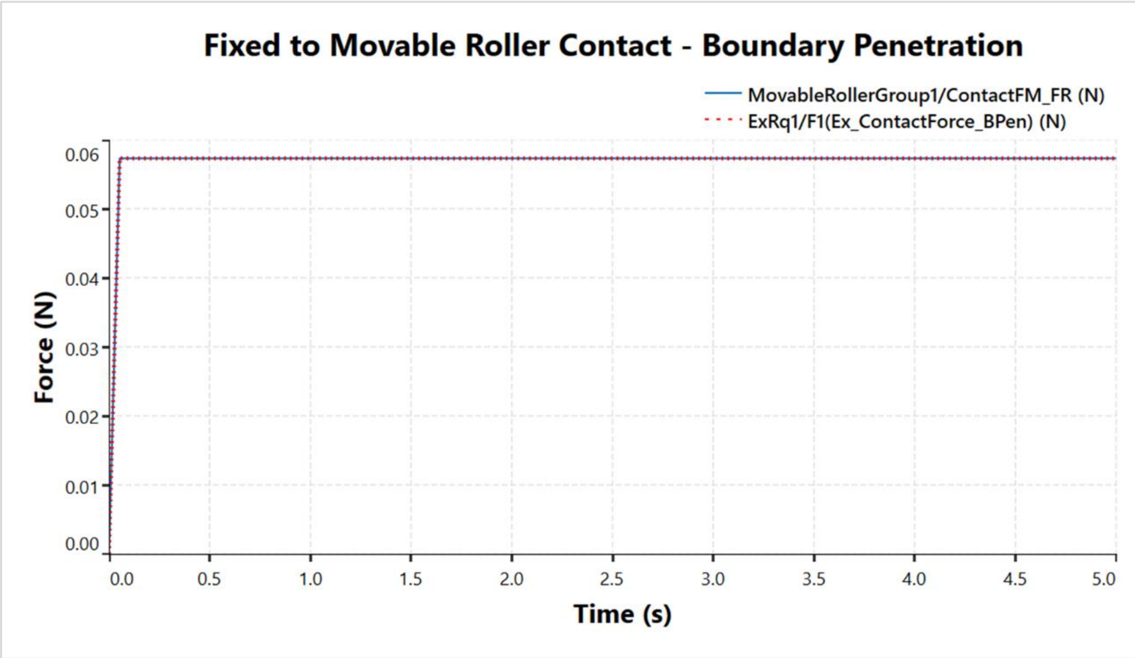
Fixed to Movable Roller simulation

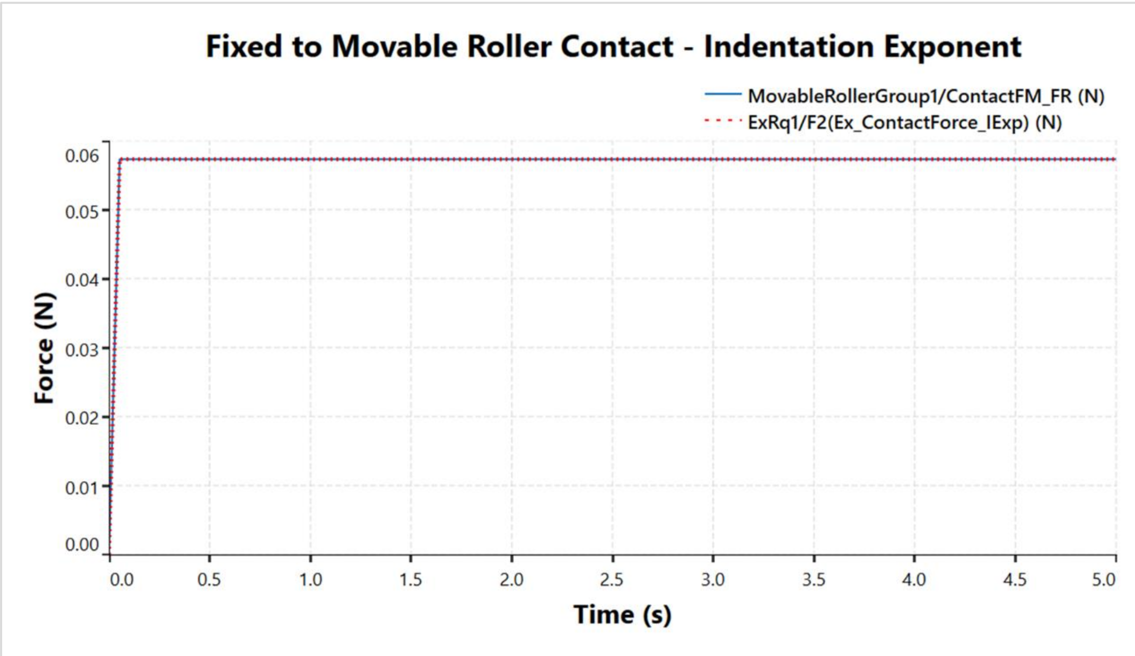


➢ Comparison of results

Plot the results

- Fixed to Movable Roller Contact normal force





Object Value	MTT2D	Expression	Error(%)
f_n $[N]$ (Boundary Penetration)	5.73e-2	5.73e-2	0
f_n $[N]$ (Indentation Exponent)	5.73e-2	5.73e-2	0

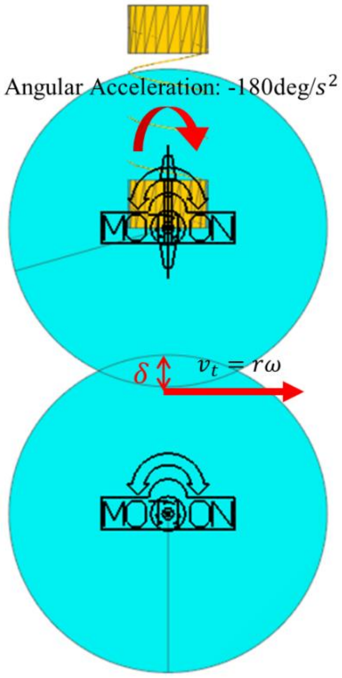
Fixed to Movable Roller Contact Friction Force

- The same formula is used for the Friction Force between the rollers.

$$f_f = \mu(v_t) f_n$$

- The penetration is maintained at 2 mm by imposing displacement motion on Translational Joint.
- The angular acceleration is increased linearly by 1 sec using the STEP function, and the acceleration after 1 sec is kept constant.
- Verify Friction Coefficient with Contact Normal Force and Friction Force when v_t is equal to Threshold Velocity.

$$\mu(v_t) = \frac{f_f}{f_n}$$



Modeling parameter

Property	Value
Fixed Roller Radius	10
Fixed Roller Center Point	30., 0, 0
Movable Roller Radius	10
Movable Roller Direction	0, 20., 0

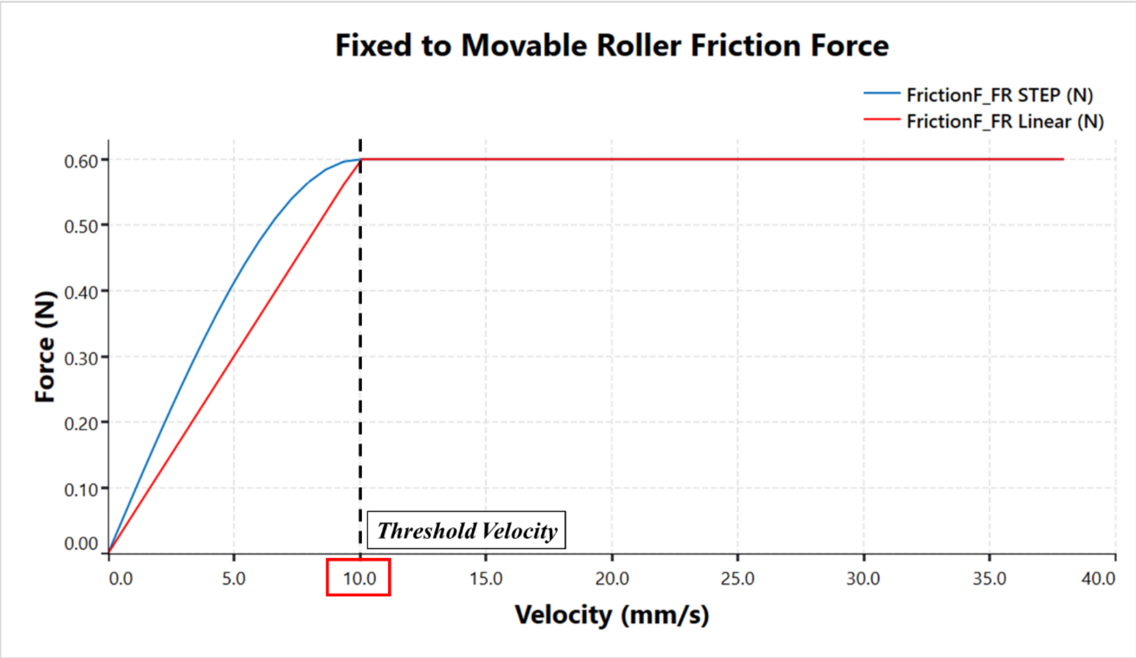
PV_Friction Coefficient	1
PV_Threshold Velocity	10

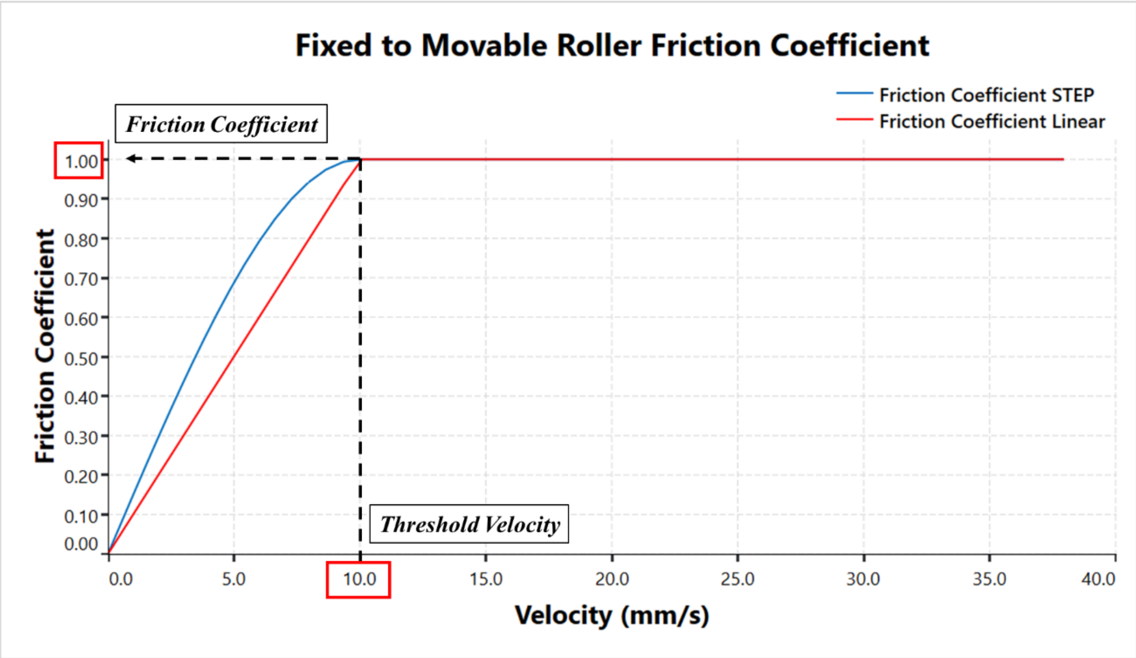
Name	Expression
Ex_RevJoint	0
EX_Penetration	-2
Ex_AngularAcc	step(time,1,0,2,-180d)
Ex_Penetration	-2
Ex_TangentialVel	WM(1,2)*(10-2)

➢ Comparison of results

Plot the results

- STEP/Linear Type Friction Force and Friction Coefficient





Object Value	STEP	Linear
$\boldsymbol{f_f}\ [N],\ \boldsymbol{v_t = 10}$	0.6	0.6
$\boldsymbol{\mu(v_t)},\ \boldsymbol{v_t = 10}$	1	1

Guide Contact

Linear Guide

⭘ Contact force

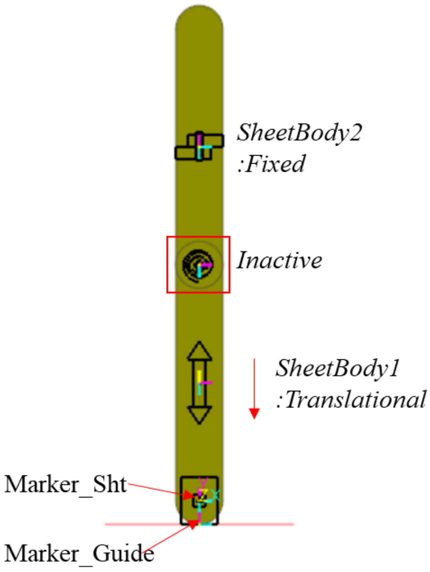
- Verify the results by comparing Expression and RPLT of the Sheet to Linear Guide contact force defined by the MTT Contact formula.

  $f_n = -k\delta^m - c(\delta)\dot{\delta}^n$: Contact Normal Force

  $c(\delta) = c_{\max}\delta^n$: using the indentation exponent

  $c(\delta) = step(\delta, 0, 0, \delta_b, c_{\max})$: using the boundary penetration

- The Sheet moving in the -Y direction comes into the Linear Guide by gravity.
- To verify the contact force of a single sheet segment and a guide, the Revolute Joint and the Rotational Spring constituting the Sheet Group are inactive.
- The sheet segment is constrained with a translational joint at the center of mass so that it can only be moved in the Y-axis direction.
- Create Marker_Sht (1) at the center of the Circular Edge of Sheet Segment and create Marker_Guide (2) at the center of Linear Guide for a Contact Expression.



Modeling parameter

Property	Value
Sheet Start Point	0, 0, 0
Sheet Direction	0, 40., 0
Number of Segment	2
Sheet Thickness	2
Sheet Marker Mother Body	SheetBody1
Sheet Marker Position	0, 0, 0
Guide Mother Body	MotherBody
Guide Start Point	-4., -1., 0
Guide End Point	4., -1., 0
Guide Marker Mother Body	MotherBody
Guide Marker Position	0, -1., 0
Guide Contact Direction	Up
PV_Stiffness	0.1
PV_BoundaryPenetration	0.5

Name	Expression
Ex_Penetration	1-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel	VY(1,2)

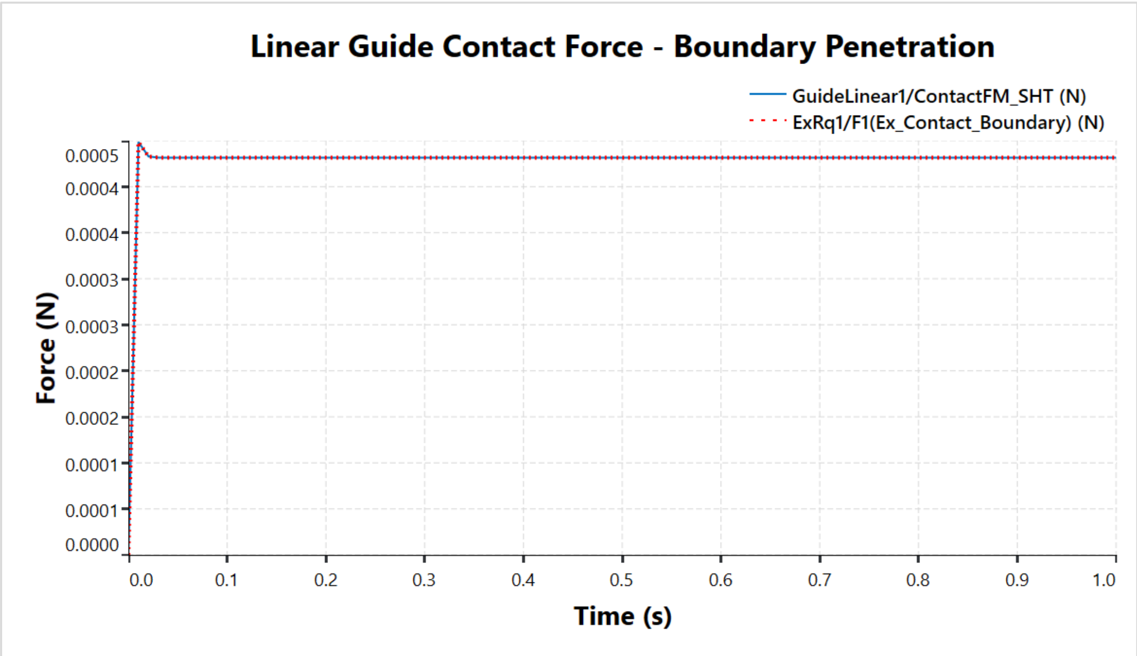
Ex_BoundaryPenetration	step(VARVAL(1),0,0,PV_BoundaryPenetration,PV_MaximumDamping)
Ex_Contact_Boundary	-(-PV_Stiffness*VARVAL(1)**PV_Exponent)-(VARVAL(3)*VARVAL(2))
Ex_Contact_Indentation	-(-PV_Stiffness*(VARVAL(1)**PV_Exponent)) -(PV_MaximumDamping*(VARVAL(1)**PV_IndentationExponent)*VARVAL(2))*2

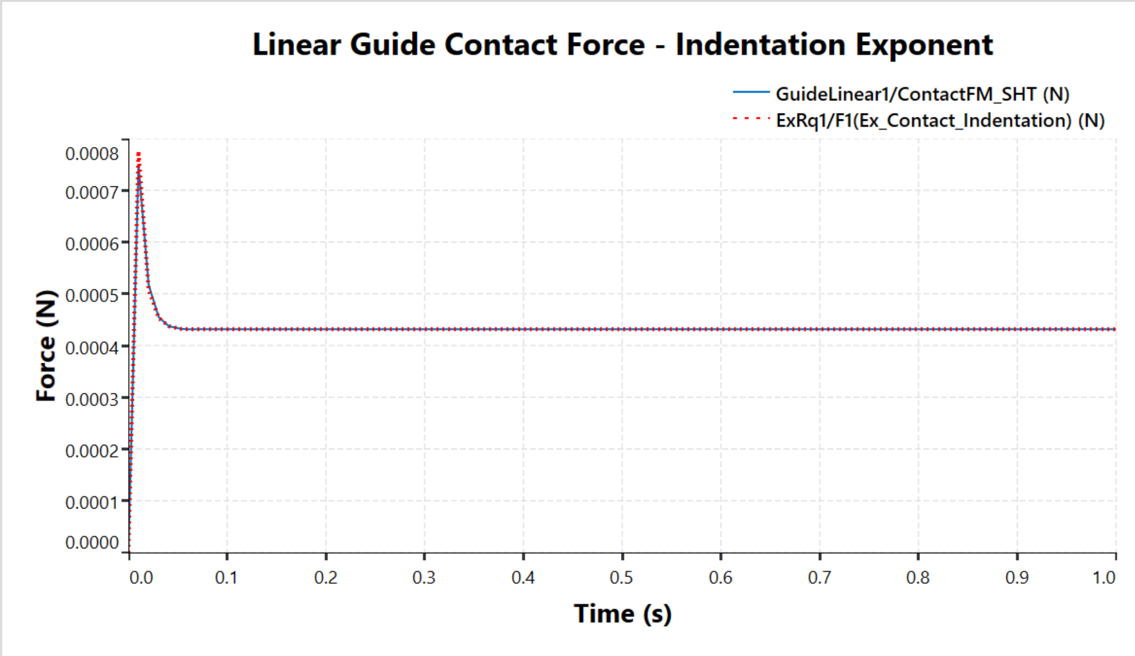
Name	Expression
VE_Penetration (1)	Ex_Penentration
VE_PenetrationVel (2)	Ex_PenetrationVel
VE_BoundaryPenetration (3)	Ex_BoundaryPenetration

➢ Comparison of results

Plot the results

- Linear Guide to Sheet Contact





Object Value	MTT2D	Expression	Error(%)
f_n ***Max*** $[N]$ (Boundary Penetration)	4.48e-4	4.48e-4	0
f_n ***Max*** $[N]$ (Indentation Exponent)	7.50e-4	7.75e-4	3.4
f_n ***End*** $[N]$ (Boundary Penetration)	4.31e-4	4.31e-4	0
f_n ***End*** $[N]$ (Indentation Exponent)	4.31e-4	4.31e-4	0

Friction force

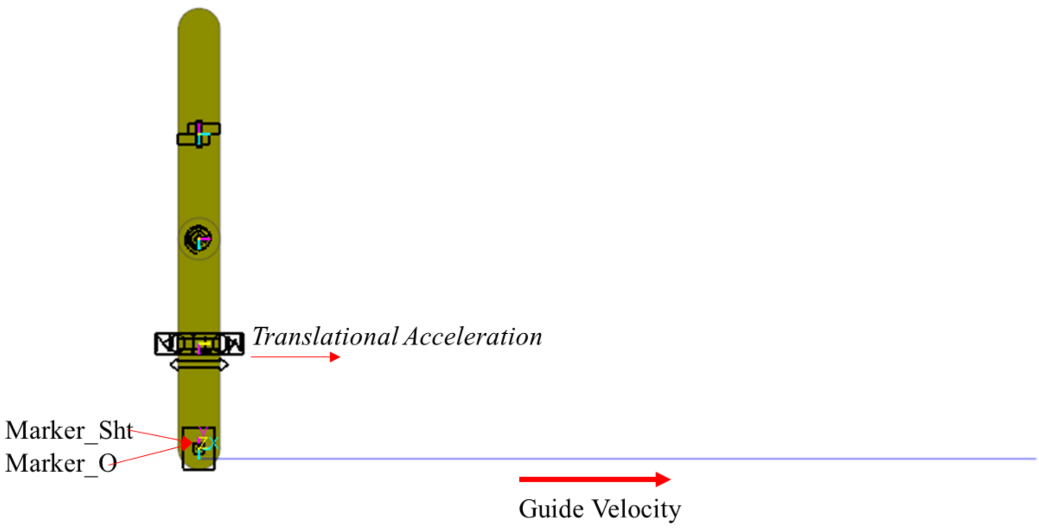
- In the case of Guide Contact, a v_g term is added to apply the influence on Guide Velocity (which implements the transfer effect of sheet such as Conveyor Belt) in the Friction Force equation.

  $$f_f = \mu(v_t - v_g) f_n$$

- Fix SheetBody2 as Fixed Joint and create +X direction Translational Joint to the center of mass of SheetBody1. The Revolute Joint and Rotational Spring is inactive.

- Create Marker_Sht (1) at the center of the circular edge of the sheet and Marker_O (2) at the origin. And calculate the value of $(v_t - v_g)$.
- Keep the penetration constant where the contact force has occurred, and verify the Friction Force and Friction Coefficient by imposing +X translational motion with constant acceleration.

$$\mu(v_t - v_g) = \frac{f_f}{f_n}$$



Modeling parameter

Property	Value
Sheet Start Point	0, 0, 0
Sheet Direction	0, 40., 0
Number of Segment	2
Sheet Thickness	2
Guide Mother Body	MotherBody
Guide Start Point	0, -0.5, 0
Guide End Point	40., -0.5, 0
Guide Contact Direction	Up
PV_Stiffness	0.1
PV_BoundaryPenetration	0.5
PV_FrictionCoeff	0.3
PV_ThresholdVel	10

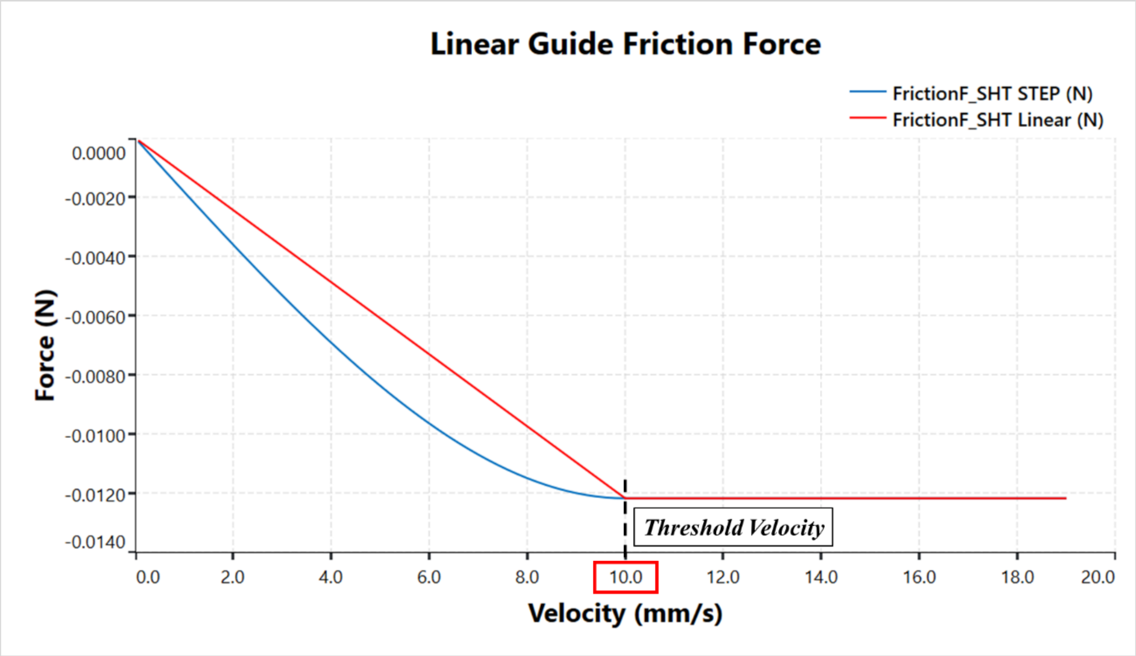
PV_GuideVelocity	1

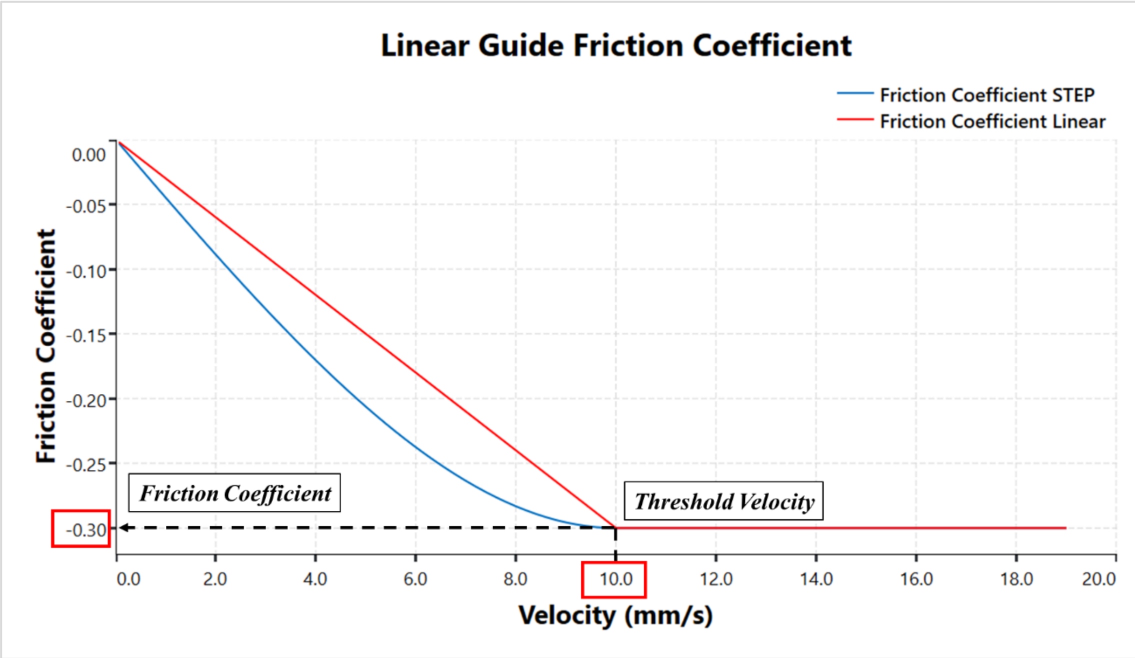
Name	Expression
Ex_ TraAcc	10
Ex_Tangential_Velocity	WM(1,2)-PV_GuideVelocity

➢ Comparison of results

Plot the results

- Linear Guide to Sheet Friction Force and Friction Coefficient





Object Value	STEP	Linear
$f_f\ [N],\ v_t = 10$	-0.012	-0.012
$\mu(v_t),\ v_t = 10$	0.3	0.3

Arc Guide

Contact force

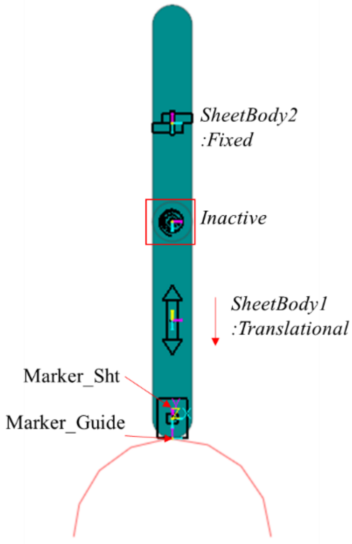
- Verify the results by comparing Expression and RPLT of the Sheet to Arc Guide contact force defined by the MTT Contact formula.

  $f_n = -k\delta^m - c(\delta)\dot{\delta}^n$: Contact Normal Force

  $c(\delta) = c_{\max}\delta^n$: using the indentation exponent

  $c(\delta) = step(\delta, 0, 0, \delta_b, c_{\max})$: using the boundary penetration

- The Sheet moving in the -Y direction comes into the Arc Guide by gravity.
- To verify the contact force of a single sheet segment and a guide, the Revolute Joint and the Rotational Spring constituting the Sheet Group are inactive.
- The sheet segment is constrained with a translational joint at the center of mass so that it can only be moved in the Y-axis direction.
- Create Marker_Sht (1) at the center of the Circular Edge of Sheet Segment and create Marker_Guide (2) at the contact point of Arc Guide for a Contact Expression.
- Sheet Properties are used in the same values as the Linear Guide.



Modeling parameter

Property	Value
Sheet Start Point	0, 0, 0
Sheet Direction	0, 40., 0
Number of Segment	2
Sheet Thickness	2
Sheet Marker Mother Body	SheetBody1
Sheet Marker Position	0, 0, 0
Guide Mother Body	MotherBody
Guide Radius	5.
Guide Angle	180
Guide Center Point	0, -6., 0
Guide Direction to Start Point	5., 0, 0
Guide Marker Mother Body	MotherBody
Guide Marker Position	0, -1., 0
Guide Contact Direction	Up
PV_Stiffness	0.1
PV_BoundaryPenetration	0.5

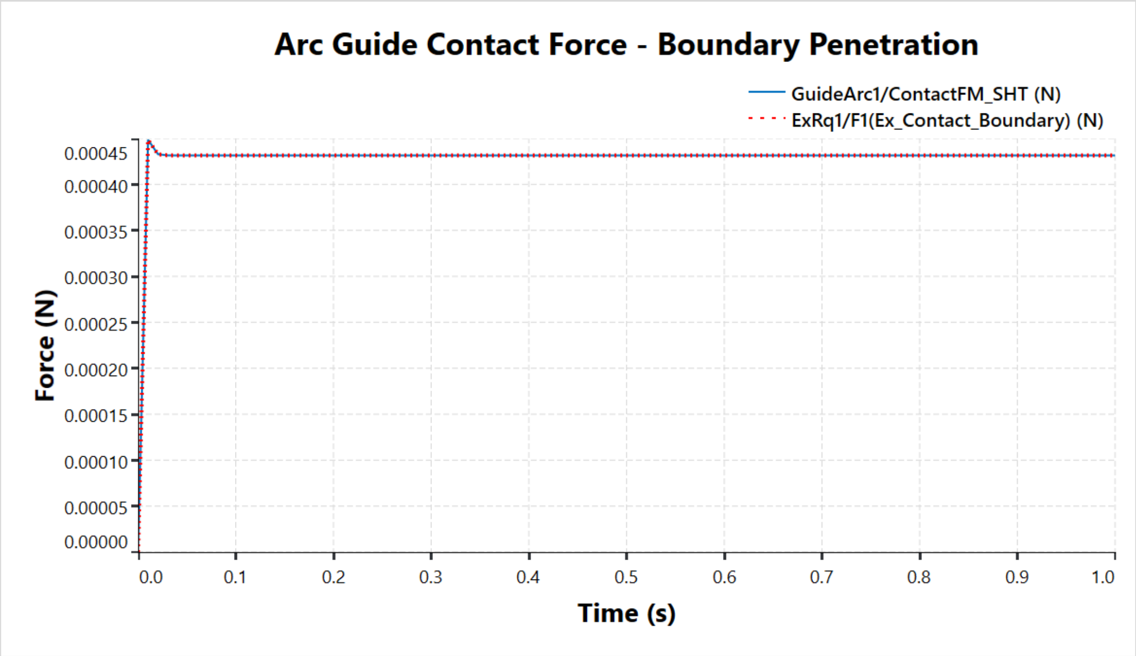
Name	Expression
Ex_Penetration	1-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel	VY(1,2)
Ex_BoundaryPenetration	step(VARVAL(1),0,0,PV_BoundaryPenetration,PV_MaximumDamping)
Ex_Contact_Boundary	-(-PV_Stiffness*VARVAL(1)**PV_Exponent)-(VARVAL(3)*VARVAL(2))
Ex_Contact_Indentation	-(-PV_Stiffness*(VARVAL(1)**PV_Exponent)) -(PV_MaximumDamping*(VARVAL(1)**PV_IndentationExponent)*VARVAL(2))*2

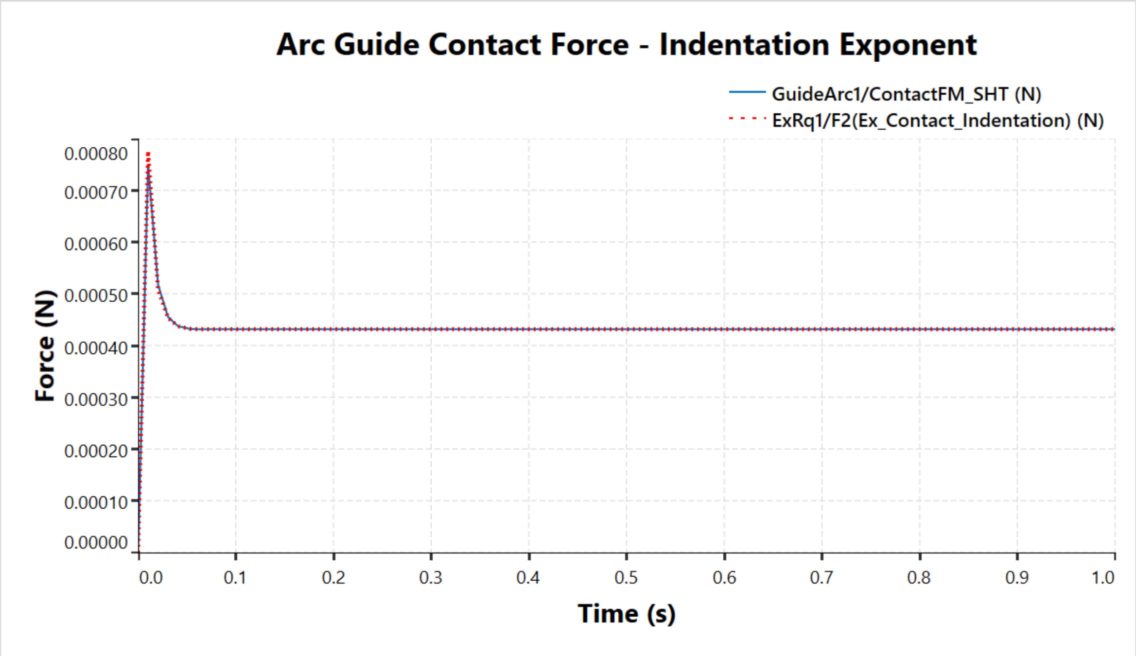
Name	Expression
VE_Penetration (1)	Ex_Penentration
VE_PenetrationVel (2)	Ex_PenetrationVel
VE_BoundaryPenetration (3)	Ex_BoundaryPenetration

➢ Comparison of results

Plot the results

- Arc Guide to Sheet Contact





Object Value	MTT2D	Expression	Error(%)

f_n Max $[N]$ (Boundary Penetration)	4.48e-4	4.48e-4	0
f_n Max $[N]$ (Indentation Exponent)	7.50e-4	7.75e-4	3.4
f_n End $[N]$ (Boundary Penetration)	4.31e-4	4.31e-4	0
f_n End $[N]$ (Indentation Exponent)	4.31e-4	4.31e-4	0

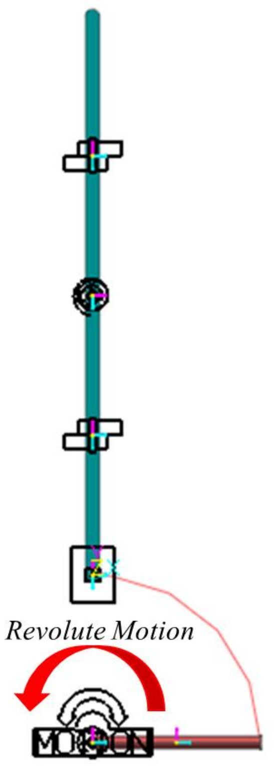
Friction force

- To verify the arc guide friction force, create the dummy cylinder as the mother body of the guide and give rotational motion. Ignore the Guide Velocity ~~of~~ v_g .

$$f_f = \mu(v_t) f_n$$

$$\mu(v_t) = \frac{f_f}{f_n}$$

- With all sheet segments fixed as fixed joints, ~~set~~ create the revolute joint at the first point of the dummy cylinder and impose the motion of the constant angular acceleration.



Modeling parameter

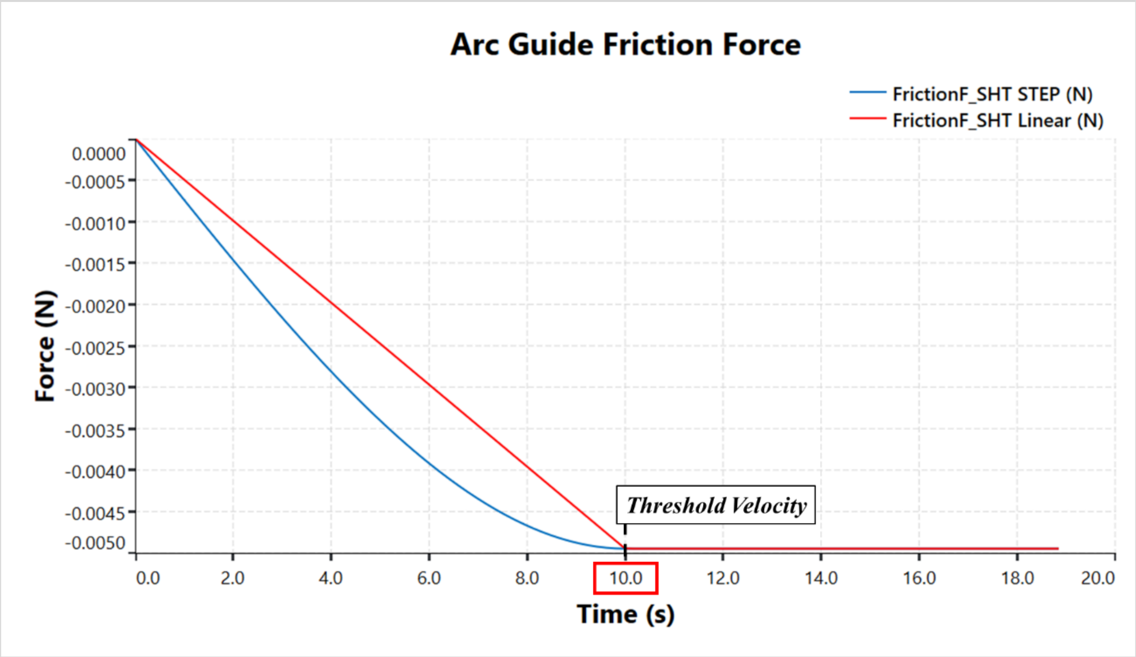
Property	Value
Sheet Start Point	0, 0, 0
Sheet Direction	0, 30., 0
Number of Segment	2
Sheet Thickness	0.5
Cylinder First Point	0, -6., 0
Cylinder Second Point	6., -6., 0
Cylinder Radius	0.25
Guide Mother Body	MotherBody
Guide Radius	6.
Guide Angle	90.
Guide Center Point	0, -6., 0
Guide Direction to Start Point	5., 0, 0
PV_FrictionCoeff	0.3
PV_ThresholdVel	10
PV_GuideVelocity	0

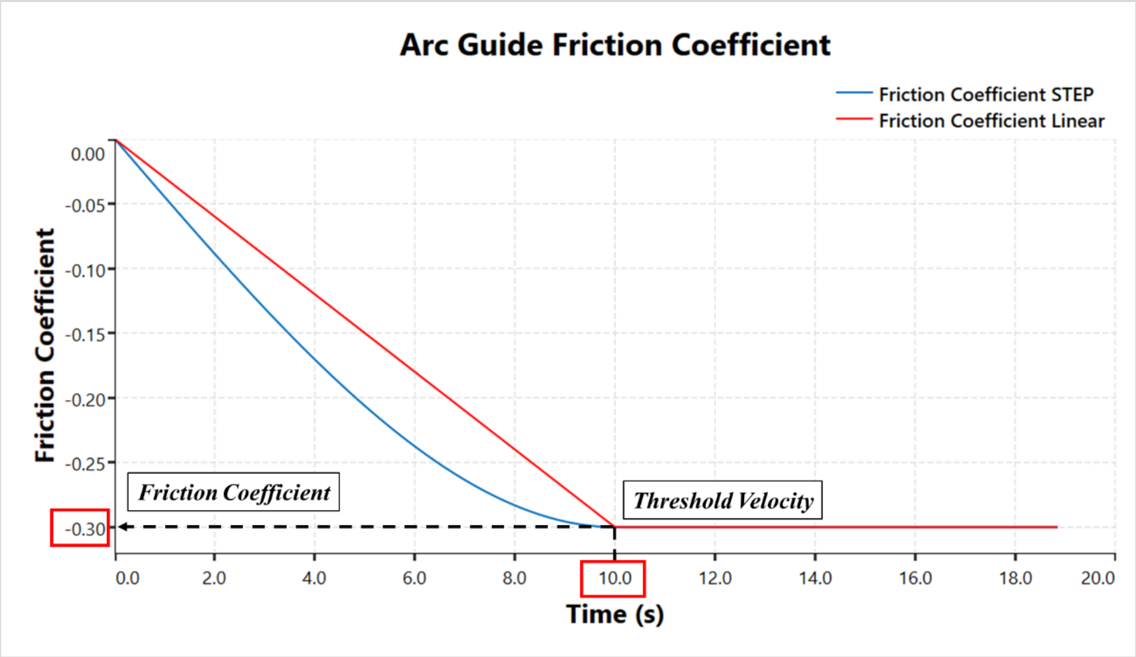
Name	Expression
Ex_ AngularAcc	180D
Ex_Tangential_Velocity	WM(1)*6

➢ Comparison of results

Plot the results

- Arc Guide to Sheet Friction Force and Friction Coefficient





Object Value	STEP	Linear
$\boldsymbol{f_f}$ $[\boldsymbol{N}]$, $\boldsymbol{v_t = 10}$	-4.94e-3	-4.94e-3
$\boldsymbol{\mu}(\boldsymbol{v_t})$, $\boldsymbol{v_t = 10}$	0.3	0.3

PEdge Guide

Contact force

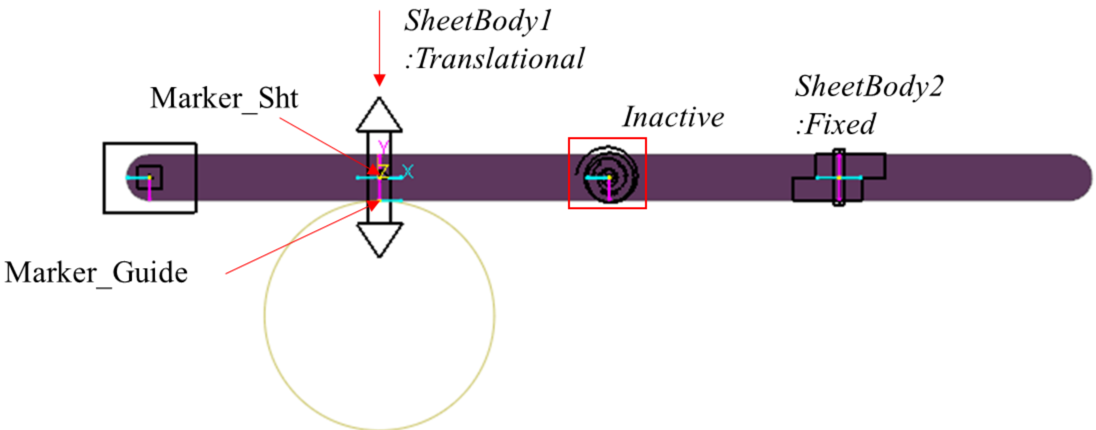
- Verify the results by comparing Expression and RPLT of the Sheet to PEdge Guide contact force defined by the MTT Contact formula.

  $f_n = -k\delta^m - c(\delta)\dot{\delta}^n$: Contact Normal Force

  $c(\delta) = c_{\max}\delta^n$: using the indentation exponent

  $c(\delta) = step(\delta, 0, 0, \delta_b, c_{\max})$: using the boundary penetration

- SheetBody1 is constrained with a Translational Joint at the center of mass so that it can only be moved in the Y-axis direction.
- SheetBody2 is constrained by Fixed Joint, Revolute Joint and Rotational Spring are Inactive.
- Marker_Sht (1) is created at the center of the mass of the sheet, and Marker_Guide (2) is created at the contact point of PEdge Guide for a Contact Expression.



Modeling parameter

Property	Value
Sheet Start Point	-5., 0, 0

Sheet Direction	7., 0, 0
Number of Segment	2
Sheet Thickness	1
Sheet Marker Mother Body	SheetBody1
Sheet Marker Position	0, 0, 0
Guide Mother Body	MotherBody
Guide Radius	5.
Guide Center Point	0, -3., 0
Guide Marker Mother Body	MotherBody
Guide Marker Position	0, -0.5, 0
PV_Stiffness	0.1
PV_BoundaryPenetration	0.5

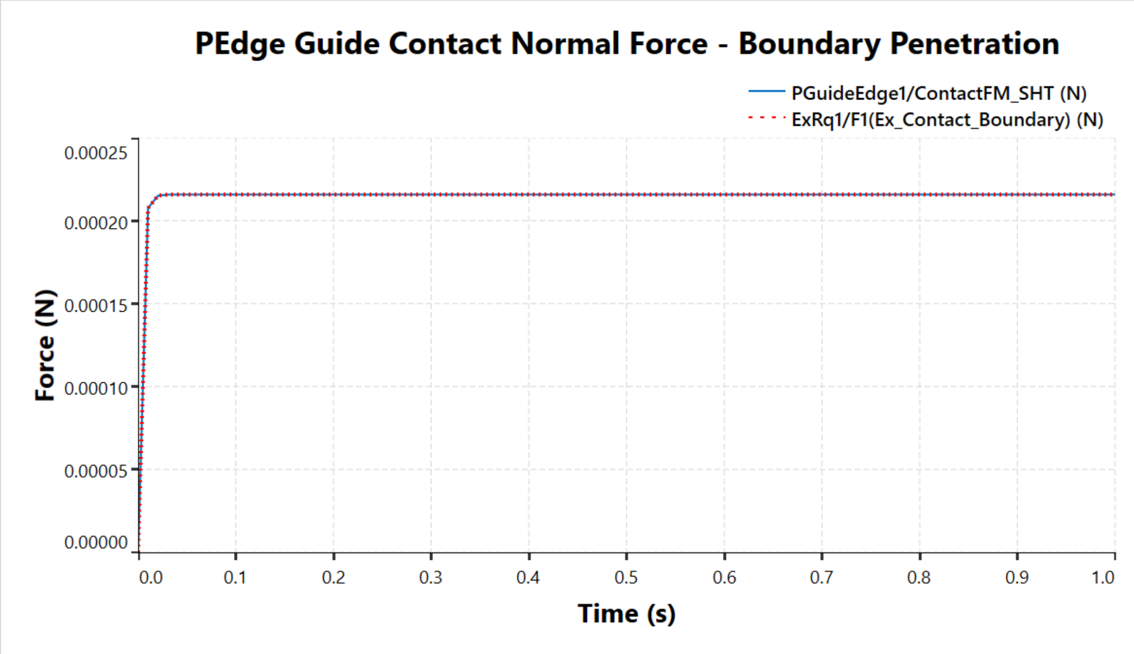
Name	Expression
Ex_Penetration	0.5-sqrt(DX(1,2)**2+DY(1,2)**2)
Ex_PenetrationVel	VY(1,2)
Ex_BoundaryPenetration	step(VARVAL(1),0,0,PV_BoundaryPenetration,PV_MaximumDamping)
Ex_Contact_Boundary	-(-PV_Stiffness*VARVAL(1)**PV_Exponent)-(VARVAL(3)*VARVAL(2))
Ex_Contact_Indentation	-(-PV_Stiffness*(VARVAL(1)**PV_Exponent)) -(PV_MaximumDamping*(VARVAL(1)**PV_IndentationExponent)*VARVAL(2))*2

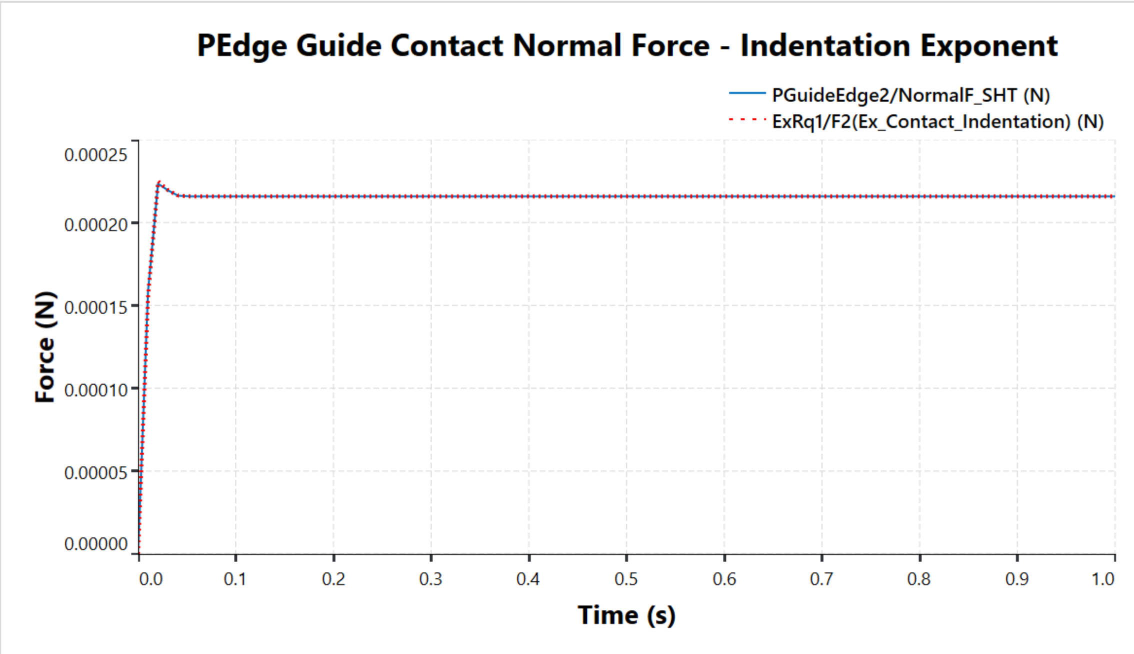
Name	Expression
VE_Penetration (1)	Ex_Penentration
VE_PenetrationVel (2)	Ex_PenetrationVel
VE_BoundaryPenetration (3)	Ex_BoundaryPenetration

➢ Comparison of results

Plot the results

- PEdge Guide to Sheet Contact





Object Value	MTT2D	Expression	Error(%)
f_n Max $[N]$ (Boundary Penetration)	2.16e-4	2.16e-4	0
f_n Max $[N]$ (Indentation Exponent)	2.23e-4	2.25e-4	0.6
f_n End $[N]$ (Boundary Penetration)	2.16e-4	2.16e-4	0

f_n ***End*** $[N]$ **(Indentation Exponent)**	2.15e-4	2.15e-4	0

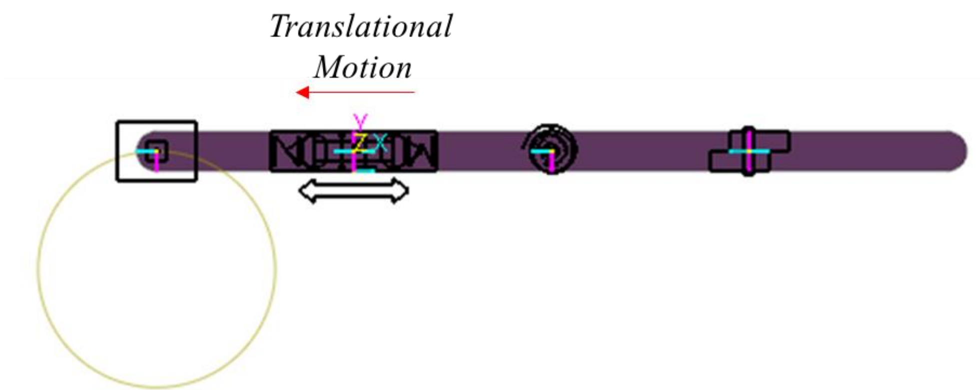
Friction force

- Keep the penetration constant where the contact force has occurred and verify the Friction Force and Friction Coefficient by imposing translational motion with constant acceleration.

$$f_f = \mu(v_t) f_n$$

$$\mu(v_t) = \frac{f_f}{f_n}$$

- Fix SheetBody2 as Fixed Joint and create -X direction Translational Joint to the center of mass of SheetBody1. The Revolute Joint and Rotational Spring is inactive.



Modeling parameter

Property	Value
Sheet Start Point	-5., 0, 0
Sheet Direction	7., 0, 0
Number of Segment	2
Sheet Thickness	1
Guide Radius	3.
Guide Center Point	-5., -3., 0
PV_FrictionCoeff	0.3

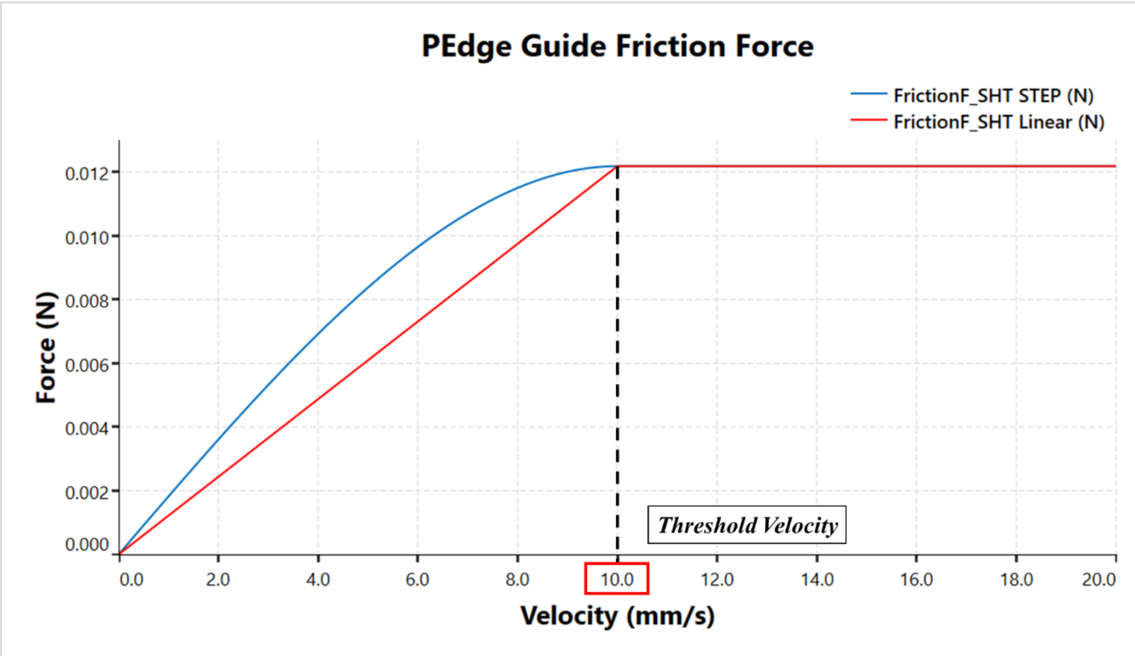
PV_ThresholdVel	10
PV_GuideVelocity	0

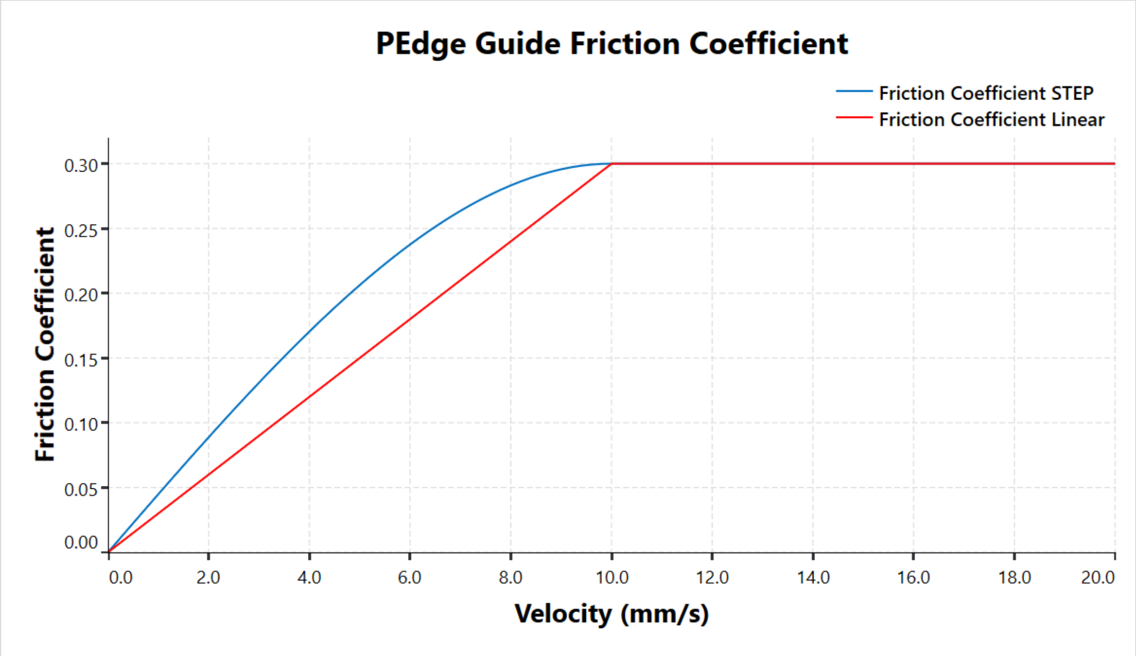
Name	Expression
Ex_ AngularAcc	-20
Ex_Tangential_Velocity	VM(SheetBody1.CM)

➢ Comparison of results

Plot the results

- PEdge Guide to Sheet Friction Force and Friction Coefficient





Object Value	STEP	Linear
$\boldsymbol{f_f}\ [N],\ \boldsymbol{v_t} = \mathbf{10}$	1.21e-2	1.21e-2
$\boldsymbol{\mu}(\boldsymbol{v_t}),\ \boldsymbol{v_t} = \mathbf{10}$	0.3	0.3