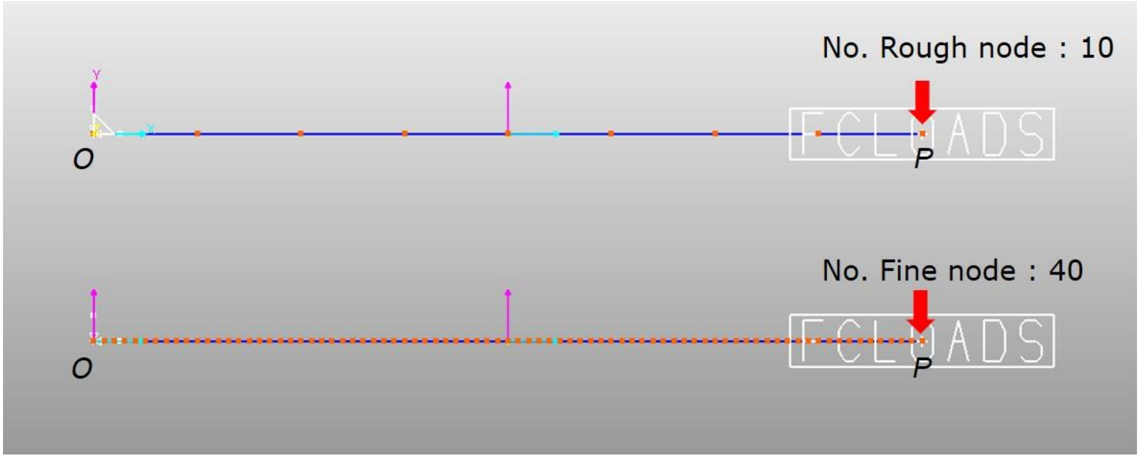
Flexible Model Validation

Bending displacement of cantilevered Beam

(Beam element) Cantilevered Beam

- The beam body is fixed to the ground with BC constraint condition.
- The cross section of the beam is 10[mm] by 10[mm] square and its length is 400 mm.
- The number of nodes when the nodes are rough and fine is 10 and 40, respectively.
- The concentrated load(*P* or *T*) is applied on the flexible body at the point of P without gravity.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	400	mm
Width of Beam	b	10	mm
Height of Beam	h	10	mm
Young's modulus	E	200000	MPa

Shear modulus	G	77821	Mpa
Area Moment of inertia	I	833.33	mm^4
Torsional Constant	J	1408	mm^4/rad
No. Rough and Fine node	-	10, 40	-

Theoretical Solution

Boundary Conditions

- The load of P in the y axis, x axis is applied at the point of P for bending and tension test, respectively.
- The torque of T in the x axis is applied at the point of P for torsion test.

Given	Symbol	Value	Unit
Concentrated Load	P	300	N
Torque	T	500	$N{\cdot}m$

Theoretical Solution

- Bending deformation

$$\delta_y = \frac{pl^3}{3El} = \frac{300 \times 400^3}{3 \times 200000 \times 833.33} = 38.4$$

- Bending deformation

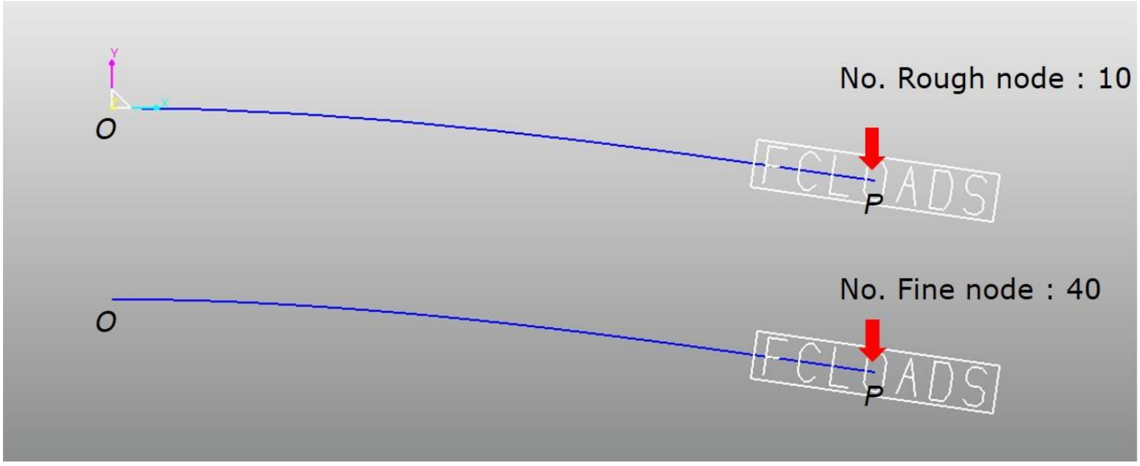
$$\delta_x = \frac{pl}{EA} = \frac{300 \times 400}{200000 \times 100} = 6.0\mathrm{e} - 3$$

- Torsional deformation

$$\theta_x = \frac{Tl}{\mathrm{GJ}} = \frac{500 \times 400}{77827 \times 1408} = 1.82\mathrm{e} - 3$$

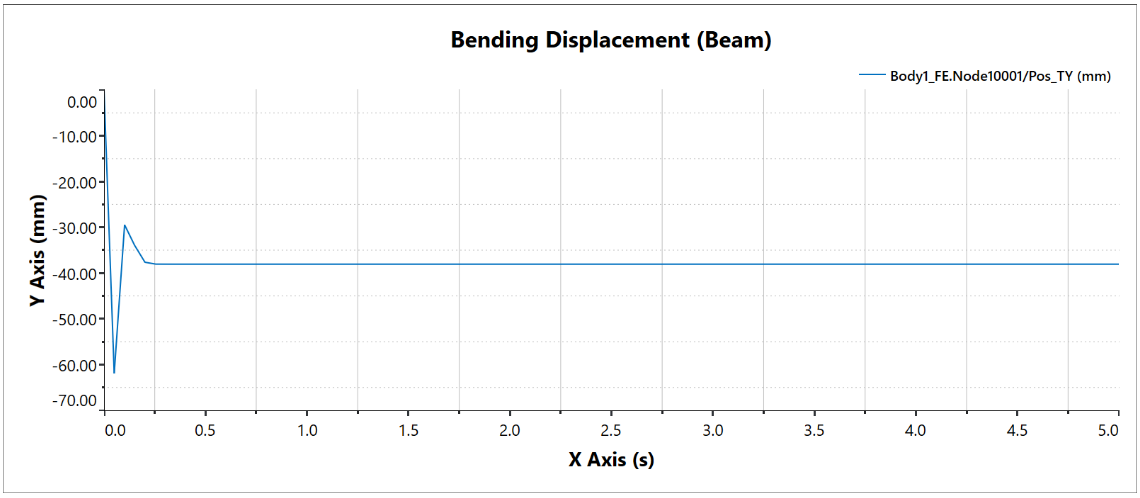
○ Numerical Solution - RecurDyn

Simulation result after bending



Plot the results

- Measure the bending displacement of the end when the analysis result converges.



○ Comparison of results

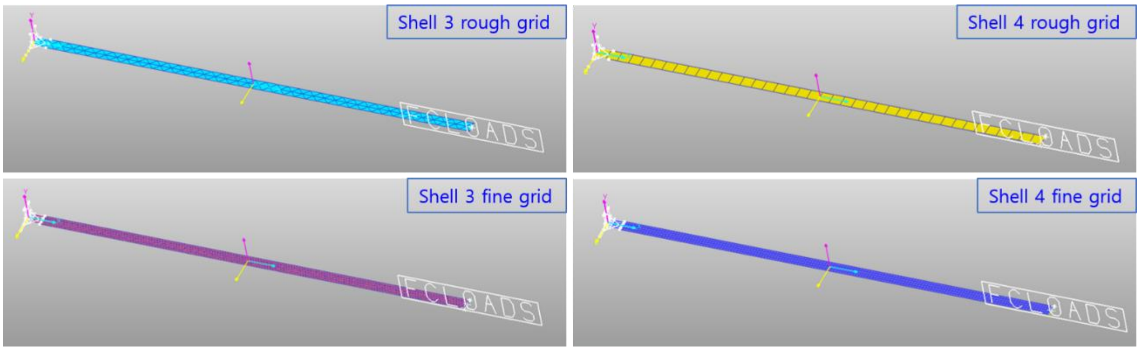
- Beam element (No. Rough and Fine nodes: 10, 40)

Object Value	Theory	RecurDyn	Error(%)
δ_{Y_Rough} $[mm]$	38.4	38.06	0.89

$\boldsymbol{\delta_{Y\_Fine}}$ $[\boldsymbol{mm}]$	38.4	38.06	0.89
$\boldsymbol{\delta_{x\_Rough}}$ $[\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\delta_{x\_Fine}}$ $[\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\theta_{Y\_Rough}}$ $[\boldsymbol{mm}]$	1.82e-3	1.82e-3	0
$\boldsymbol{\theta_{Y\_Fine}}$ $[\boldsymbol{mm}]$	1.82e-3	1.82e-3	0

(Shell 3 and 4 element) Cantilevered Beam

- The flexible sheet is fixed to the ground with BC constraint condition.
- The width of the sheet is determined as 10[mm] with 10[mm] thickness and its length is 400 mm.
- The number of elements and nodes are specified by a rough and fine grid.
- The concentrated load(P) is applied on the flexible body at the point of P without gravity.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	400	mm
Width of Sheet	b	10	mm
Sheet Thickness	h	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	833.33	mm^4
Torsional Constant	J	1408	mm^4/rad
No. shell 3 element	-	162, 1288	-
No. shell 4 element	-	40, 642	-

Theoretical Solution

Boundary Conditions

- The load of P in the y axis, x axis is applied at the point of P for bending and tension test, respectively.
- The torque of T in the x axis is applied at the point of P for torsion test.

Given	Symbol	Value	Unit
Concentrated Load	P	300	N
Torque	T	500	$N{\cdot}m$

Theoretical Solution

- Bending deformation

$$\delta_y = \frac{pl^3}{3El} = \frac{300 \times 400^3}{3 \times 200000 \times 833.33} = 38.4$$

- Bending deformation

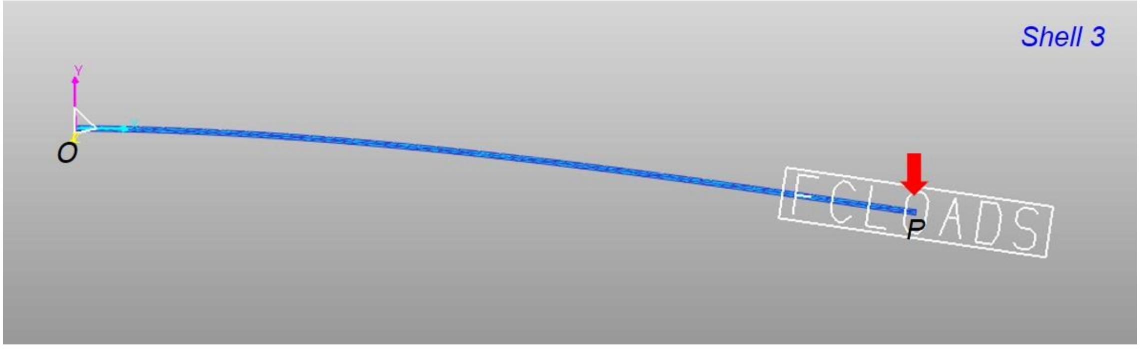
$$\delta_x = \frac{pl}{EA} = \frac{300 \times 400}{200000 \times 100} = 6.0\mathrm{e} - 3$$

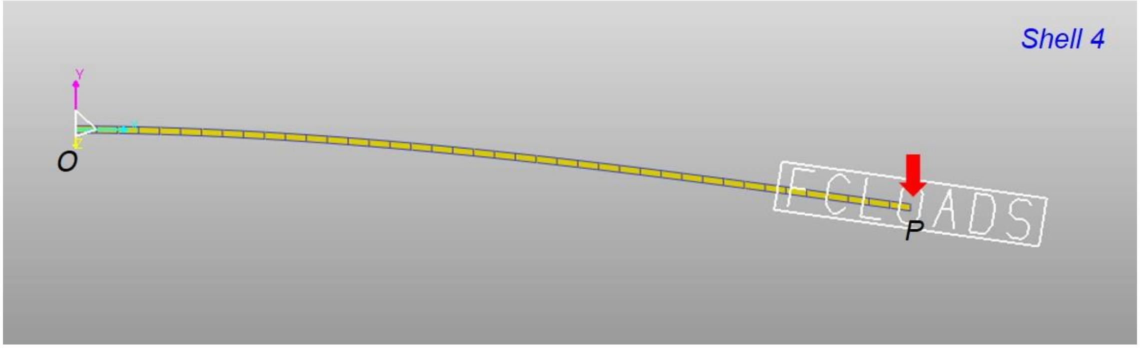
- Torsional deformation

$$\theta_x = \frac{Tl}{\mathrm{GJ}} = \frac{500 \times 400}{77827 \times 1408} = 1.82\mathrm{e} - 3$$

⭘ Numerical Solution - RecurDyn

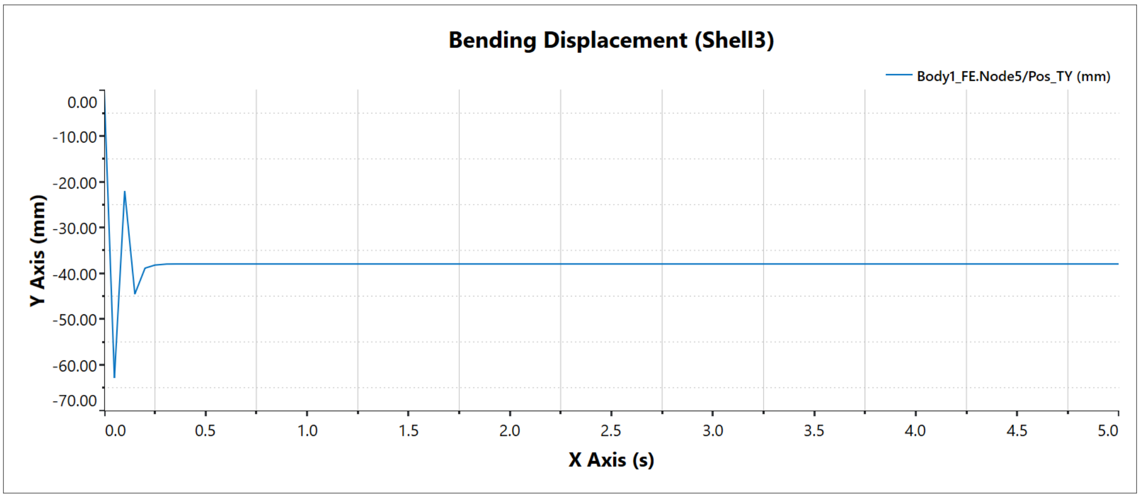
Simulation result after bending

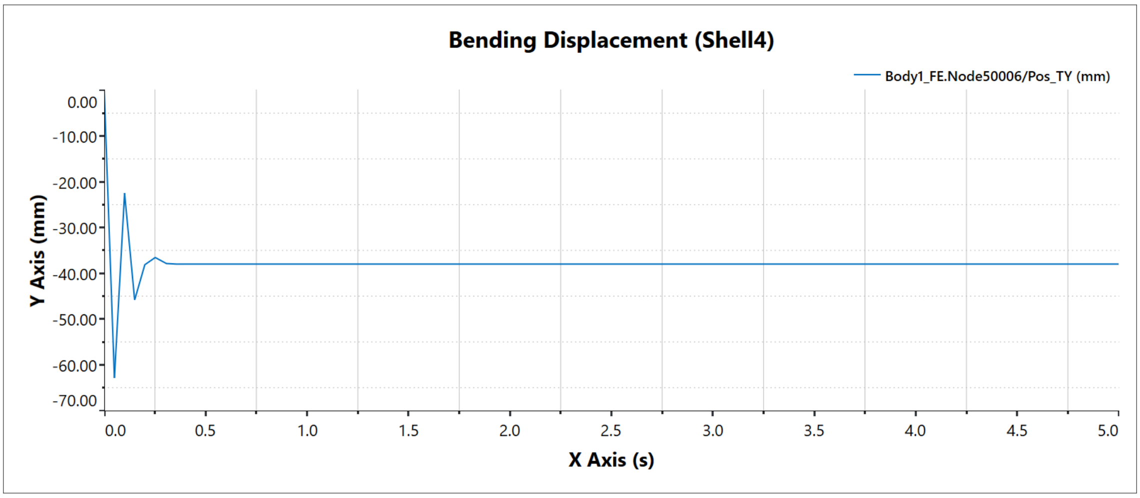




Plot the results

- Measure the bending displacement of the end when the analysis result converges.





Comparison of results

- Shell 3 element (No. Rough and Fine elements: 162, 1288)

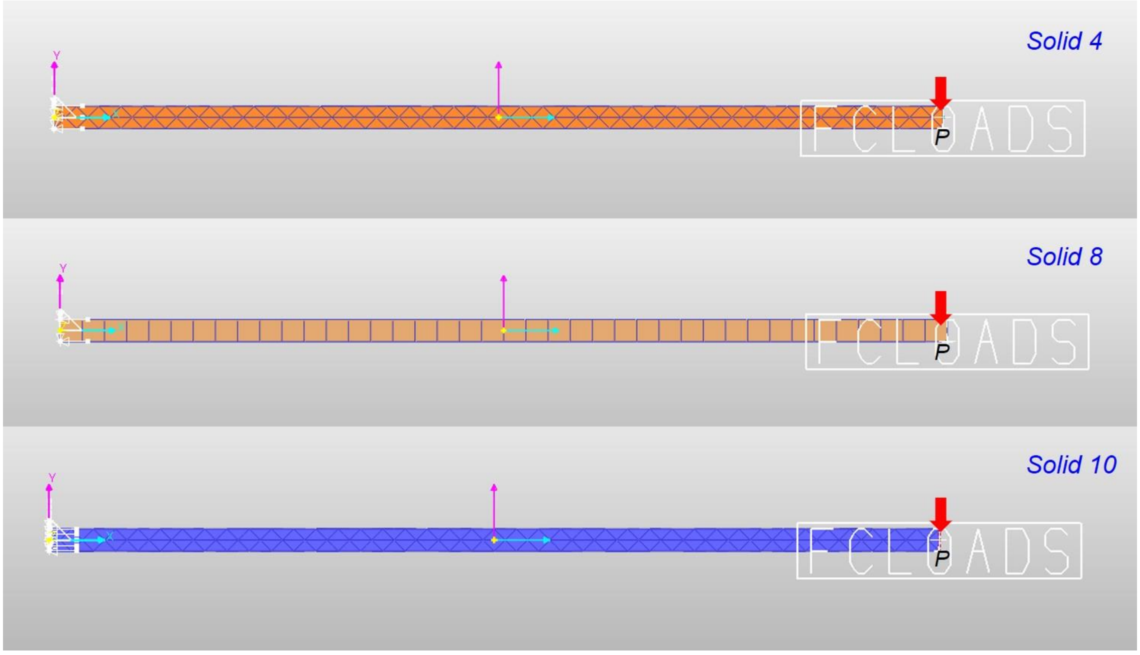
Object Value	Theory	RecurDyn	Error(%)
δ_{Y_Rough} $[mm]$	38.4	37.96	1.14
δ_{Y_Fine} $[mm]$	38.4	38.01	1.01
δ_{x_Rough} $[mm]$	6.0e-3	6.0e-3	0
δ_{x_Fine} $[mm]$	6.0e-3	6.0e-3	0
θ_{Y_Rough} $[mm]$	1.82e-3	1.82e-3	0
θ_{Y_Fine} $[mm]$	1.82e-3	1.82e-3	0

- Shell 4 element (No. Rough and Fine elements: 40, 642)

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{\delta_{Y\_Rough}}$ $[\boldsymbol{mm}]$	38.4	39.98	1.09
$\boldsymbol{\delta_{Y\_Fine}}$ $[\boldsymbol{mm}]$	38.4	38.01	1.01
$\boldsymbol{\delta_{x\_Rough}}$ $[\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\delta_{x\_Fine}}$ $[\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\theta_{Y\_Rough}}$ $[\boldsymbol{mm}]$	1.82e-3	1.82e-3	0
$\boldsymbol{\theta_{Y\_Fine}}$ $[\boldsymbol{mm}]$	1.82e-3	1.82e-3	0

(Solid 4, 8, and 10 element) Cantilevered Beam

- The flexible block body is fixed to the ground with BC constraint condition.
- The width and thickness of the block are determined by 10 [mm] and the length is 400 mm.
- The number of elements and nodes are specified by a rough and fine grid.
- The concentrated load(P) is applied on the flexible body at the point of P without gravity.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	400	mm
Width of Block	b	10	mm
Height of Block	h	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa

Area Moment of inertia	I	833.33	mm^4
Torsional Constant	J	1408	mm^4/rad
No. solid 4 element	-	730, 17077	-
No. solid 8 element	-	40, 2561	-
No. solid 10 element	-	730, 17075	-

Theoretical Solution

Boundary Conditions

- The load of P in the y axis, x axis is applied at the point of P for bending and tension test, respectively.
- The torque of T in the x axis is applied at the point of P for torsion test.

Given	Symbol	Value	Unit
Concentrated Load	P	300	N
Torque	T	500	$N{\cdot}m$

Theoretical Solution

- Bending deformation

$$\delta_y = \frac{pl^3}{3El} = \frac{300 \times 400^3}{3 \times 200000 \times 833.33} = 38.4$$

- Bending deformation

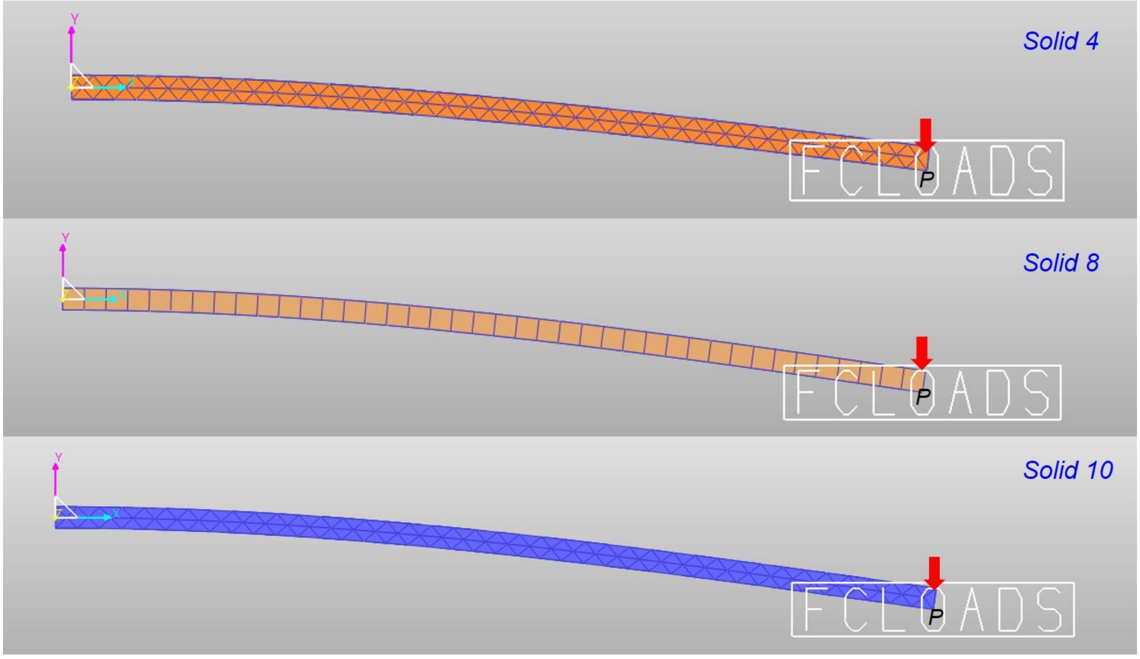
$$\delta_x = \frac{pl}{EA} = \frac{300 \times 400}{200000 \times 100} = 6.0\mathrm{e} - 3$$

- Torsional deformation

$$\theta_x = \frac{Tl}{\mathrm{GJ}} = \frac{500 \times 400}{77827 \times 1408} = 1.82\mathrm{e} - 3$$

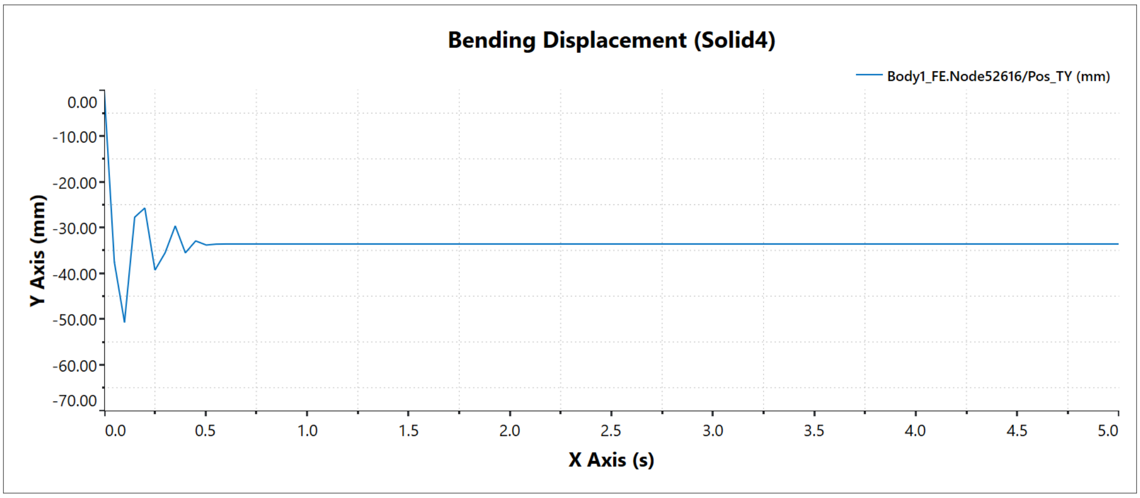
● Numerical Solution - RecurDyn

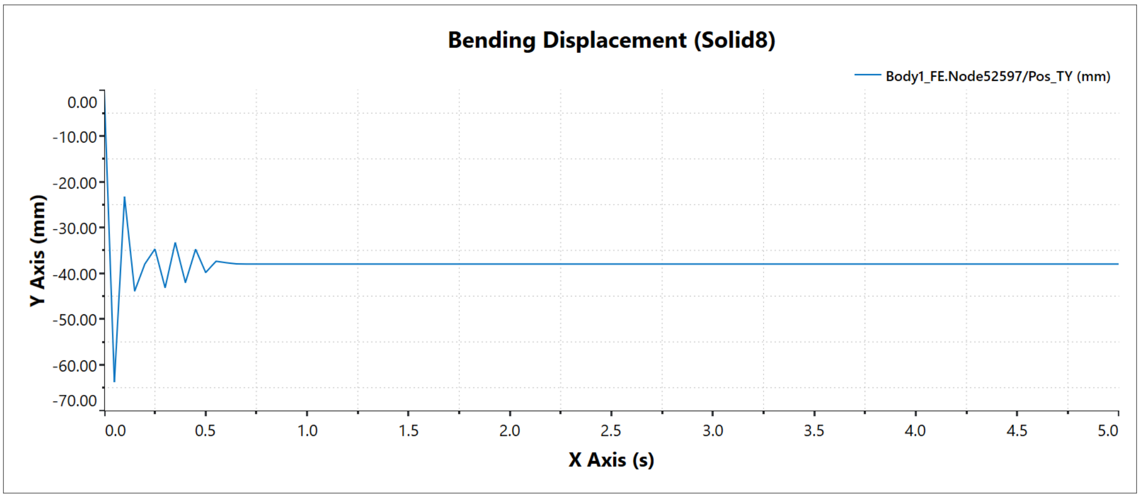
Simulation result after bending

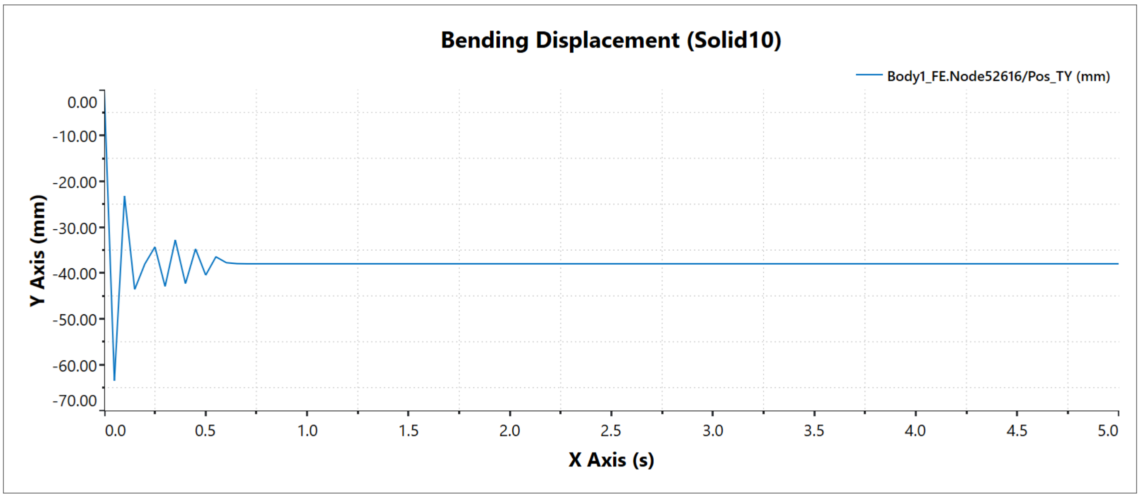


Plot the results

- Measure the bending displacement of the end when the analysis result converges.







⭘ Comparison of results

- Solid 4 element (No. Rough and Fine elements: 730, 17077)

Object Value	Theory	RecurDyn	Error(%)
δ_{Y_Rough} $[mm]$	38.4	38.6	0.52
δ_{Y_Fine} $[mm]$	38.4	38.6	0.52
δ_{x_Rough} $[mm]$	6.0e-3	6.0e-3	0
δ_{x_Fine} $[mm]$	6.0e-3	6.0e-3	0
θ_{Y_Rough} $[mm]$	1.82e-3	1.82e-3	0
θ_{Y_Fine} $[mm]$	1.82e-3	1.82e-3	0

- Solid 8 element (No. Rough and Fine elements: 40, 2561)

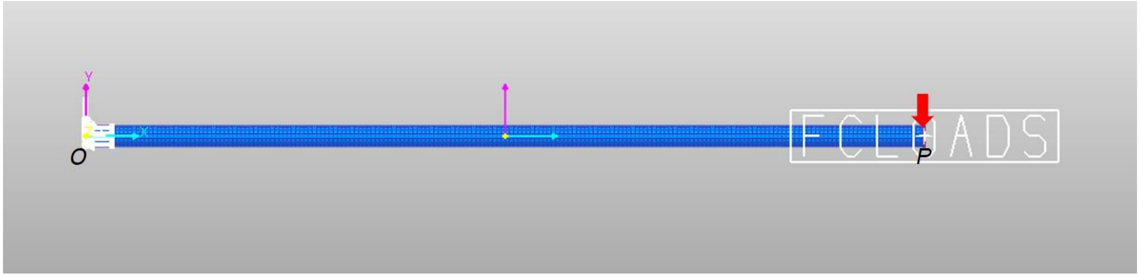
Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{\delta_{Y\_Rough}}\ [\boldsymbol{mm}]$	38.4	37.98	1.09
$\boldsymbol{\delta_{Y\_Fine}}\ [\boldsymbol{mm}]$	38.4	38.036	0.947
$\boldsymbol{\delta_{x\_Rough}}\ [\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\delta_{x\_Fine}}\ [\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\theta_{Y\_Rough}}\ [\boldsymbol{mm}]$	1.82e-3	1.82e-3	0
$\boldsymbol{\theta_{Y\_Fine}}\ [\boldsymbol{mm}]$	1.82e-3	1.82e-3	0

- Solid 10 element (No. Rough and Fine elements: 730, 17075)

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{\delta_{Y\_Rough}}\ [\boldsymbol{mm}]$	38.4	37.91	1.28
$\boldsymbol{\delta_{Y\_Fine}}\ [\boldsymbol{mm}]$	38.4	37.99	1.07
$\boldsymbol{\delta_{x\_Rough}}\ [\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\delta_{x\_Fine}}\ [\boldsymbol{mm}]$	6.0e-3	6.0e-3	0
$\boldsymbol{\theta_{Y\_Rough}}\ [\boldsymbol{mm}]$	1.82e-3	1.82e-3	0
$\boldsymbol{\theta_{Y\_Fine}}\ [\boldsymbol{mm}]$	1.82e-3	1.82e-3	0

(Solid 4, 5, 6 element) Cantilevered Beam

- The cylinder shape of flexible body is fixed to the ground with BC constraint condition.
- The radius is determined by 5 [mm] and the length is 400 mm.
- The number of elements and nodes are specified by a rough and fine grid.
- The concentrated load(P) is applied on the flexible body at the point of P without gravity.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	400	mm
Width of Block	b	10	mm
Height of Block	h	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	833.33	mm^4
Torsional Constant	J	1408	mm^4/rad
No. solid 4,5,6 element	-	3905, 37123	-

⭘ Theoretical Solution

Boundary Conditions

- The load of P in the y axis, x axis is applied at the point of P for bending and tension test, respectively.
- The torque of T in the x axis is applied at the point of P for torsion test.

Given	Symbol	Value	Unit
Concentrated Load	P	300	N
Torque	T	500	$N{\cdot}m$

Theoretical Solution

- Bending deformation

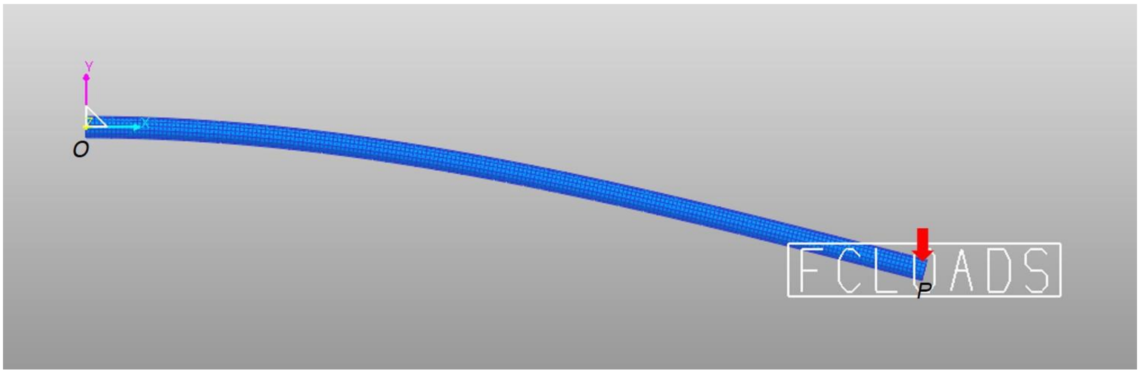
$$\delta_y = \frac{pl^3}{3El} = \frac{300 \times 400^3}{3 \times 200000 \times 490.87} = 65.19$$

- Bending deformation

$$\delta_x = \frac{pl}{EA} = \frac{300 \times 400}{200000 \times 78.539} = 7.64e-3$$

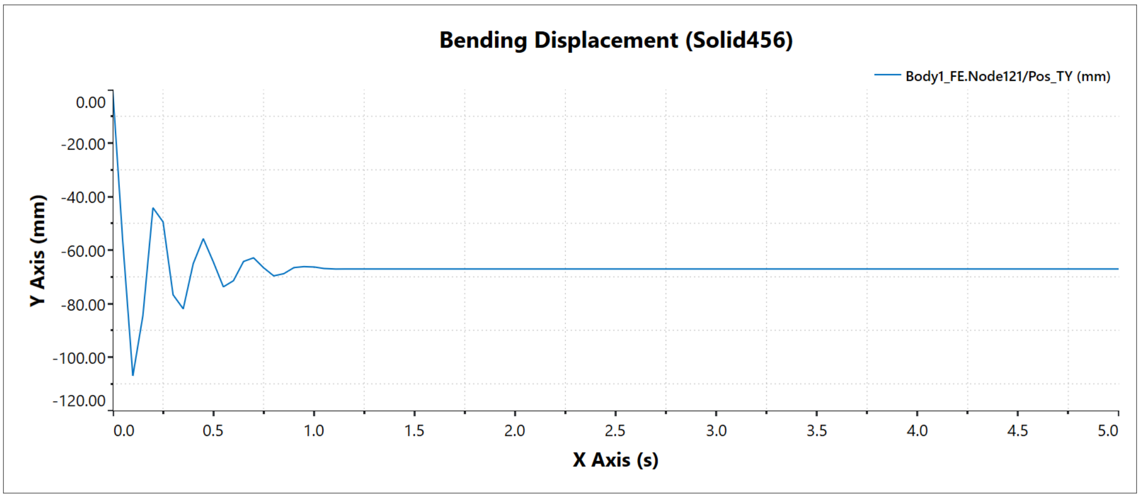
Numerical Solution - RecurDyn

Simulation result after bending



Plot the results

- Measure the bending displacement Y of the end when the analysis result converges.



○ Comparison of results

- Solid 4,5,6 element (No. Rough and Fine elements : 3905, 37123)

Object Value	Theory	RecurDyn	Error(%)
δ_{Y_Rough} $[mm]$	65.19	67.21	2.88
δ_{Y_Fine} $[mm]$	65.19	64.25	1.44
δ_{x_Rough} $[mm]$	7.64e-3	7.83e-3	2.48
δ_{x_Fine} $[mm]$	7.64e-3	7.67e-3	0.39

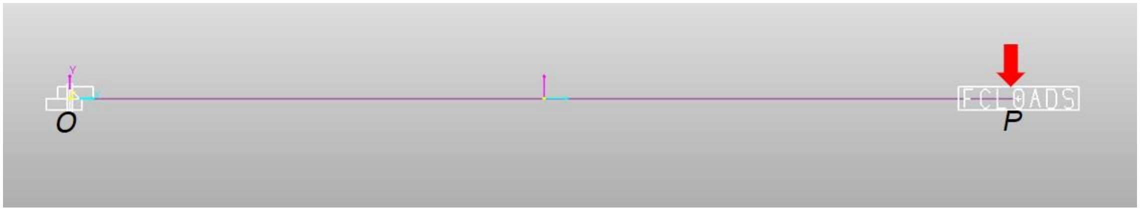
Natural Frequency (f_n)

The natural frequency, also known as eigen frequency, is the frequency of the oscillation, measured in hertz (Hz). All objects have a natural frequency or set of frequencies at which they vibrate. The frequency is dependent upon the properties of the material the object is made of (this affects the speed of the wave) and the length of the material (this affects the wavelength).

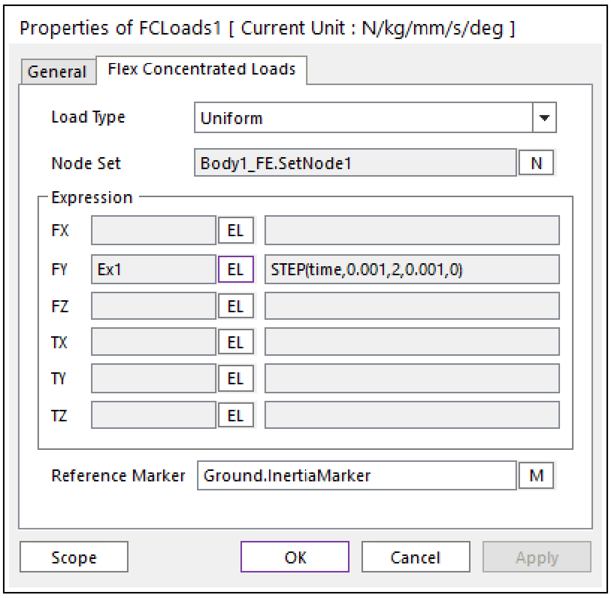
In this chapter, the natural frequency of flexible body derived from the FFT analysis of the body motion is compared with a calculated value based on the theory.

(Beam element) Cantilevered Beam

- The sheet body is connected to the ground with fixed joint at the point of O.
- The cross section of the beam is 1[mm] by 1[mm] square and its length is 1000 mm.
- The number of elements are 50 and 500 for rough and fine grids, respectively.



- The load of P in the y axis is applied at the point of P during a very short initial period.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	1000	mm
Width of Beam	b	1	mm
Height of Beam	h	1	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	8.333e-2	mm^4
Poisson's Ratio	nu	0.285	-
Density	ρ	7.85e-6	kg/mm^3

⭘ Theoretical Solution

- First critical frequency (k = 1.875)

$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e} - 6 \times 1000^4}} = 0.82$$

- 2nd critical frequency (k = 4.694)

$$f_{n2} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{4.694^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e} - 6 \times 1000^4}} = 5.11$$

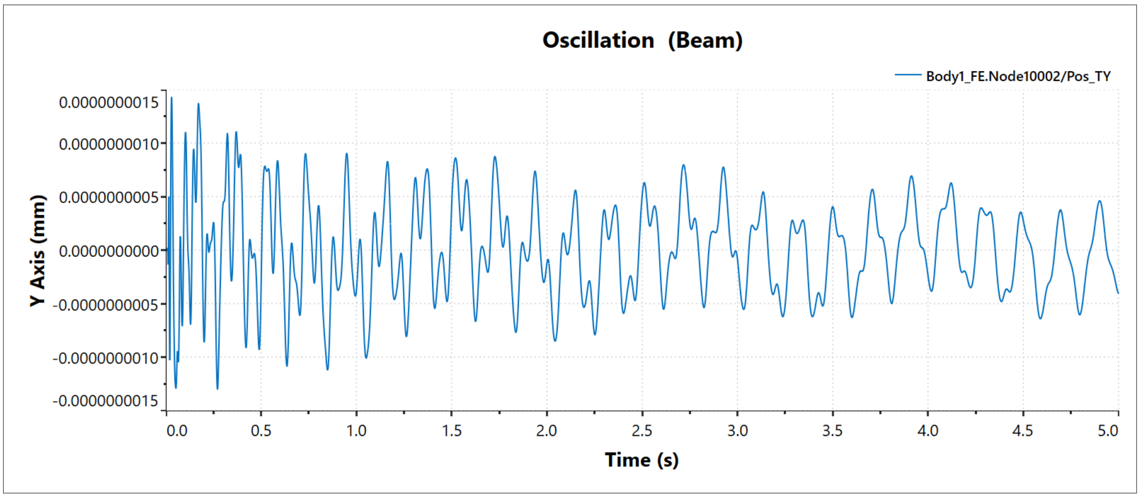
- 3rd critical frequency (k = 7.855)

$$f_{n3} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{7.855^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e} - 6 \times 1000^4}} = 14.31$$

Numerical Solution - RecurDyn

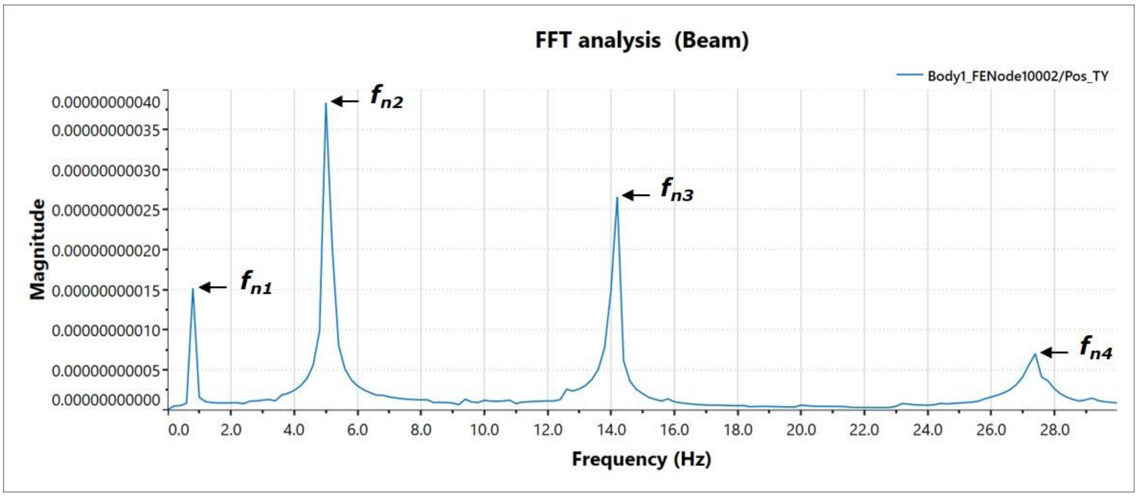
Plot the oscillation of P

- Multiple frequencies are superimposed to show complex graphs.



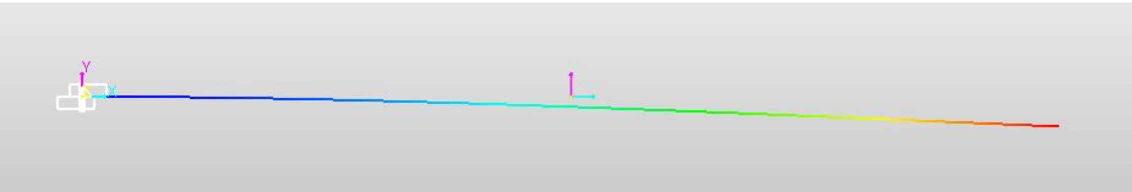
FFT Analysis

- Various frequencies can be visualized through FFT analysis.



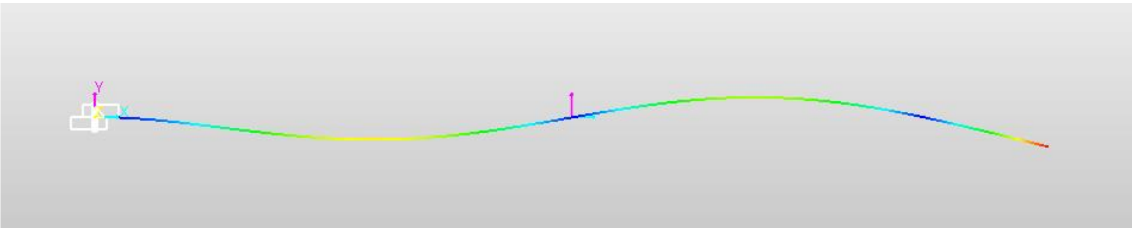
Mode frequency of RFlexGen

- First mode frequency = 0.82



- Second mode frequency = 5.11

- Third mode frequency = 14.31



Comparison of results

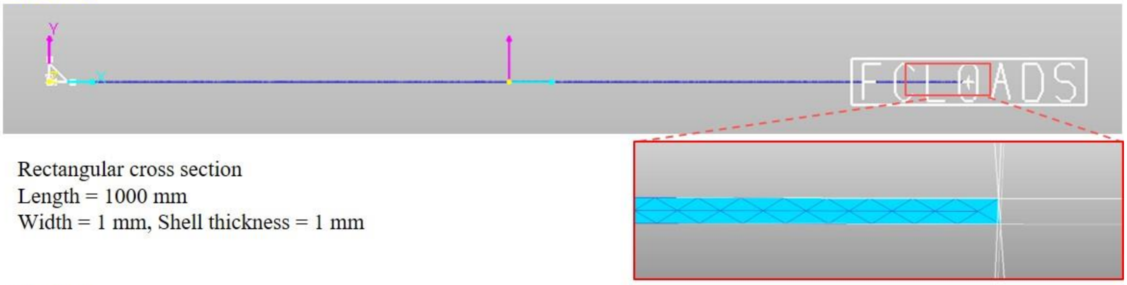
- Comparison with theoretical result and RFlexGen

Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1_Rough} $[Hz]$	0.82	0.82	0.8	2.5	2.5
f_{n1_Fine} $[Hz]$	0.82	0.82	0.8	2.5	2.5
f_{n2_Rough} $[Hz]$	5.11	5.11	4.99	2.4	2.4
f_{n2_Fine} $[Hz]$	5.11	5.11	4.99	2.4	2.4
f_{n3_Rough} $[Hz]$	14.31	14.29	14.2	0.77	0.63
f_{n3_Fine} $[Hz]$	14.31	14.31	14.2	0.77	0.77

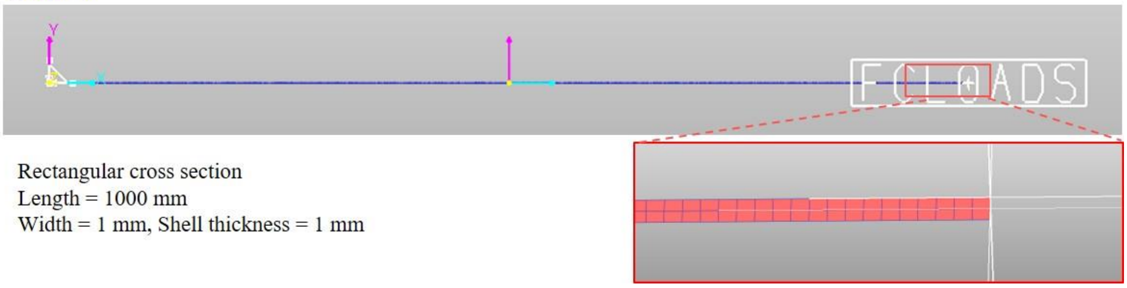
(Shell 3 & 4) Cantilevered Beam

- The flexible sheet is connected to the ground with BC constraint condition at the Ground.
- The width of the sheet is determined as 1[mm] with 1[mm] thickness and its length is 1000 mm.
- The number of elements and nodes are specified by a rough and fine grid.

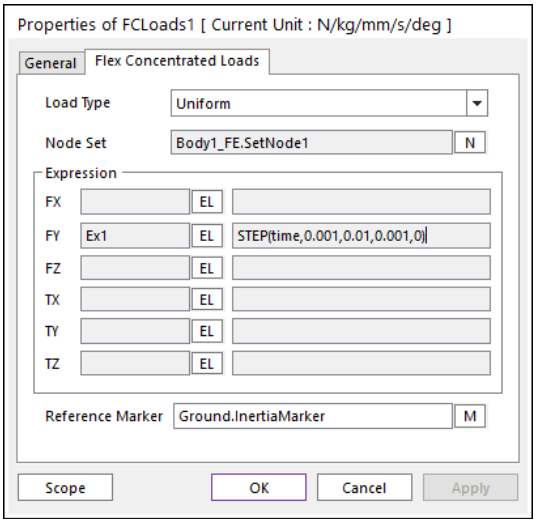
Shell 3



Shell 4



- The load of P in the y axis is applied at the point of P during a very short initial period.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	1000	mm
Width of Sheet	b	1	mm
Sheet Thickness	h	1	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	8.333e-2	mm^4
Poisson's Ratio	nu	0.285	-
Density	ρ	7.85e-6	kg/mm^3

Theoretical Solution

- First critical frequency (k = 1.875)

$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e}-6 \times 1000^4}} = 0.82$$

- 2nd critical frequency (k = 4.694)

$$f_{n2} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{4.694^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e}-6 \times 1000^4}} = 5.11$$

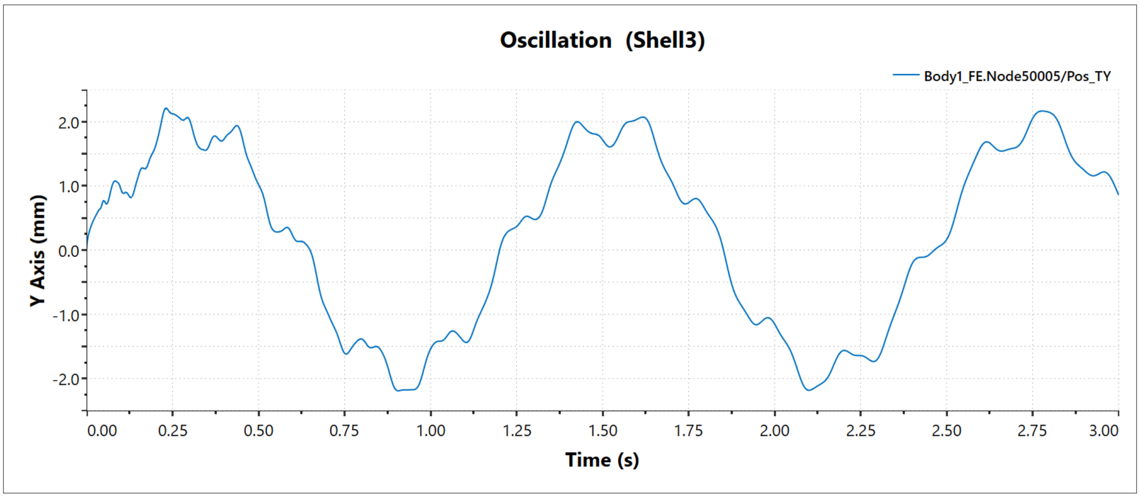
- 3rd critical frequency (k = 7.855)

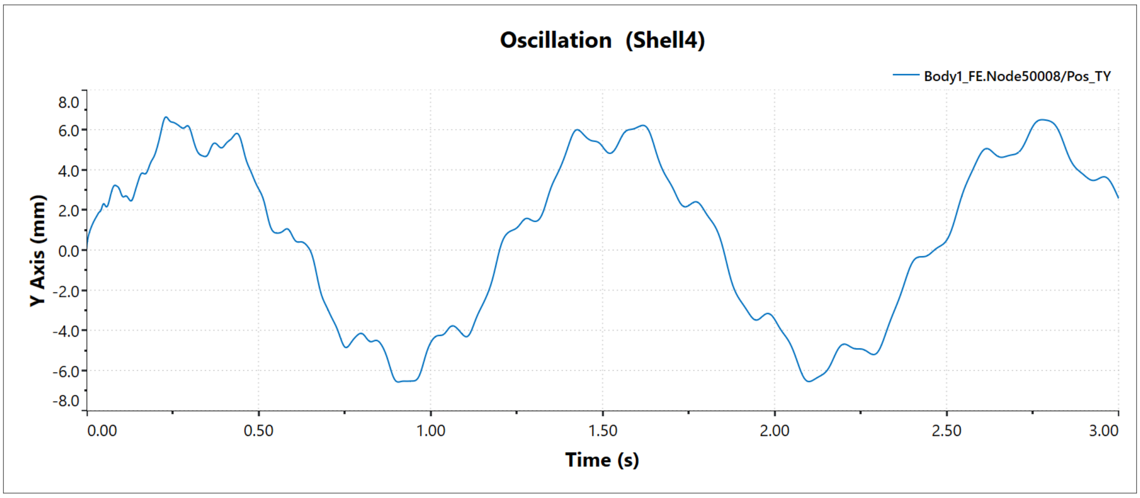
$$f_{n3} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{7.855^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e}-6 \times 1000^4}} = 14.31$$

Numerical Solution - RecurDyn

Plot the oscillation of P

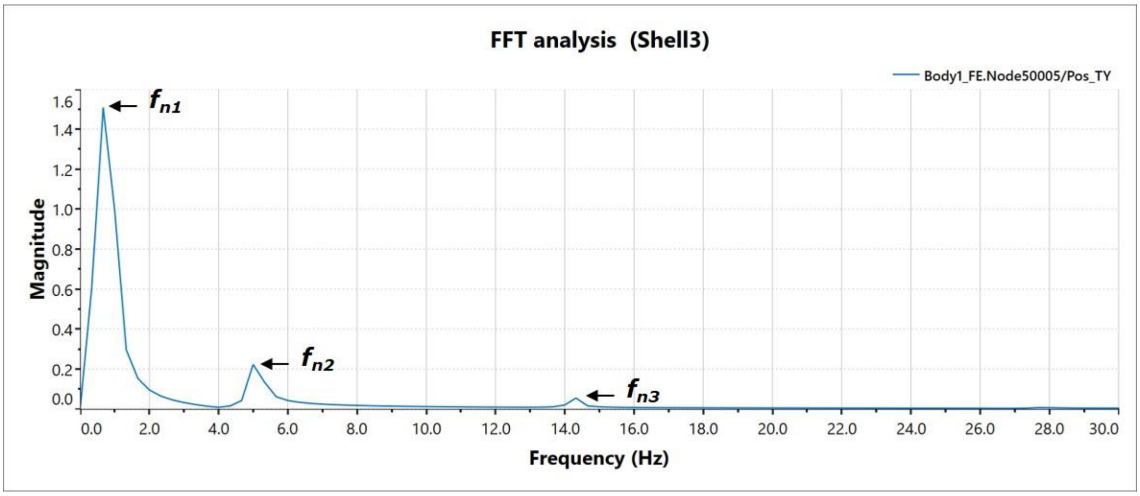
- Multiple frequencies are superimposed to show complex graphs.

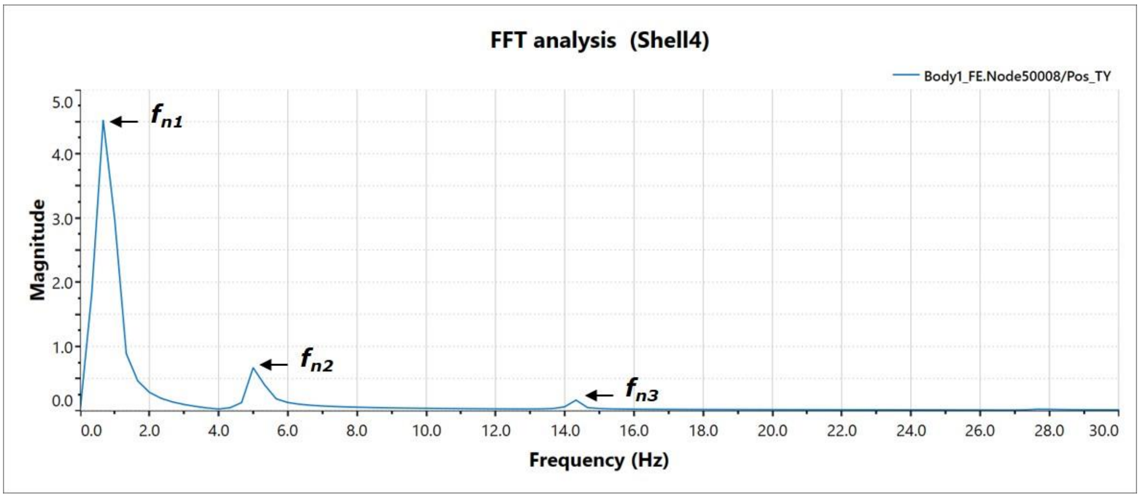




FFT Analysis

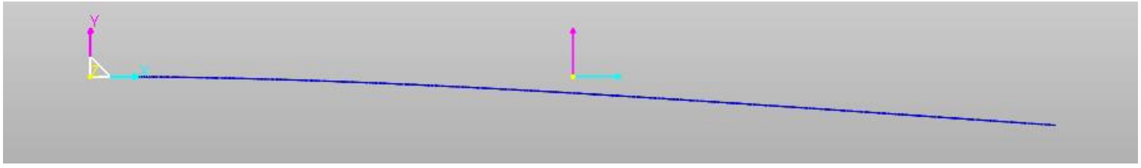
- Various frequencies can be visualized through FFT analysis.



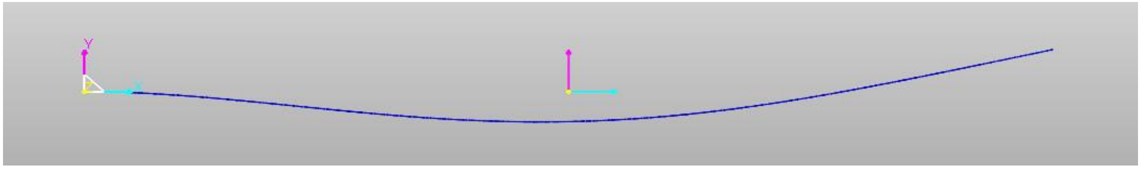


Mode frequency of RFlexGen of Shell 3 element

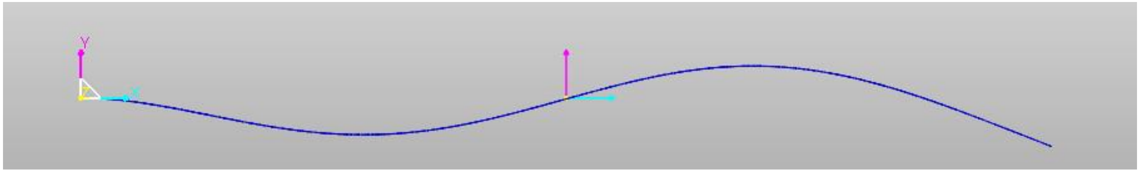
- First mode frequency = 0.82



- Second mode frequency = 5.11



➢ Third mode frequency = 14.31



Comparison of results

- Shell 3 element (No. Rough and Fine elements: 4000, 32000)

Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1_Rough} $[Hz]$	0.82	0.82	0.7	17	17
f_{n1_Fine} $[Hz]$	0.82	0.82	0.7	17	17
f_{n2_Rough} $[Hz]$	5.11	5.11	4.99	2.4	2.4
f_{n2_Fine} $[Hz]$	5.11	5.11	4.99	2.4	2.4
f_{n3_Rough} $[Hz]$	14.31	14.29	14.32	0.07	0.2
f_{n3_Fine} $[Hz]$	14.31	14.31	14.32	0.07	0.07

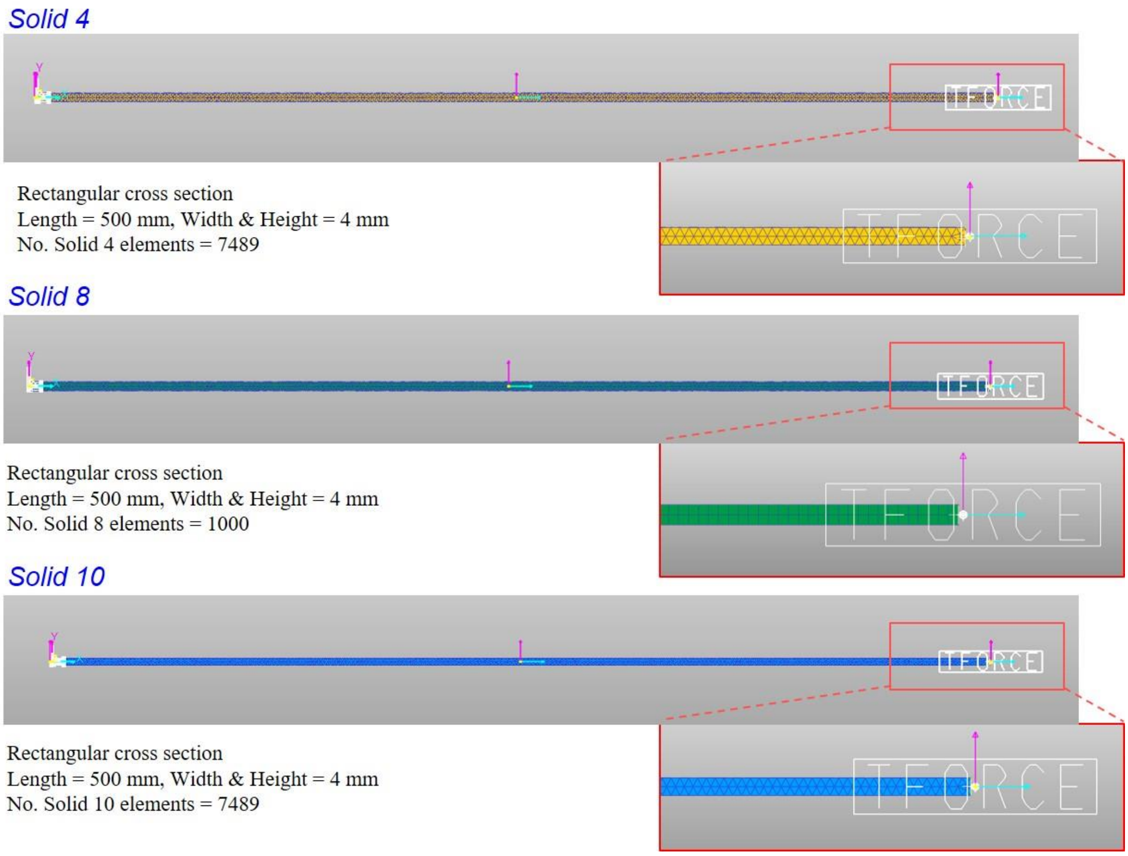
- Shell 4 element (No. Rough and Fine elements: 4000, 16000)

Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1_Rough} $[Hz]$	0.82	0.82	0.7	17	17
f_{n1_Fine} $[Hz]$	0.82	0.82	0.7	17	17
f_{n2_Rough} $[Hz]$	5.11	5.11	4.99	2.4	2.4
f_{n2_Fine} $[Hz]$	5.11	5.11	4.99	2.4	2.4

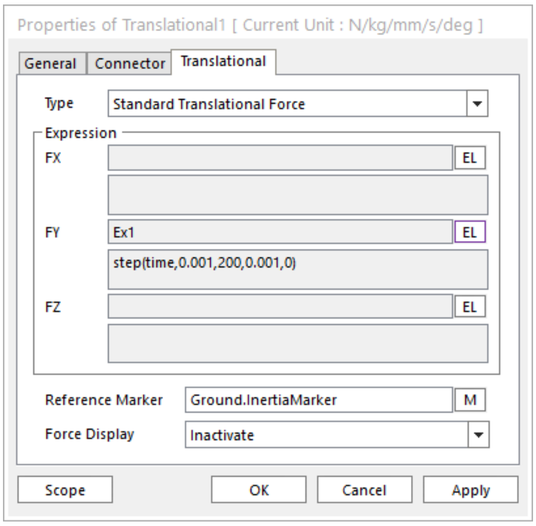
f_{n3_Rough} $[Hz]$	14.31	14.29	14.32	0.07	0.2
f_{n3_Fine} $[Hz]$	14.31	14.31	14.32	0.07	0.07

(Solid 4, 8 and 10) Cantilevered Beam

- The flexible block body is connected to the ground with BC constraint condition at the Ground.
- The width and thickness of the block are determined by 4[mm] and the length is 500 mm.
- The number of elements and nodes are specified by a rough and fine grid.
- The concentrated load(*P*) is applied on the flexible body at the point of P without gravity.



- The translation force in the y axis is applied at the end point of beam during a very short initial period.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	500	mm
Width of Sheet	b	4	mm
Sheet Thickness	h	4	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	21.333	mm^4
Poisson's Ratio	nu	0.285	-
Density	ρ	7.85e-6	kg/mm^3

Theoretical Solution

- First critical frequency (k = 1.875)

$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 21.333}{1 \times 7.85e - 6 \times 1000^4}} = 13$$

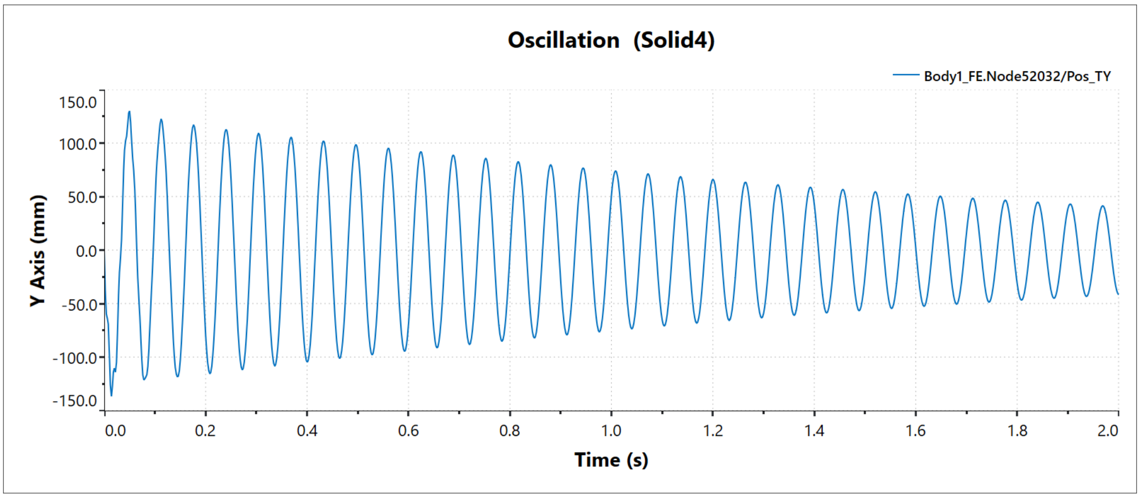
- 2nd critical frequency (k = 4.694)

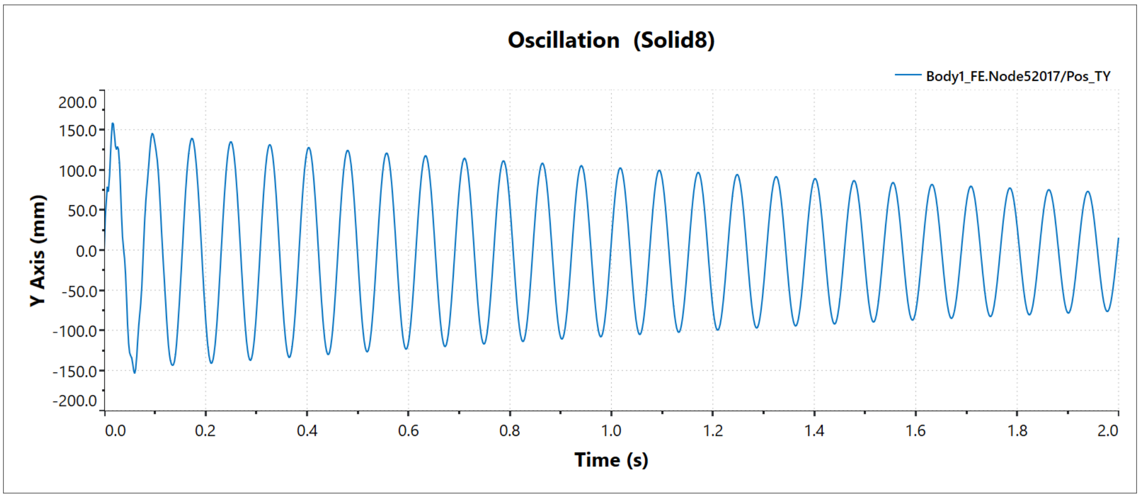
$$f_{n2} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{4.694^2}{2\pi}\sqrt{\frac{200000000 \times 21.333}{1 \times 7.85e-6 \times 1000^4}} = 81.7$$

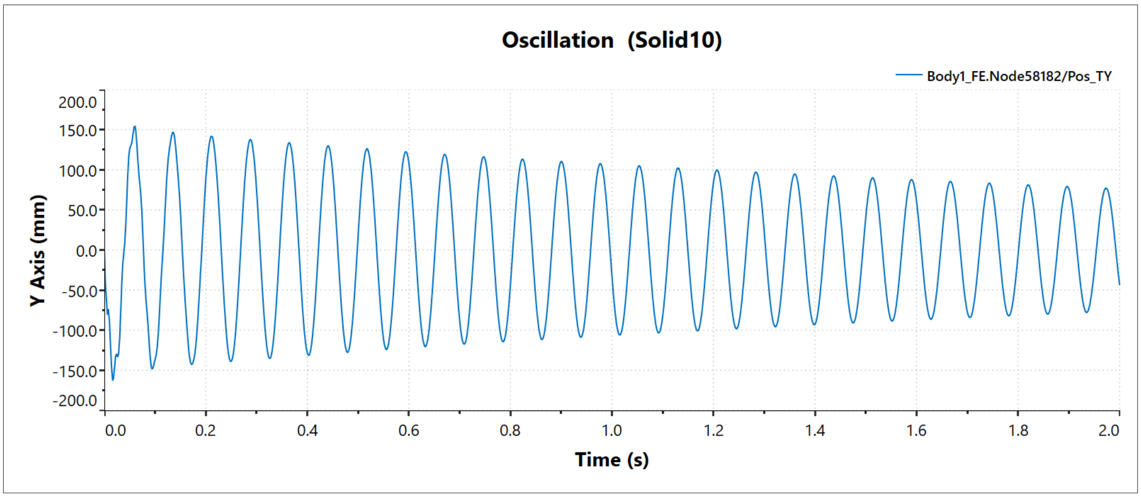
Numerical Solution - RecurDyn

Plot the oscillation of P

- Multiple frequencies are superimposed to show complex graphs.

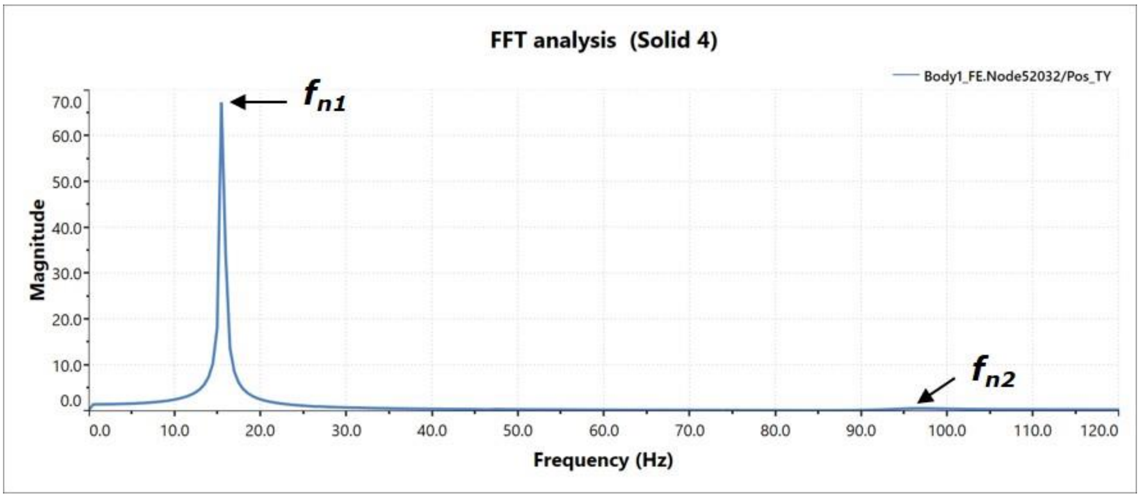


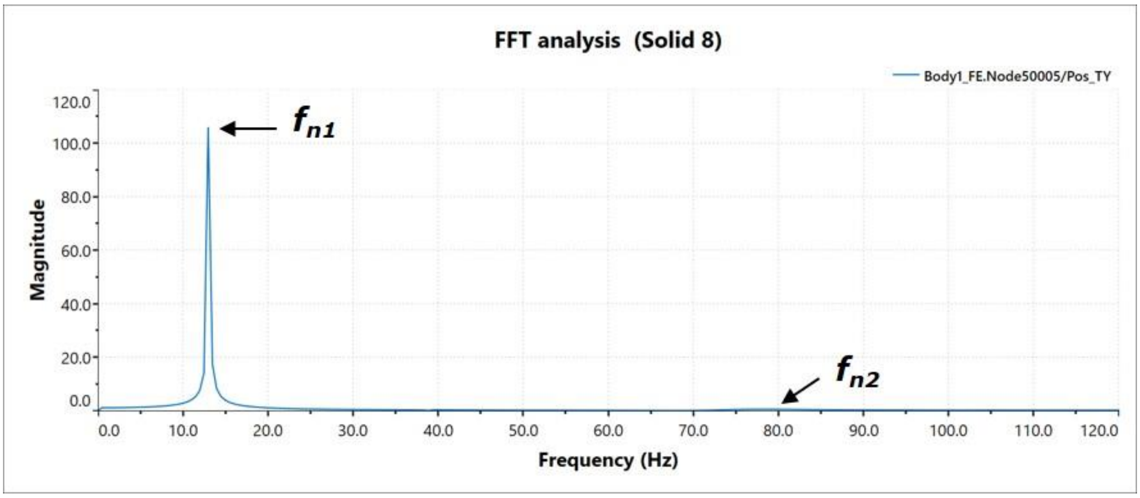


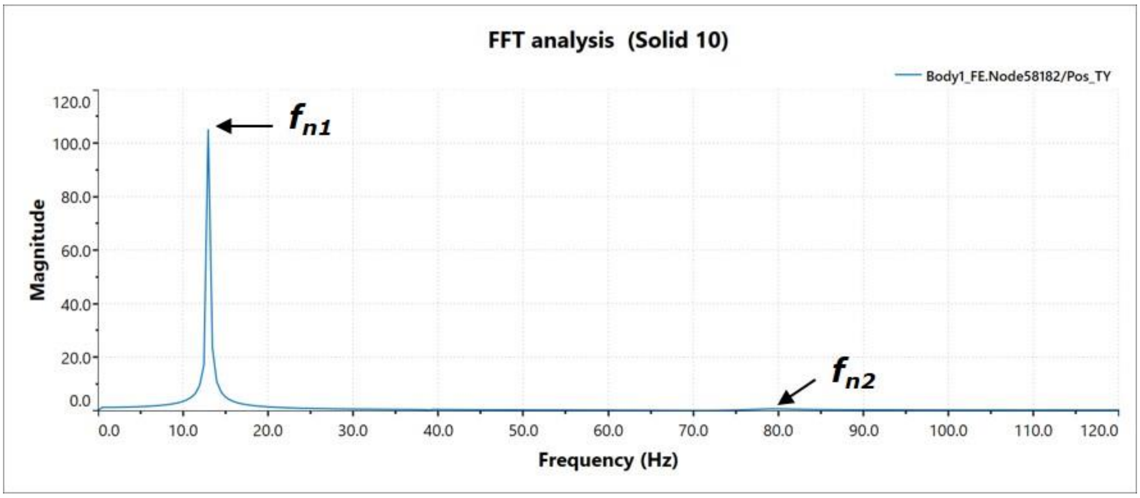


FFT Analysis

- Various frequencies can be visualized through FFT analysis.

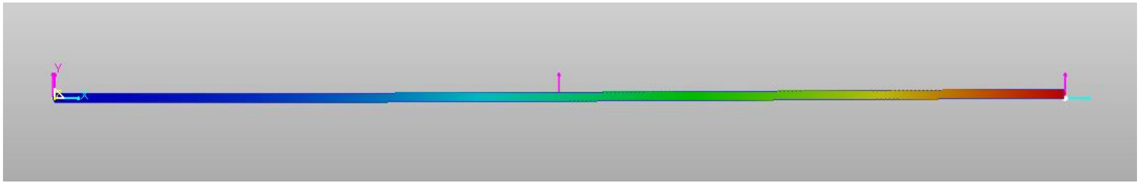




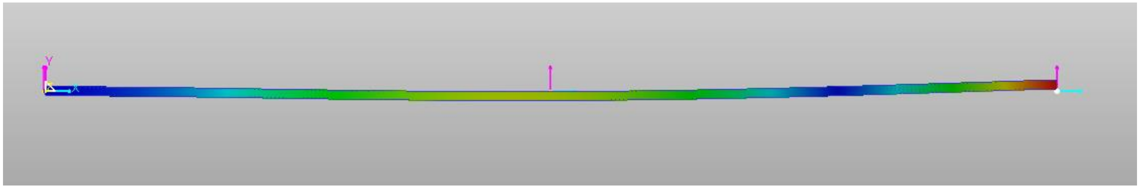


Mode frequency of RFlexGen of Solid 8 element

➢ First mode frequency = 13



➢ Second mode frequency = 81.8



Comparison of results

- Solid 4 element

Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1} $[Hz]$	13	15.55	15.4	18.5	0.97
f_{n2} $[Hz]$	81.7	97.45	97.3	19	0.15

- Solid 8 element

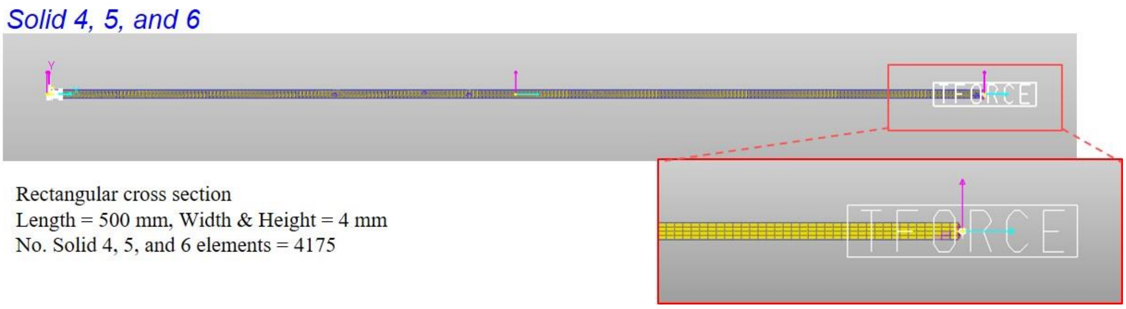
Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1} $[Hz]$	13	13	12.97	0.23	0.23
f_{n2} $[Hz]$	81.7	81.8	78.34	4.11	4.41

- Solid 10 element

Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1} $[Hz]$	13	13	12.97	0.23	0.23
f_{n2} $[Hz]$	81.7	81.8	79.84	2.27	2.45

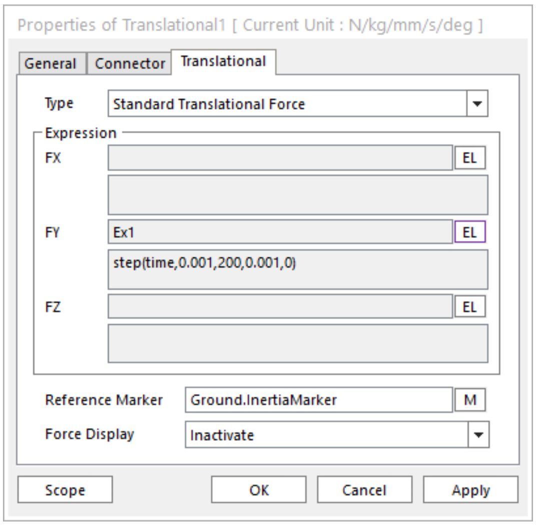
(Solid 4, 5, 6 element) Cantilevered Beam

- The flexible block body is connected to the ground with BC constraint condition at the Ground.
- The width and thickness of the block are determined by 4[mm] and the length is 500 mm.
- The flexible body consists of solid element of 4, 5, 6, and 8.
- The concentrated load(*P*) is applied on the flexible body at the point of P without gravity.



Modeling parameter

- The translation force in the y axis is applied at the end point of beam during a very short initial period.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	400	mm
Width of Block	b	10	mm
Height of Block	h	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	77821	Mpa
Area Moment of inertia	I	21.333	mm^4
Torsional Constant	J	1408	mm^4/rad
No. solid 4,5,6 element	-	4175	-

Theoretical Solution

- First critical frequency (k = 1.875)

$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 21.333}{1 \times 7.85\mathrm{e}-6 \times 1000^4}} = 13$$

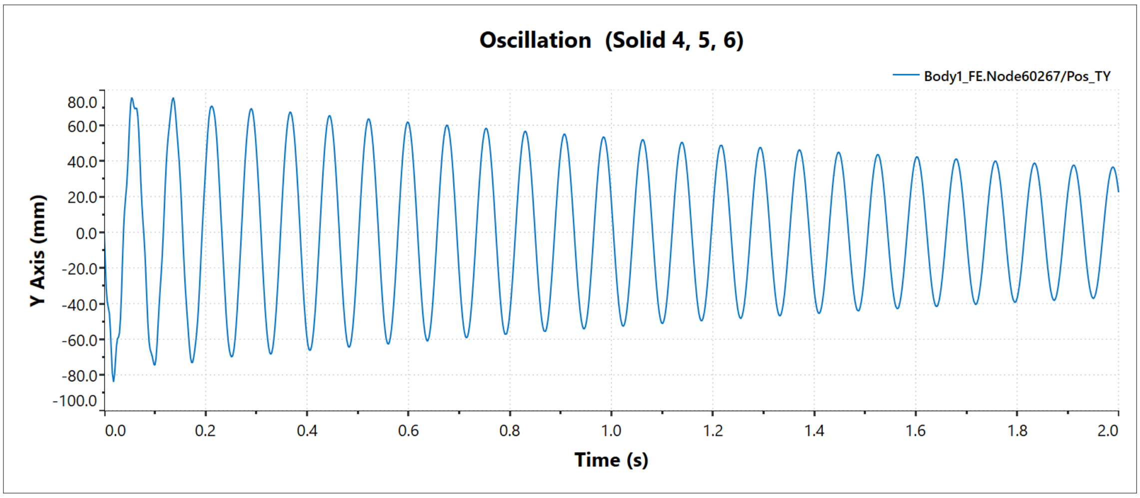
- 2nd critical frequency (k = 4.694)

$$f_{n2} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{4.694^2}{2\pi}\sqrt{\frac{200000000 \times 21.333}{1 \times 7.85\mathrm{e}-6 \times 1000^4}} = 81.7$$

Numerical Solution - RecurDyn

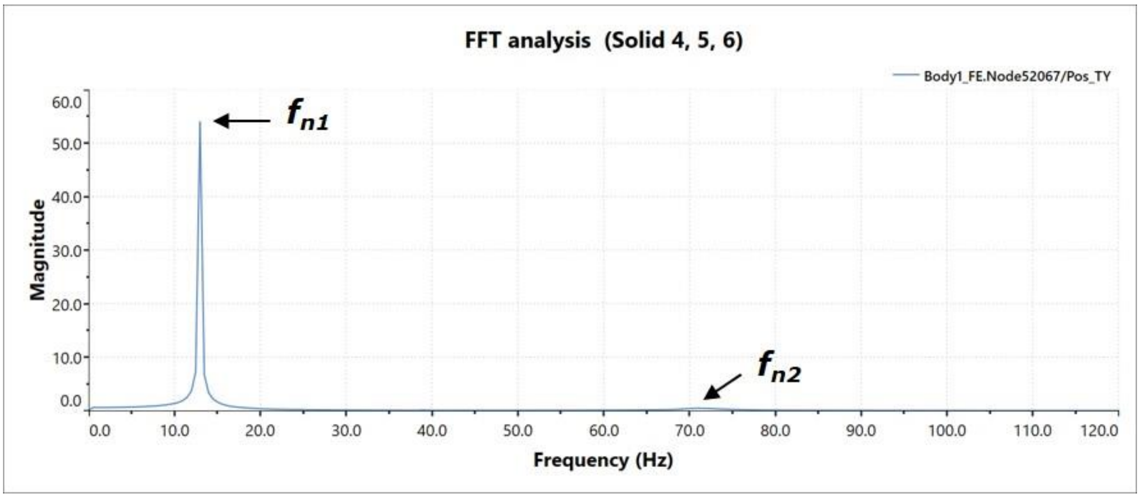
Plot the oscillation of P

- Multiple frequencies are superimposed to show complex graphs.



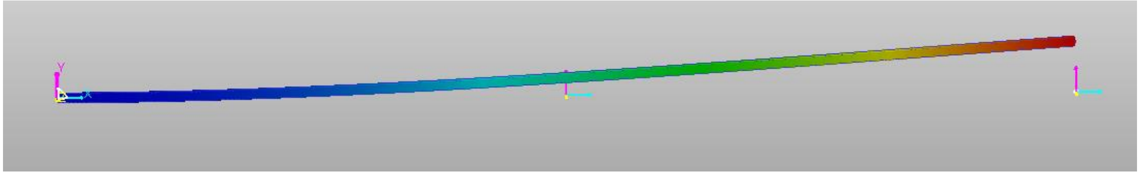
FFT Analysis

- Various frequencies can be visualized through FFT analysis.

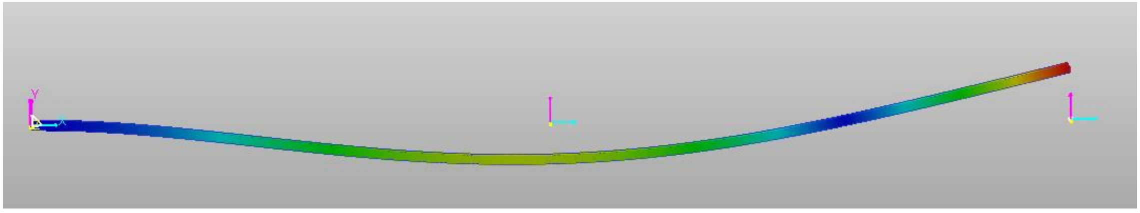


Mode frequency of RFlexGen of Solid 8 element

- ➢ First mode frequency = 13



- ➢ Second mode frequency = 82.2



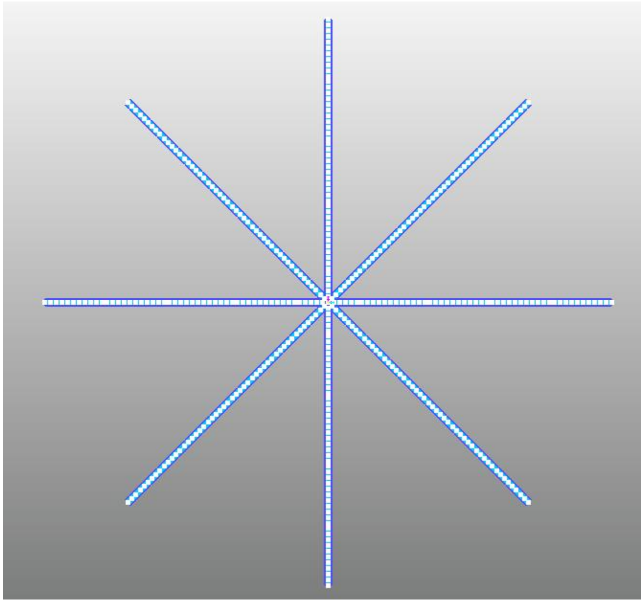
Comparison of results

- Solid 4,5, and 6 elements

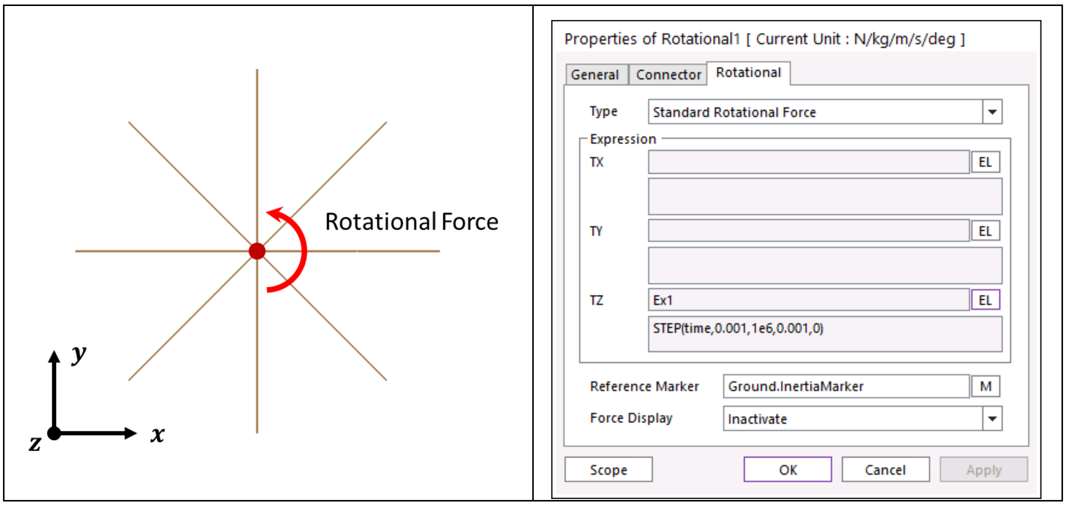
Object Value	Theory	RFlexGen	RecurDyn Fflex	Error with theory (%)	Error with RflexGen (%)
f_{n1} $[Hz]$	13	13	12.97	0.23	0.23
f_{n2} $[Hz]$	81.7	82.2	74	9.24	11

(Beam element) Pin-ended double cross

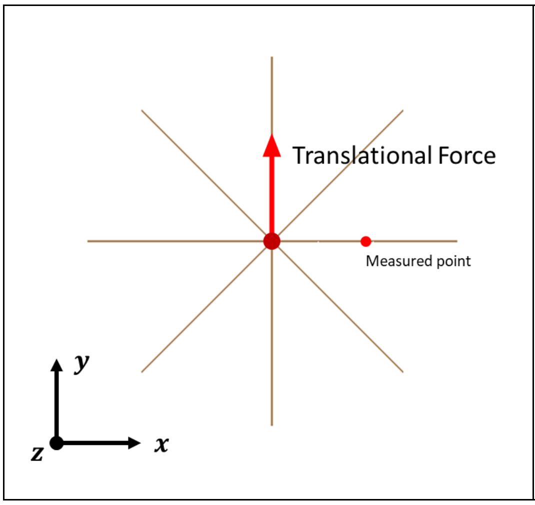
- The body is connected to the ground with fixed joint at the eight endpoints.
- The cross section of the beam is 125[mm] by 125[mm] square and its length (from the center point to the endpoint) is 5000 mm.
- The number of elements is 400.

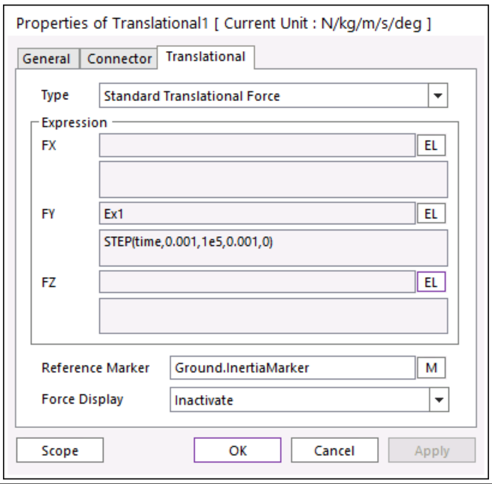


- The rotational and translational impulse are applied at the center point.
  - A torque impulse was applied at the center node in the z-direction.



■ An impulse was applied at the center node in the y-direction.





Modeling parameter

Given	Symbol	Value	Unit
Initial arm length	l_0	5000	mm
Width of Block	b	125	mm
Height of Block	h	125	mm
Young's modulus	E	200000	MPa
Poisson's Ratio	nu	0.3	-
Density	ρ	8000	kg/m^3

Theoretical Solution

- The classical beam theory prediction of the natural frequencies

$$f_i = \frac{\lambda_i^{\,2}}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}$$

- Solution table

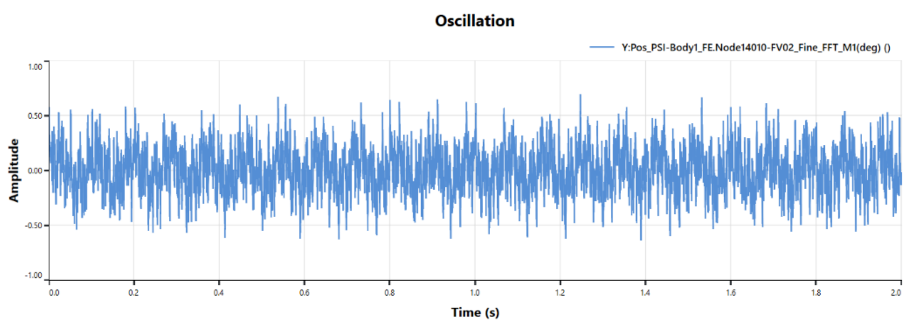
Mode No.	Mode type	λ_i	f_i (Hz)
1	Pin-ended (1st rotational mode)	π	11.336
2,3	Essentially propped (1st translational mode)	3.92660231	17.709
4 to 8	Propped (Complex mode)	3.92660231	17.709
9	Pin-ended (2nd rotational mode)	2π	45.345

Ref. Blevins, Robert D. Formulas for natural frequency and mode shape. 1979.

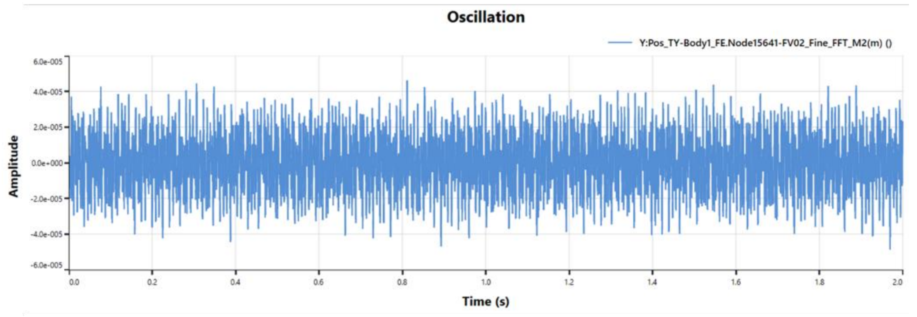
● Numerical Solution - RecurDyn

Plot of the oscillations by rotational/translational impulse

- Multiple frequencies are superimposed to show complex graphs.



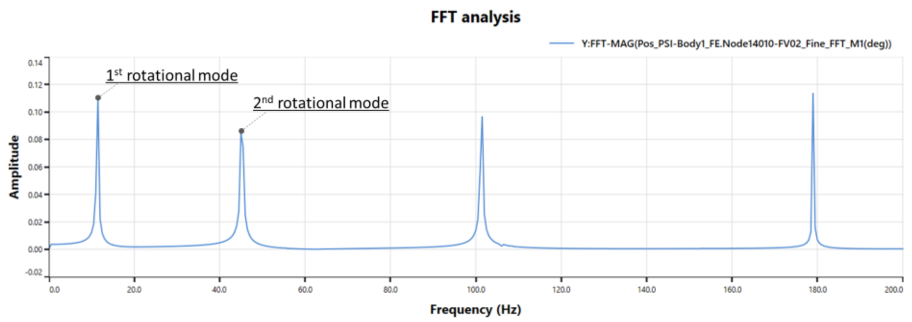
Rotational oscillations



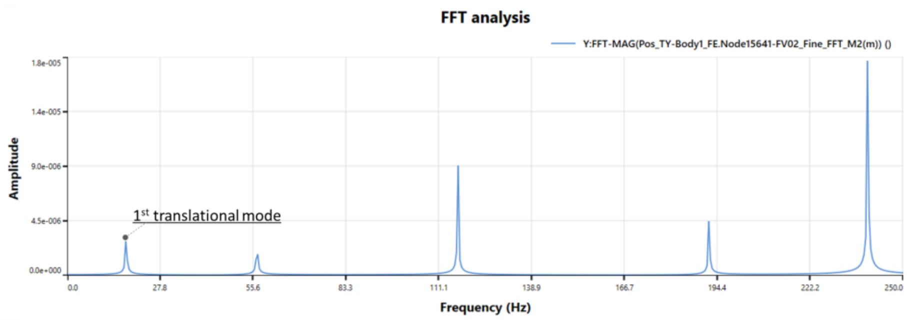
Translational oscillations

FFT Analysis

- Various frequencies can be visualized through FFT analysis.



Rotational mode



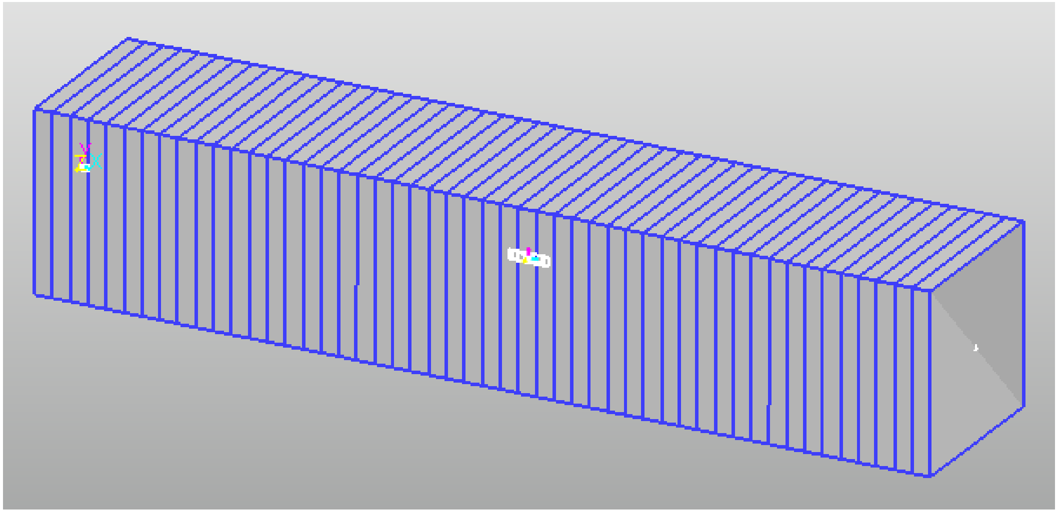
Translational mode

⭘ Comparison of results

Mode No./Type	Theory [Hz]	RecurDyn FFlex [Hz]	Error with theory (%)
Mode 1 / 1st Rotational	11.336	11.5	1.45
Mode 2&3 / 1st Translational	17.709	17.5	1.18
Mode 4-8 / Complex	17.709	-	-
Mode 9 / 2nd Rotational	45.345	45.0	0.76

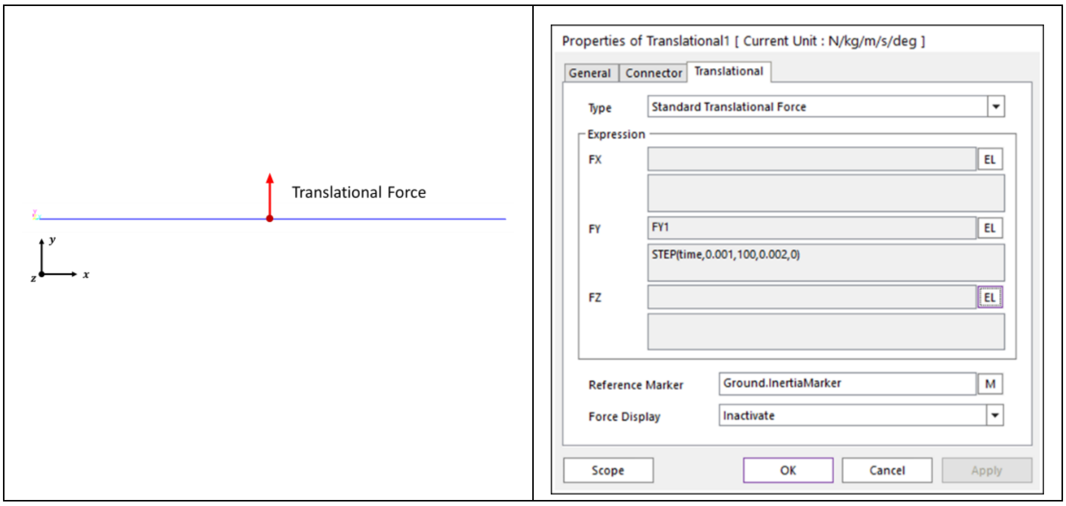
(Beam element) Deep simply-supported beam

- The body is connected to the ground with fixed joint at the left endpoint.
- The cross section of the beam is 2[m] by 2[m] square and its length is 10 [m].
- The number of elements is 50.

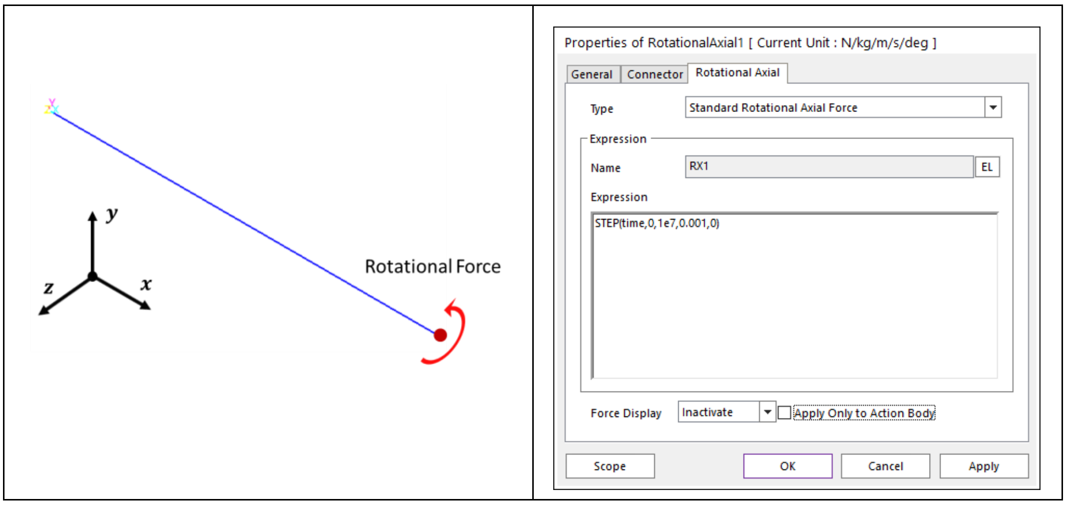


- The rotational and translational impulse are applied.

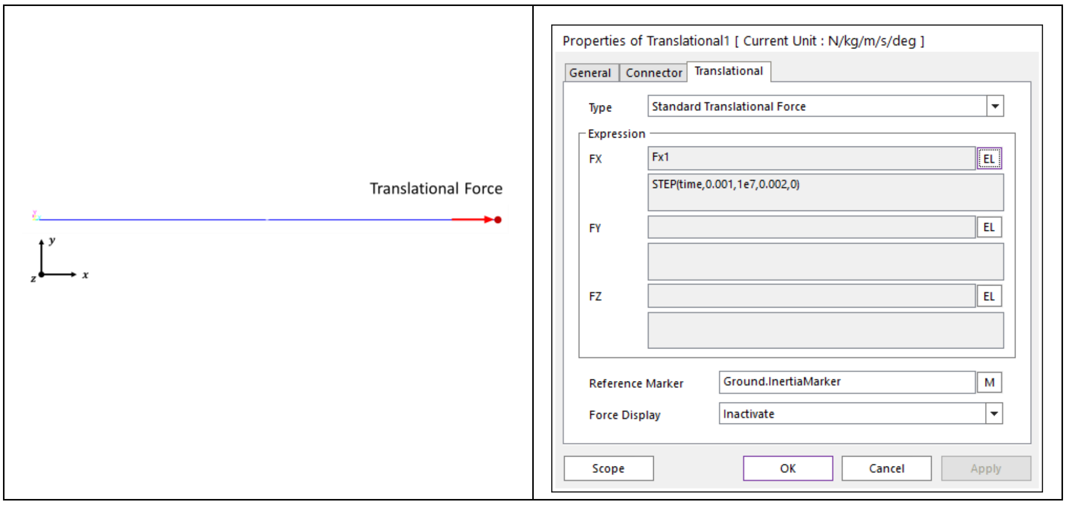
  - An impulse was applied at the center node in the y-direction.



  - A torque impulse was applied at the end node in the x-direction.



- An impulse was applied at the center node in the y-direction.



Modeling parameter

Given	Symbol	Value	Unit
Initial length	l_0	10000	mm
Width of Block	b	2000	mm
Height of Block	h	2000	mm
Young's modulus	E	200000	MPa
Poisson's Ratio	nu	0.3	-
Density	ρ	8000	kg/m^3

Theoretical Solution

- Extensional (Axial) mode

$$f_i = \frac{\lambda_i}{2\pi L}\sqrt{\frac{E}{\rho}}$$

, where

$$\lambda_i = \frac{(2i-1)\pi}{2}$$

- Torsional mode

$$f_i = \frac{\lambda_i}{2\pi L}\sqrt{\frac{CG}{\rho I_p}}$$

, where

$$\lambda_i = \frac{(2i-1)\pi}{2}$$

C = torsional constant of the cross section

= $0.1406b^4$

I_p = 2nd moment of area of the cross-section about axis of torsion

= $\frac{1}{6}b^4$, for a square cross section

- Flexural mode
  - Classical beam theory

$$f_i = \frac{{\lambda_i}^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}$$

  - Timoshenko beam theory

$$\frac{r^2\rho}{kG}{\omega_i}^4 - \left[\frac{i^2\pi^2r^2}{L^2}\frac{E}{kG} + \frac{i^2\pi^2r^2}{L^2} + 1\right]{\omega_i}^2 + \left(\frac{Er^2}{\rho}\right)^2\frac{i^4\pi^4}{L^4} = 0$$

, where

r = radius of gyration of the cross-section

= $\sqrt{\frac{I}{A}}$,

k = shear factor = $\frac{10(1+\nu)}{12+11\nu}$ = for a rectangular cross-section

- Solution table

Mode No.	Mode type	λ_i	f_i (Hz)
1,2	Flexure (1st mode)	-	42.62
3	Torsional (1st mode)	0.5π	71.20
4	Axial (Complex mode)	0.5π	125.00

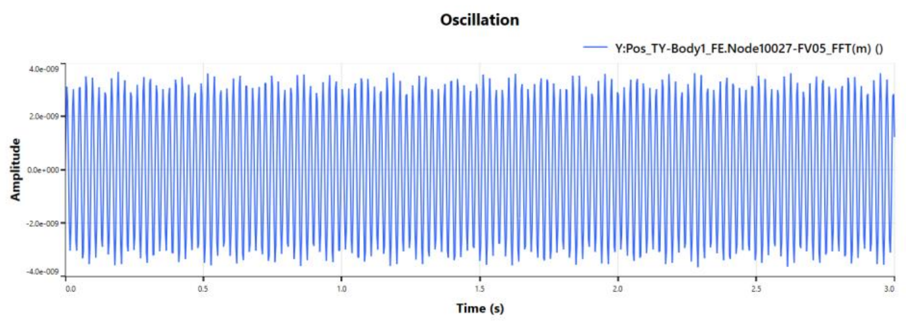
Ref. Blevins, Robert D. Formulas for natural frequency and mode shape. 1979.

Ref. Timoshenko, S., and D. H. Young. Vibration problems in Engineering. 1955.

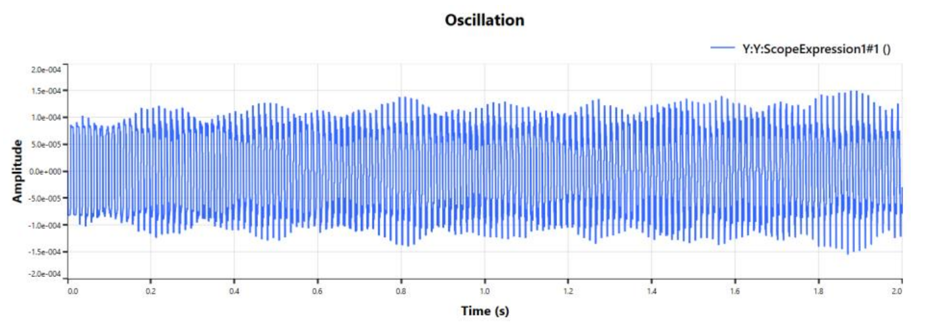
Numerical Solution - RecurDyn

Plot of the oscillations by rotational/translational impulse

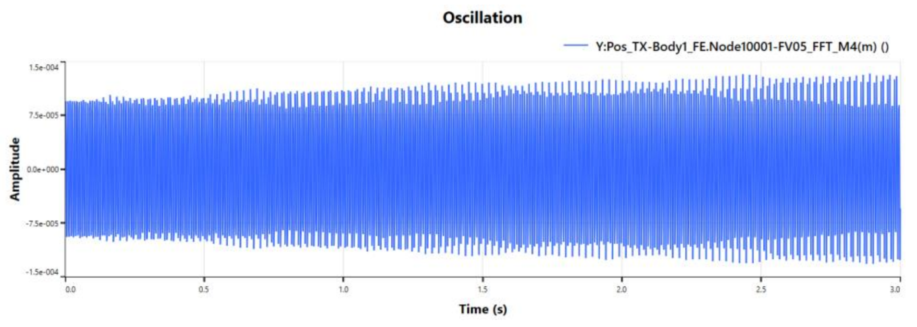
- Multiple frequencies are superimposed to show complex graphs.



Flexural oscillations



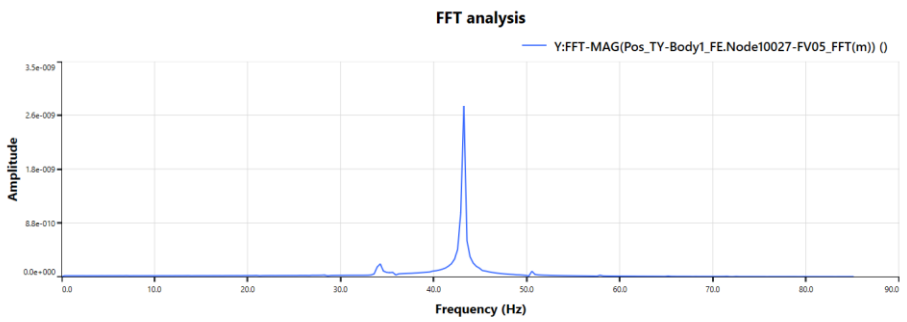
Torsional oscillations



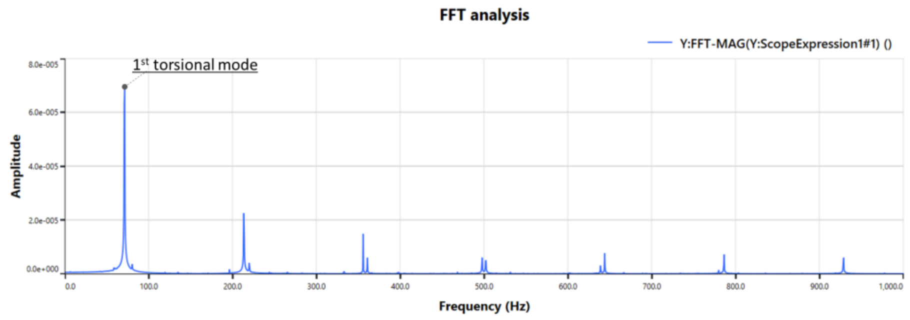
Axial oscillations

FFT Analysis

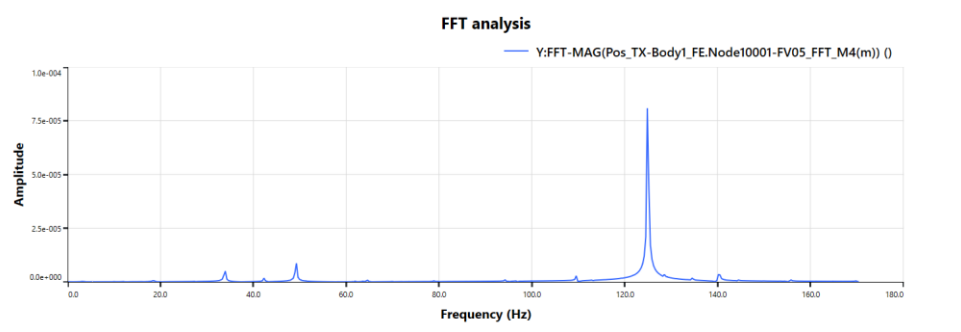
- Various frequencies can be visualized through FFT analysis.



Flexural mode



Torsional mode



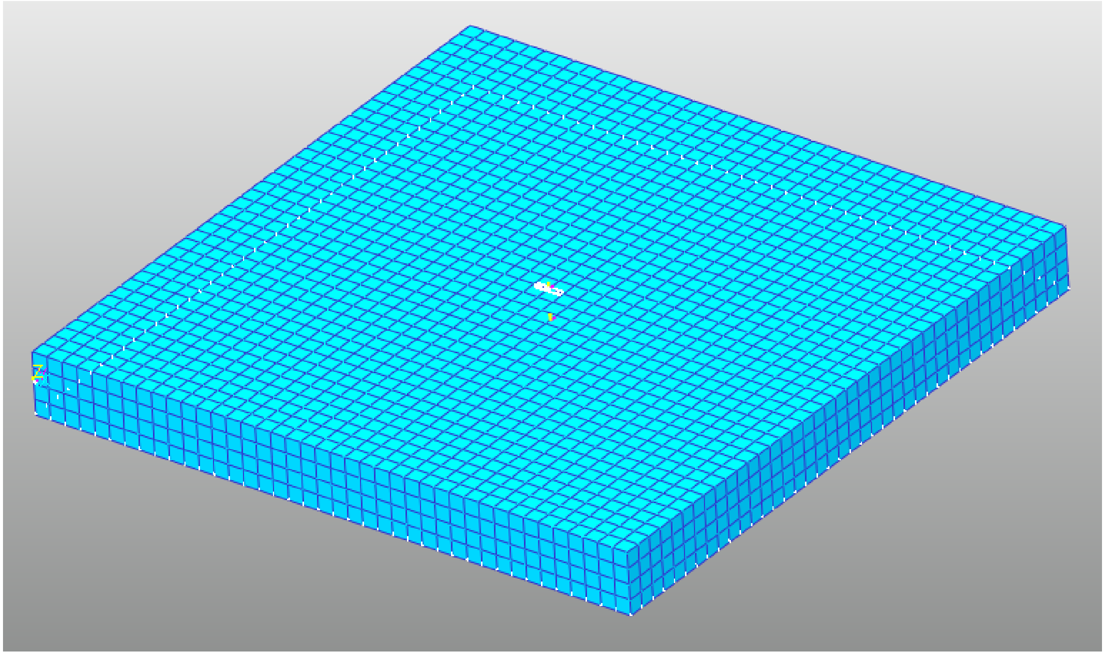
Axial mode

Comparison of results

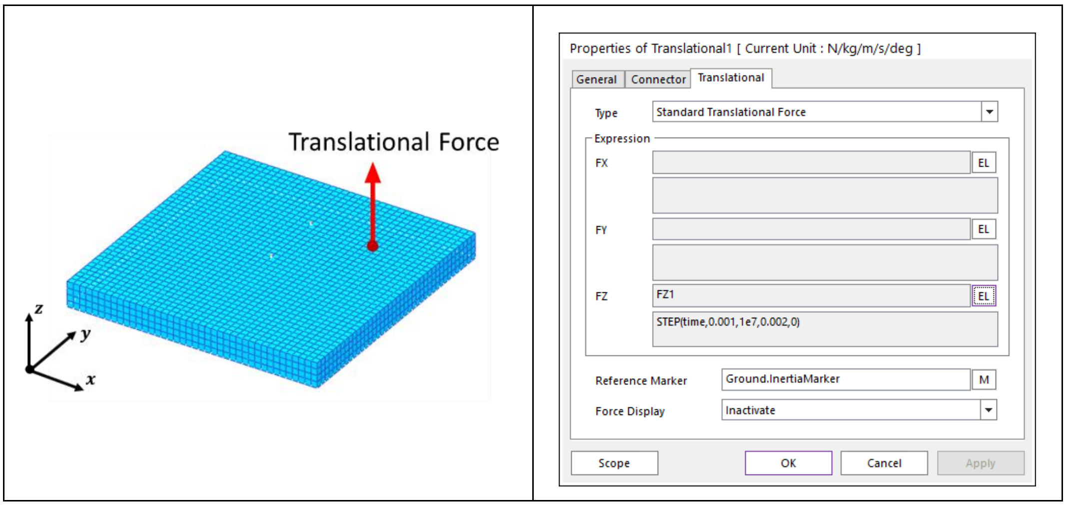
Mode No./Type	Theory [Hz]	RecurDyn FFlex [Hz]	Error with theory (%)
Mode 1&2 / 1st Flexural	42.649	43.25	1.41
Mode 3 / 1st Torsional	71.201	71.50	0.42
Mode 4 / 1st Extensional	125	124.88	0.10

(Solid 8) Simply-supported 'solid' square

- The body is connected to the ground with fixed joint at the 4 bottom edges.
- The width and height of the solid are 10[m]s each and its depth is 1[m].
- The number of elements is 6400.



- The translational impulse is applied.



Modeling parameter

Given	Symbol	Value	Unit
Width	w	10	m
Height	h	10	m
Depth	d	1	m
Young's modulus	E	200000	MPa
Poisson's Ratio	nu	0.3	-
Density	ρ	8000	kg/m^3

Theoretical Solution

- Flexural mode

$$f_{ij} = \frac{\lambda_{ij}}{2\pi}\sqrt{\frac{E}{2(1+\nu)\rho t^2}}$$

- Solution table for a/t=10

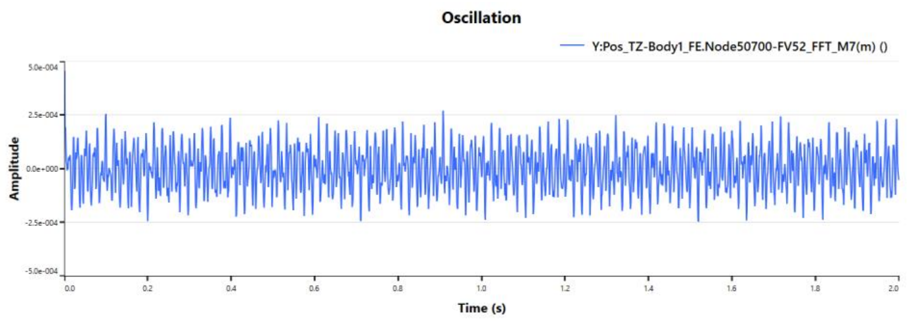
Mode No.	Mode type	λ_i	f_i (Hz)
4	1st flexural mode	0.09315	45.971
5&6	2nd flexural mode	0.22260	109.857
7	3rd flexural mode	0.3420	167.002

Ref. Rock, T., and E. Hinton. Free vibration and transient response of thick and thin plates using the finite element method. Earthquake Engineering & Structural Dynamics 3.1 (1974): 51-63.

Numerical Solution - RecurDyn

Plot of the oscillations by rotational/translational impulse

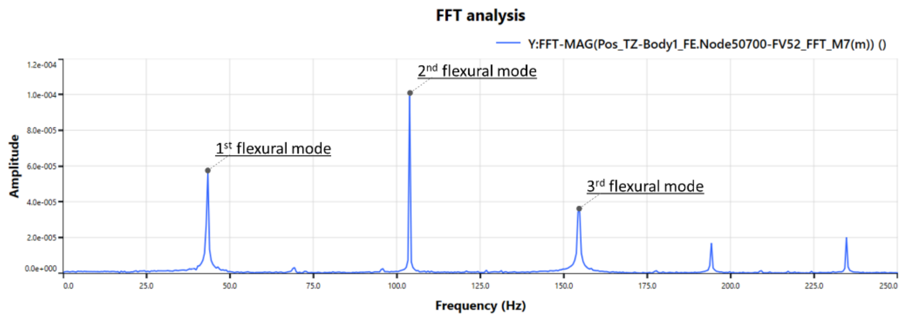
- Multiple frequencies are superimposed to show complex graphs.



Flexural oscillations

FFT Analysis

- Various frequencies can be visualized through FFT analysis.



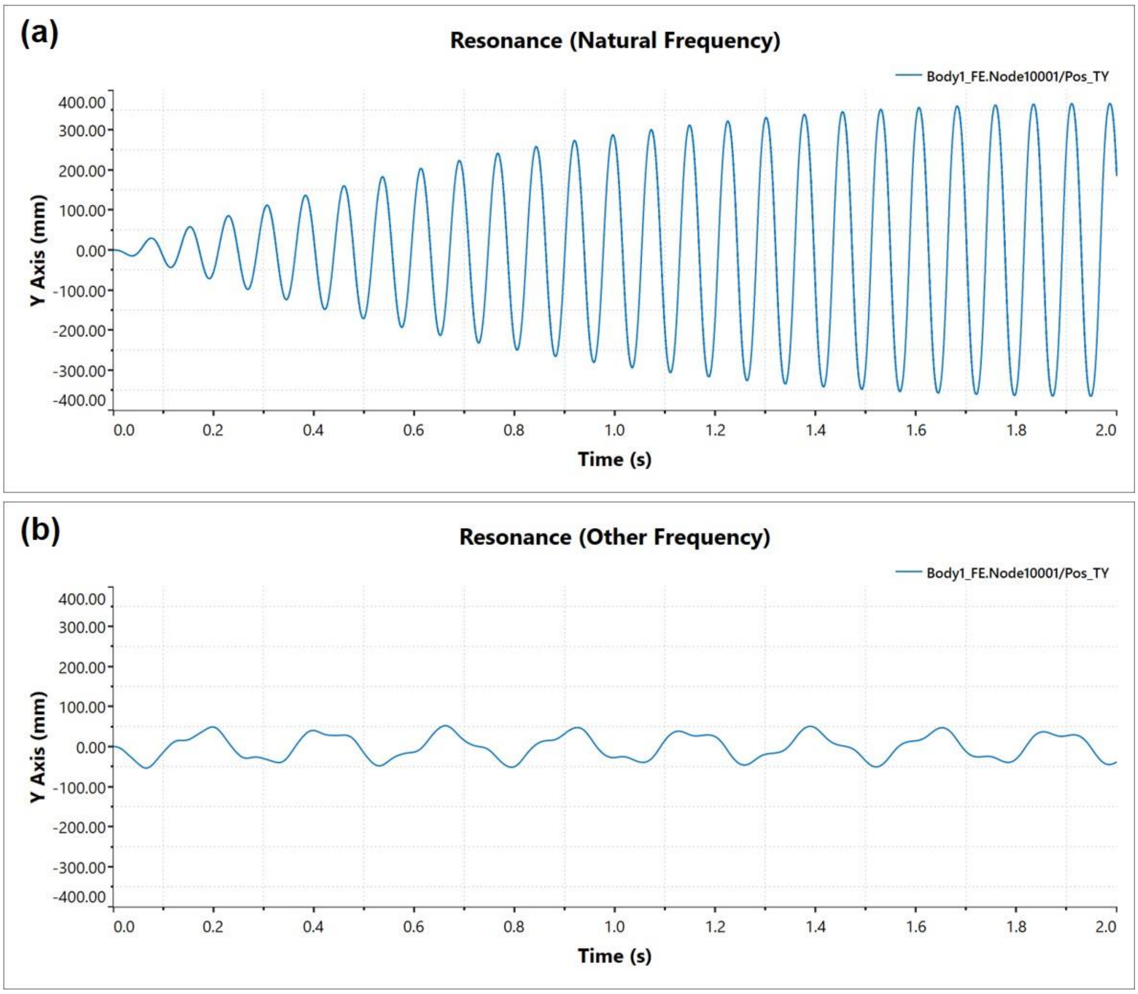
Flexural mode

⭘ Comparison of results

Mode No./Type	Theory [Hz]	RecurDyn FFlex [Hz]	Error with theory (%)
Mode 4 / 1st Flexural	45.971	43.46	5.46
Mode 5&6 / 2nd Flexural	109.857	103.90	5.42
Mode 7 / 3rd Flexural	167.002	154.85	7.28

Resonance phenomena

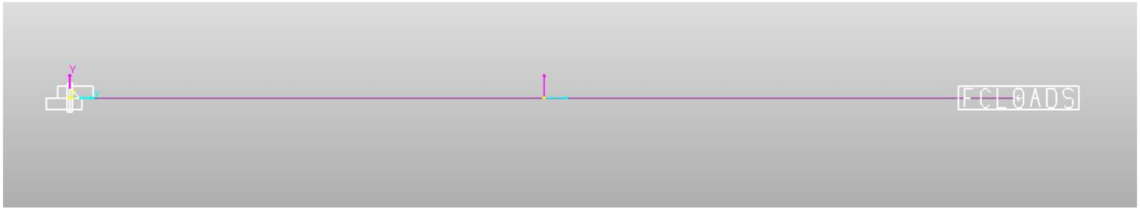
Resonance phenomenon occurs when the frequency (applied periodical force) is nearly equal to one of the natural frequencies of the system. It causes the system to oscillate with larger amplitude than when the force is applied at other frequency. The figure below shows the amplitude of the Y-axis motion when applying the system natural frequency of (a) or other frequency of (b). The amplitude becomes 10 times larger with time in (a) than that of (b). Thus, in this chapter, we will verify the natural frequency of the system by inputting theoretically calculated or proven resonance frequency as external force.



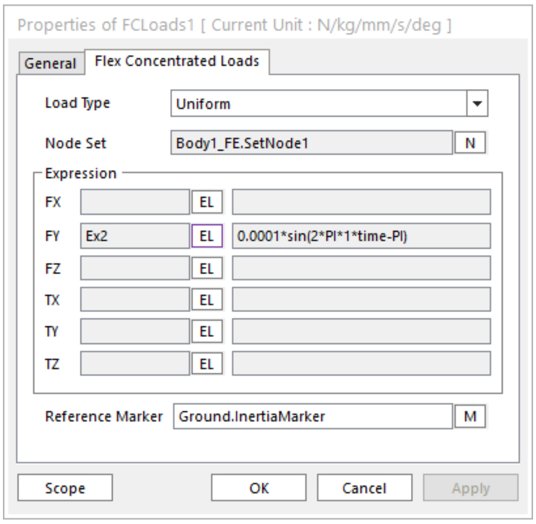
Cantilevered Beam

Beam element

- The geometry of beam and its modeling properties are equals to the shell element in previous chapter.
- The number of nodes of beam element is set as 500.



- The load of P in the y axis is applied at the point of P as periodical force like below.



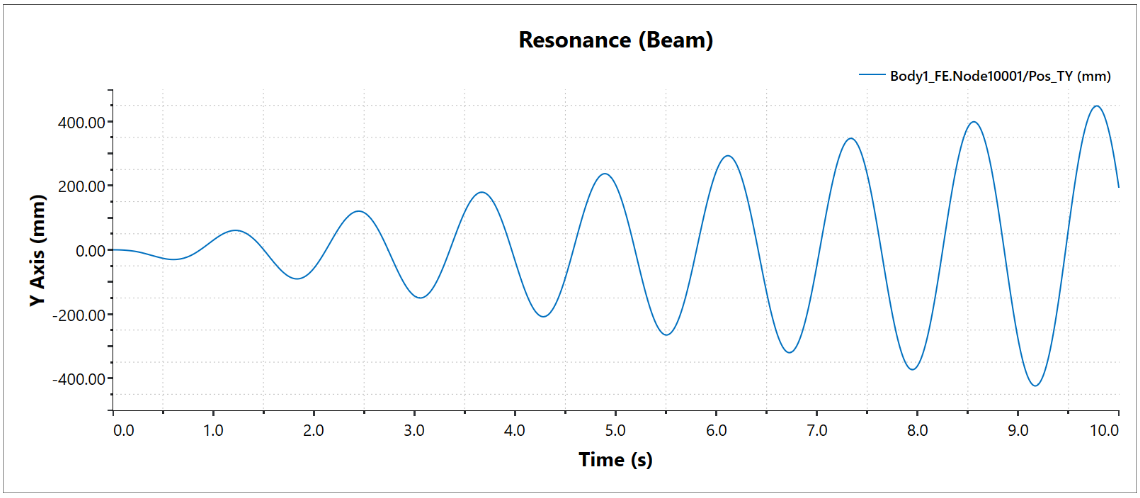
Theoretical Solution

- First critical frequency (k = 1.875)

$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\mathrm{e}-6 \times 1000^4}} = 0.82$$

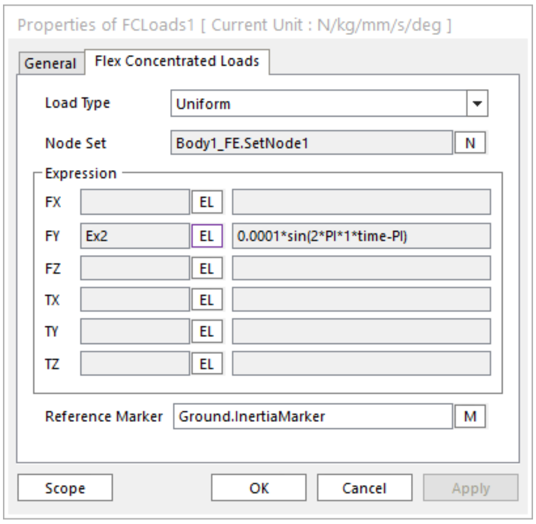
Numerical Solution - RecurDyn

➢ Beam element with First mode frequency = 0.82



Shell element (Shell 3, 4)

- The geometry of beam and its modeling properties are equals to the shell element in previous chapter.
- The number of nodes of shell 3 and 4 elements are set as 3000 and 6000, respectively.
- The load of P in the y axis is applied at the point of P as periodical force like below.



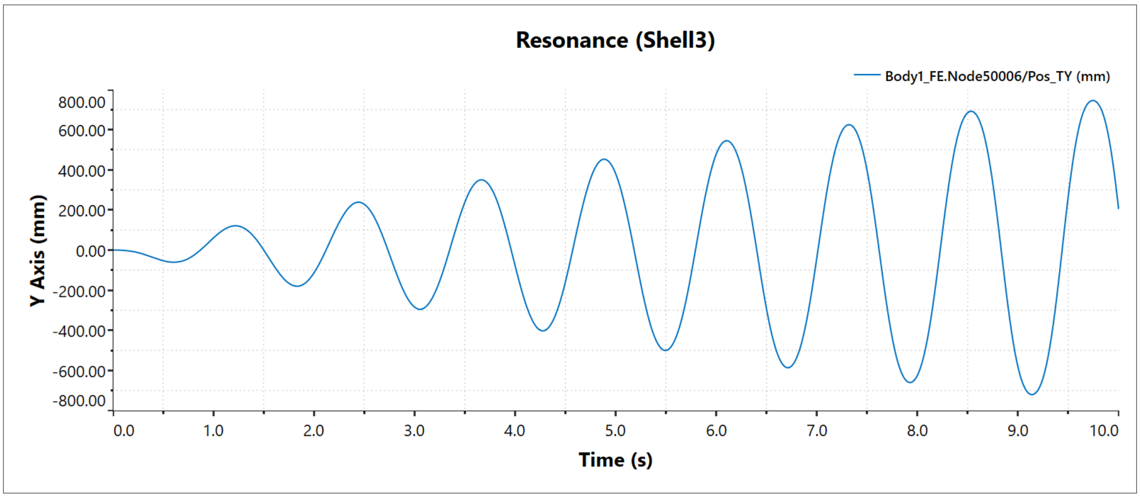
Theoretical Solution

- First critical frequency (k = 1.875)

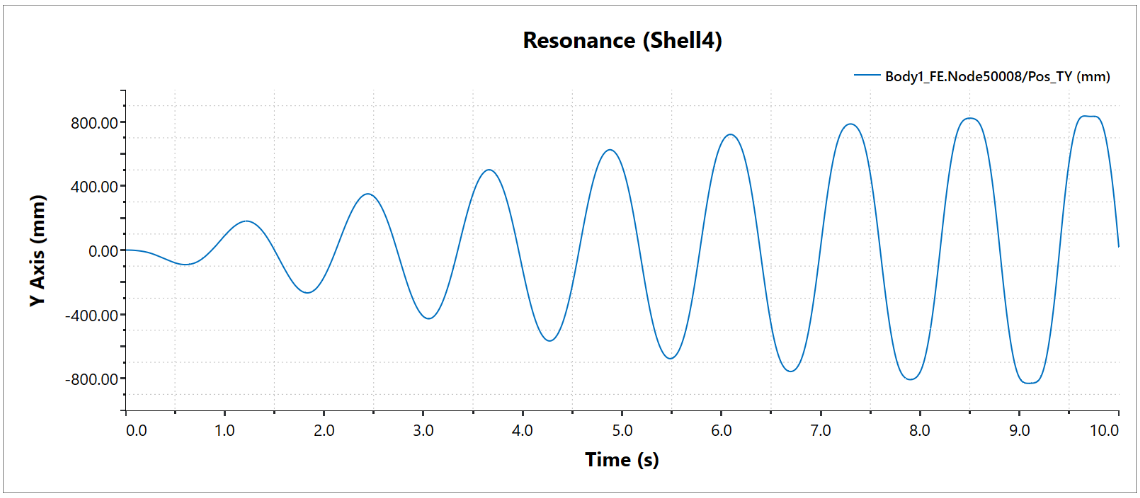
$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 8.333e-2}{1 \times 7.85\text{e}-6 \times 1000^4}} = 0.82$$

Numerical Solution - RecurDyn

➢ Shell 3 element with First mode frequency = 0.82

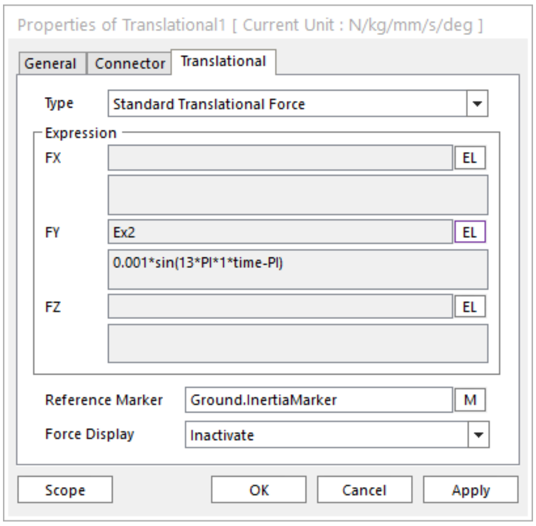


➢ Shell 4 element with First mode frequency = 0.82



Solid element (Solid 4, 8, and 10)

- The geometry of beam and its modeling properties are equals to the solid element in previous chapter.
- The number of nodes of shell 4, 8, and 10 elements are set as 7489, 1000 and 7489, respectively.
- The load of P in the y axis is applied at the point of P as periodical force like below.



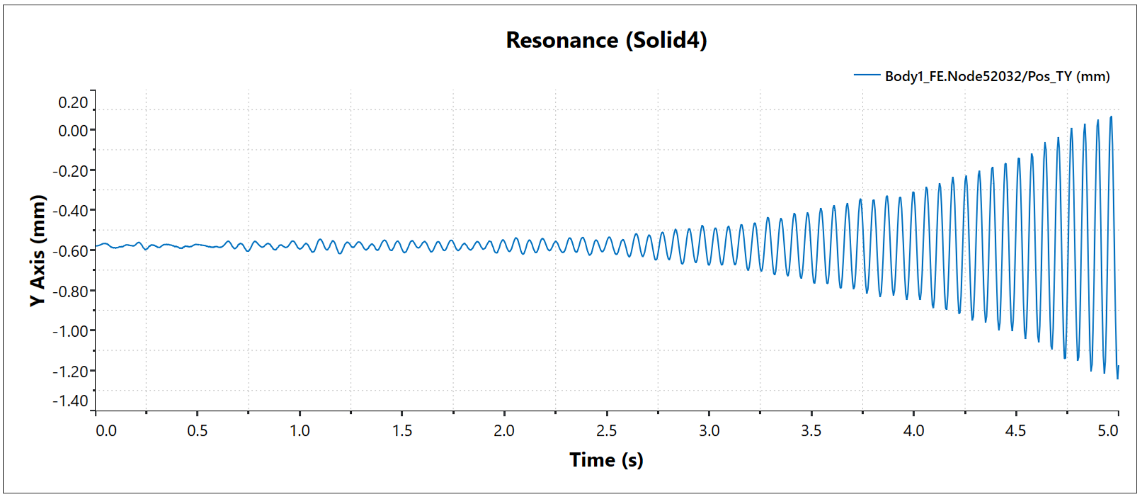
Theoretical Solution

- First critical frequency (k = 1.875)

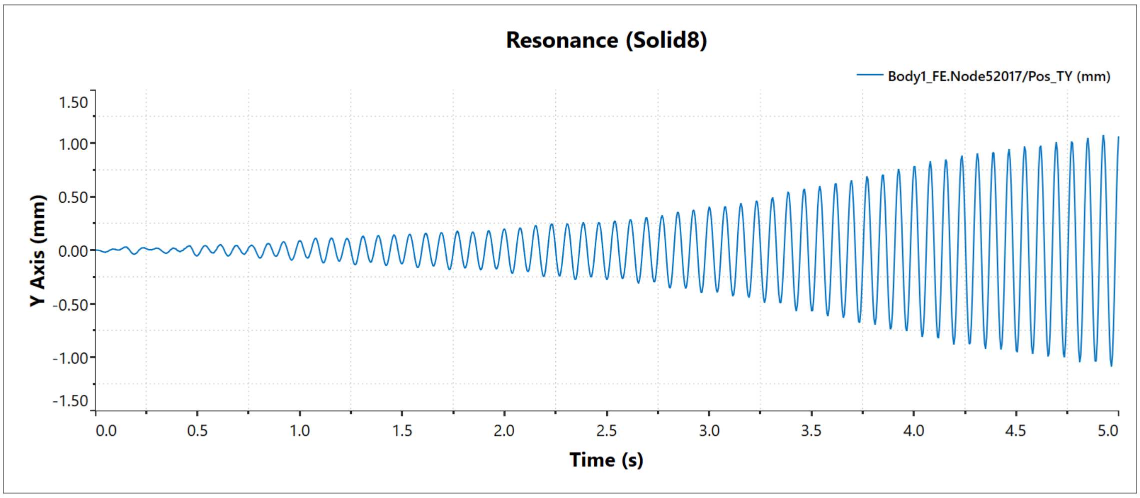
$$f_{n1} = \frac{k^2}{2\pi}\sqrt{\frac{EI}{A\rho L^4}} = \frac{1.875^2}{2\pi}\sqrt{\frac{200000000 \times 21.333}{1 \times 7.85e-6 \times 1000^4}} = 13$$

Numerical Solution - RecurDyn

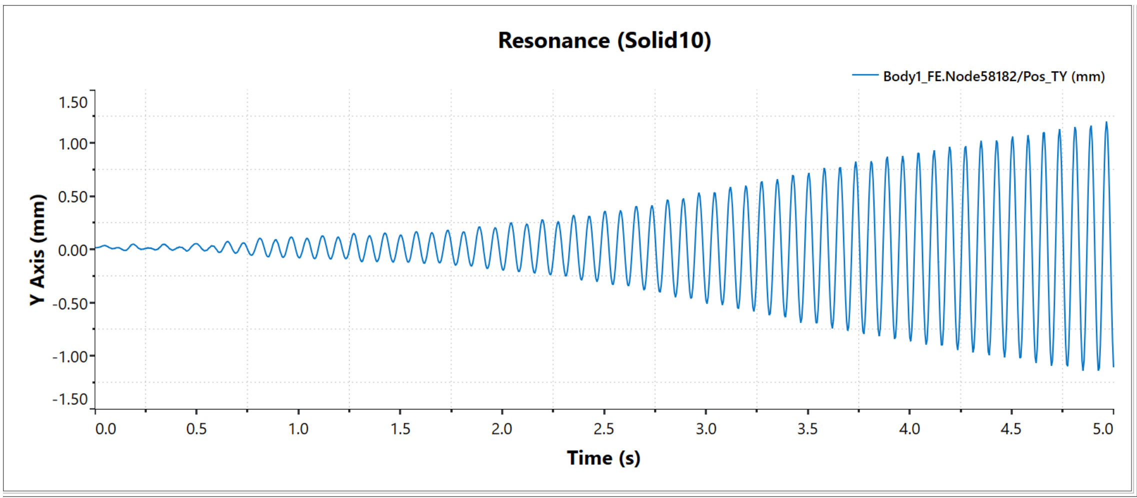
➢ Solid 4 element with First mode frequency = 13



➢ Solid 8 element with First mode frequency = 13



➢ Solid 10 element with First mode frequency = 13

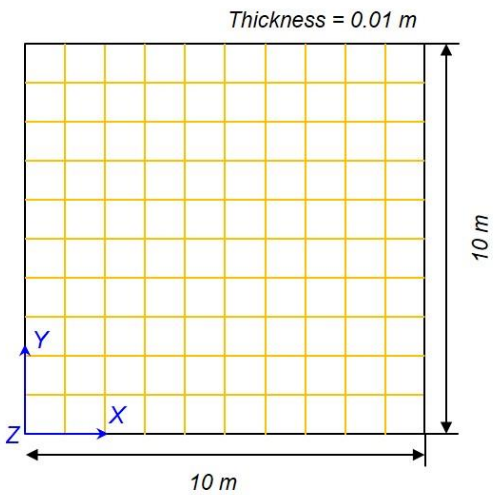


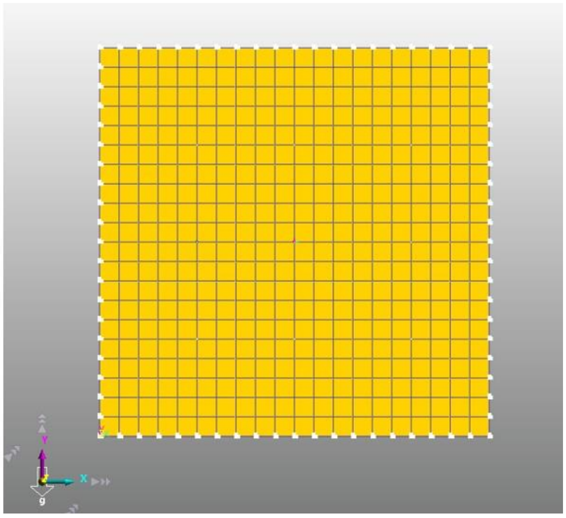
Square shape of rectangular sheet

The natural frequency of rectangular plate sheet is verified by using resonance phenomena.

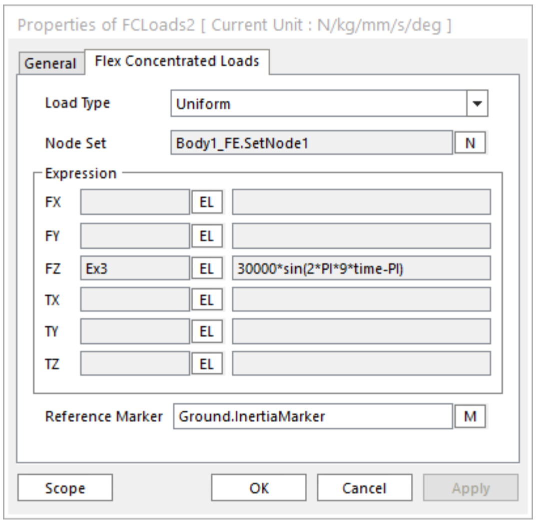
The Rayleigh method is used in this verification to determine the fundamental bending frequency. (“Vibration of Continuous Systems”, W. Leissa, 2011)

- The plate has a rectangular shape.
- All four of edges are fixed with BC boundary condition on the Ground.
- The flexible sheet consists of shell 4 element and the number of elements is set to 400.





- The concentrated load in the z axis is applied at the center as periodical force like below.



Modeling parameter

Given	Symbol	Value	Unit
Width of Sheet	w	10000	mm
Height of Sheet	h	10000	mm
Sheet Thickness	T	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	76923	Mpa
Poisson's Ratio	nu	0.3	-
Density	ρ	1.0e-5	kg/mm^3

Theoretical Solution

- First critical frequency (λ = 35.99)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{35.99}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0e-5 \times 10 \times (1-0.3^2)}} = 2.46$$

- 2nd & 3rd critical frequency (λ = 73.41)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{73.41}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0\mathrm{e} - 5 \times 10 \times (1-0.3^2)}} = 5$$

- 4th critical frequency (λ = 108.3)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{108.3}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0\mathrm{e} - 5 \times 10 \times (1-0.3^2)}} = 7.26$$

- 5th critical frequency (λ = 131.6)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{131.6}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0\mathrm{e} - 5 \times 10 \times (1-0.3^2)}} = 8.96$$

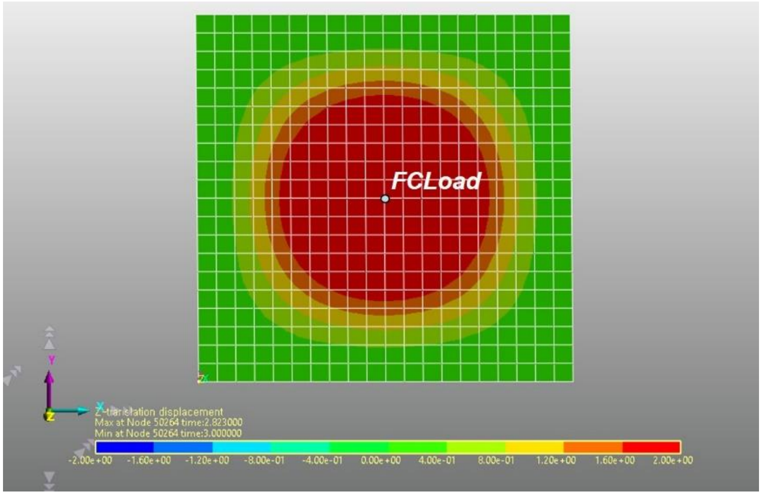
- 6th critical frequency (λ = 132.2)

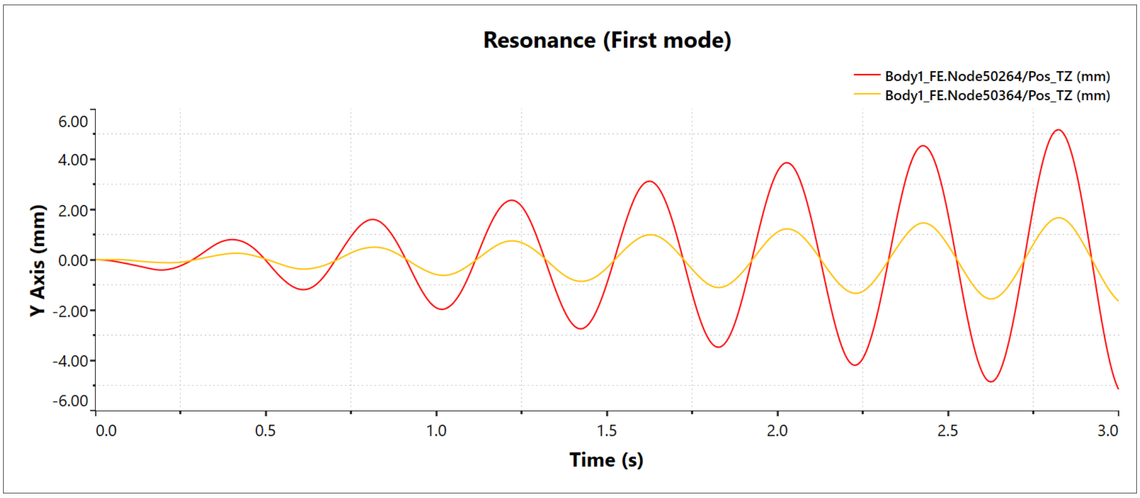
$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{132.2}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0\mathrm{e} - 5 \times 10 \times (1-0.3^2)}} = 9$$

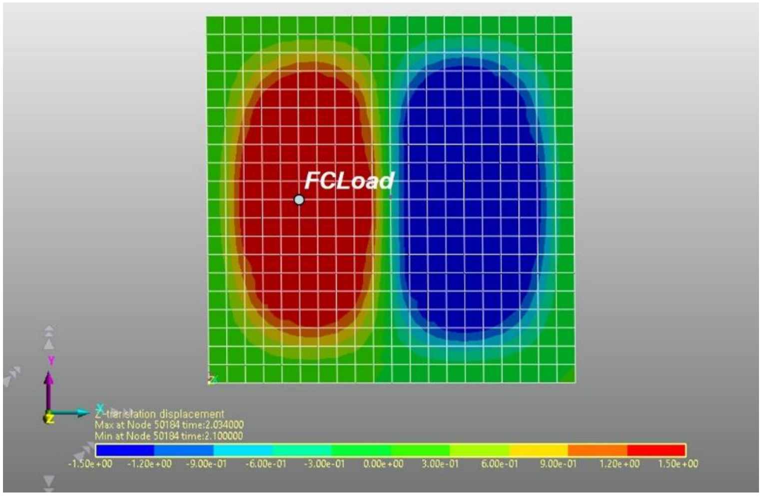
Numerical Solution - RecurDyn

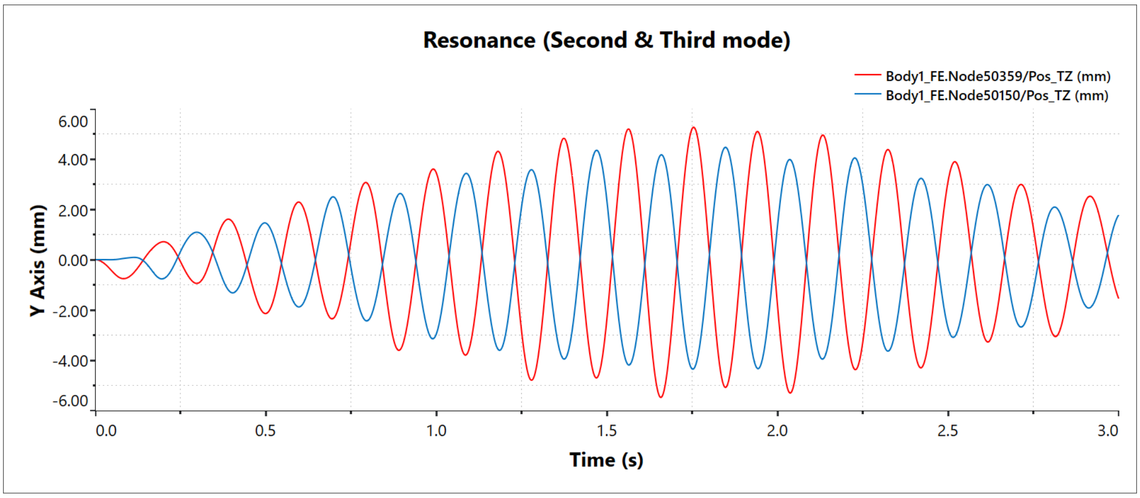
- First mode frequency = 2.46



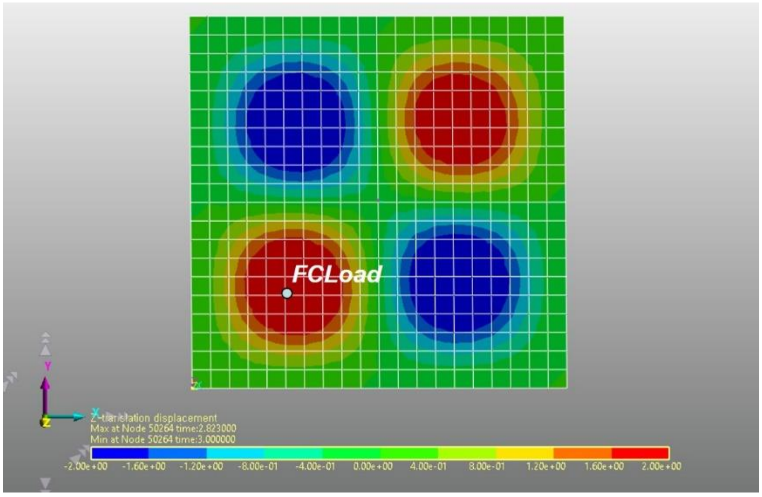


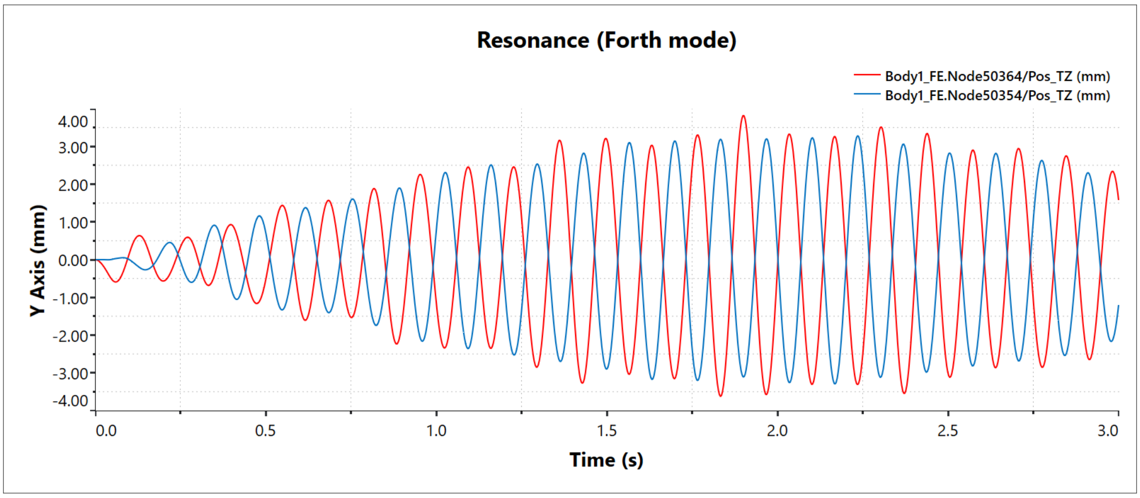
➢ Second and third mode frequency = 5



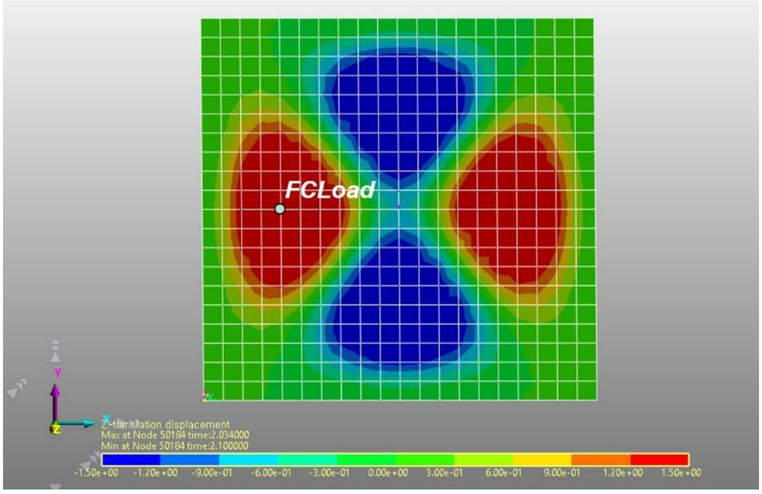


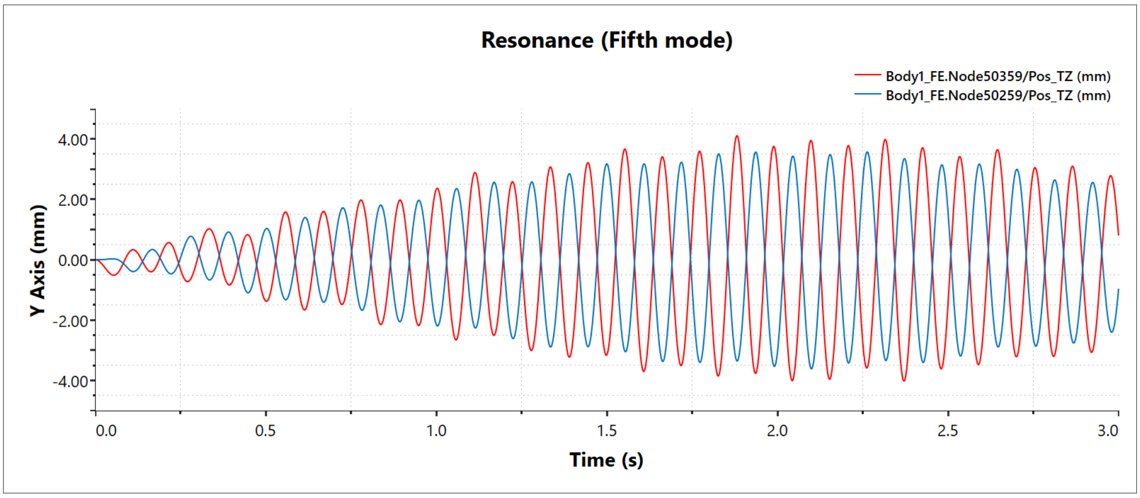
➤ Forth mode frequency = 7.26



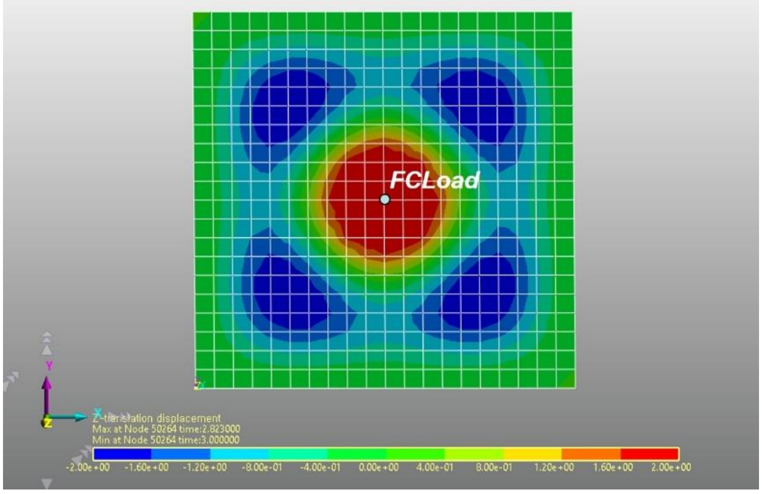


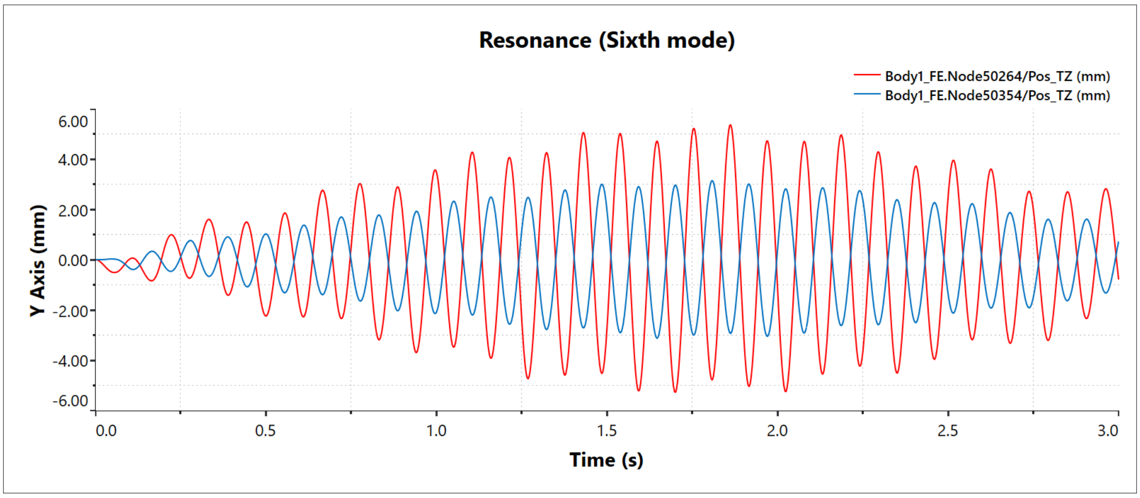
➤ Fifth mode frequency = 8.96





- Sixth mode frequency = 9





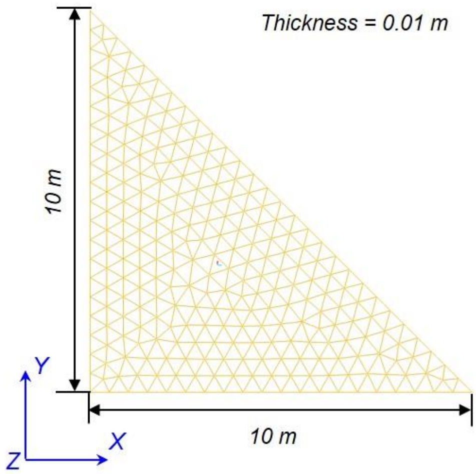
Resonance (Sixth mode)
Body1_FE.Node50264/Pos_TZ (mm)
Body1_FE.Node50354/Pos_TZ (mm)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
Y Axis (mm)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time (s)

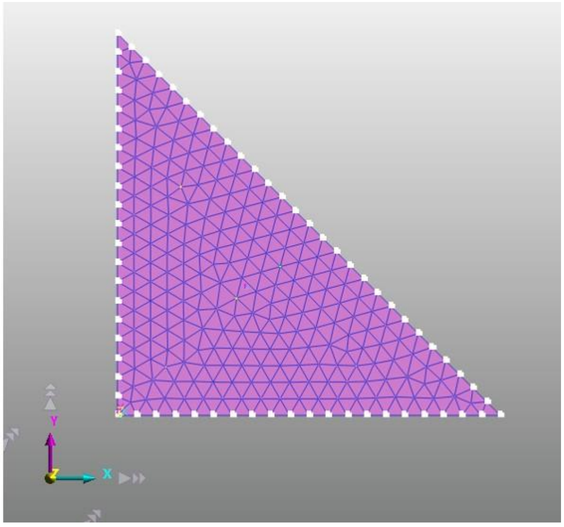
Triangular shape of sheet

The natural frequency of triangular sheet is verified by using resonance phenomena.

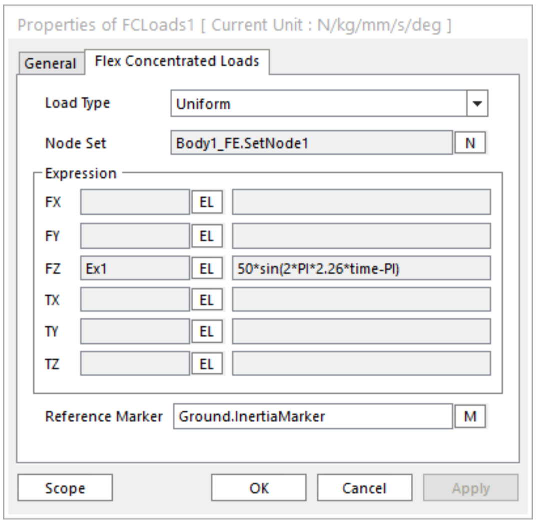
The Rayleigh method is used in this verification to determine the fundamental bending frequency. ("Vibration of Plates", S. Chakraverty, 2009)

- The plate has a triangular shape.
- All four of edges are fixed with BC boundary condition on the Ground.
- The flexible sheet consists of shell 3 element and the number of elements is set to 452.





- The concentrated load in the z axis is applied at the center as periodical force like below.



Modeling parameter

Given	Symbol	Value	Unit
Width of Sheet	w	10000	mm
Height of Sheet	h	10000	mm
Sheet Thickness	T	10	mm
Young's modulus	E	200000	MPa
Shear modulus	G	76923	Mpa
Poisson's Ratio	nu	0.3	-
Density	ρ	1.0e-6	kg/mm^3

Theoretical Solution

- First critical frequency (λ = 33.2)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{33.2}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0\mathrm{e}-5 \times 10 \times (1-0.3^2)}} = 2.26$$

- 2nd critical frequency (λ = 56.8)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{56.8}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0e-5 \times 10 \times (1-0.3^2)}} = 56.8$$

- 3rd critical frequency (λ = 68.6)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{68.6}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0e-5 \times 10 \times (1-0.3^2)}} = 4.67$$

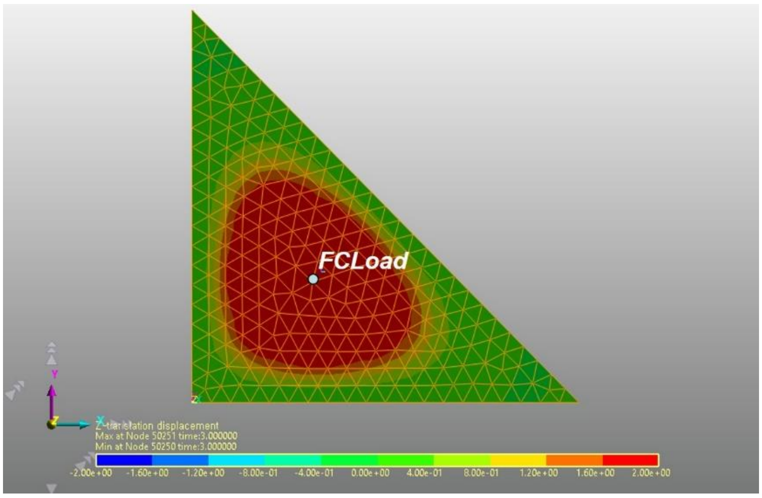
- 4th critical frequency (λ = 85.4)

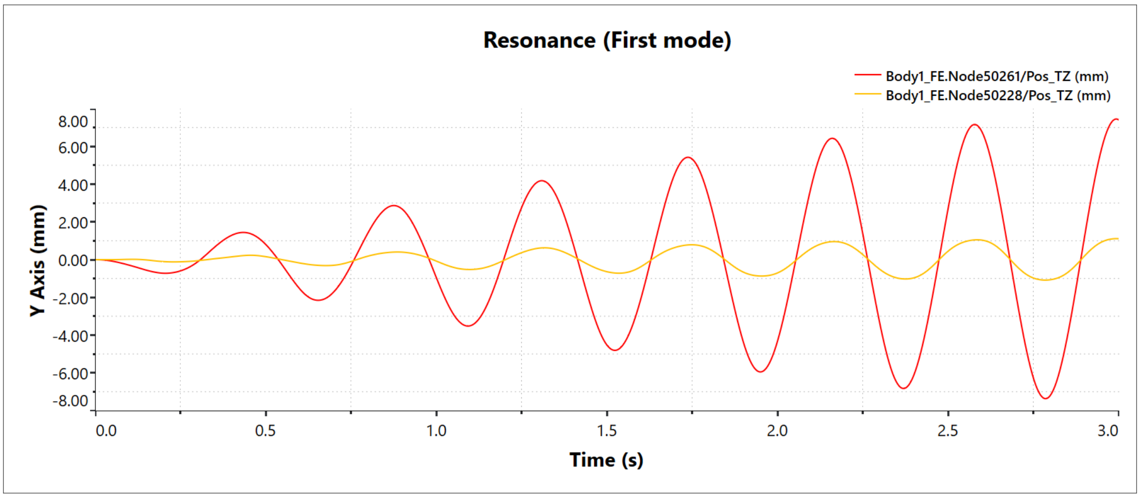
$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{85.4}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 10^3}{12 \times 1.0e-5 \times 10 \times (1-0.3^2)}} = 5.82$$

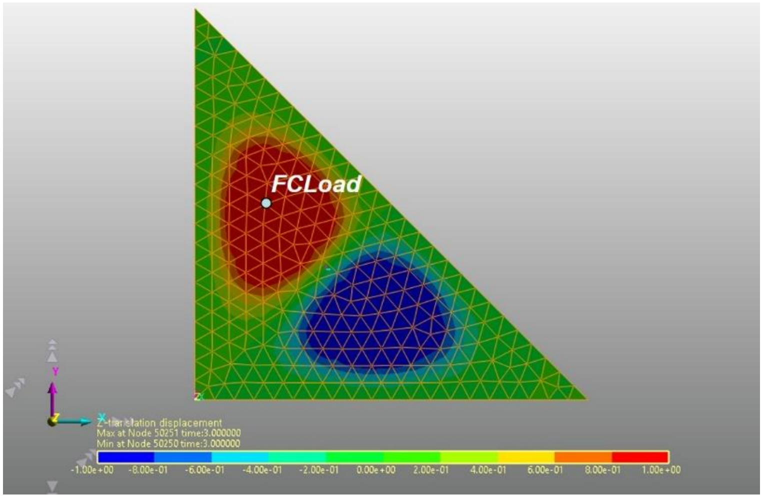
Numerical Solution - RecurDyn

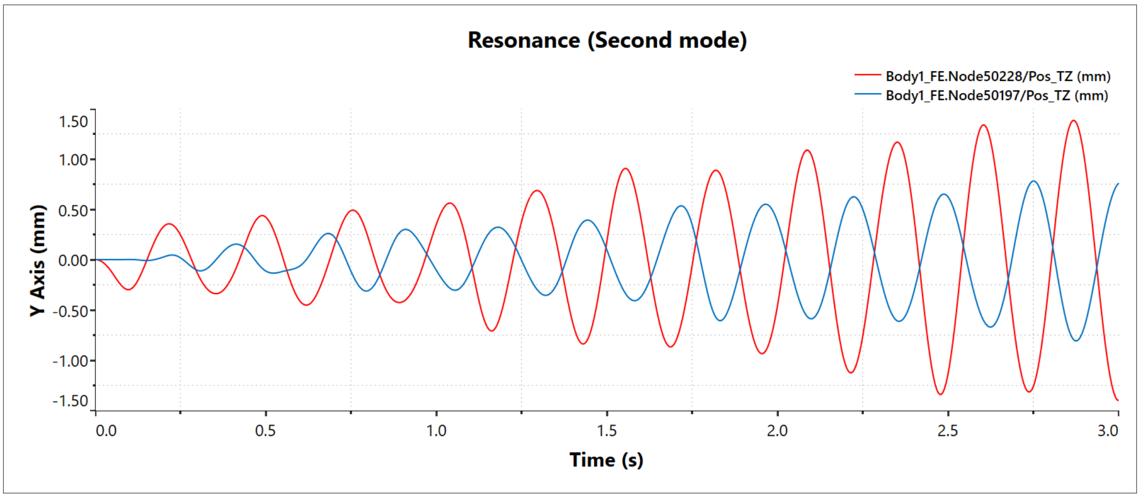
- First mode frequency = 2.26



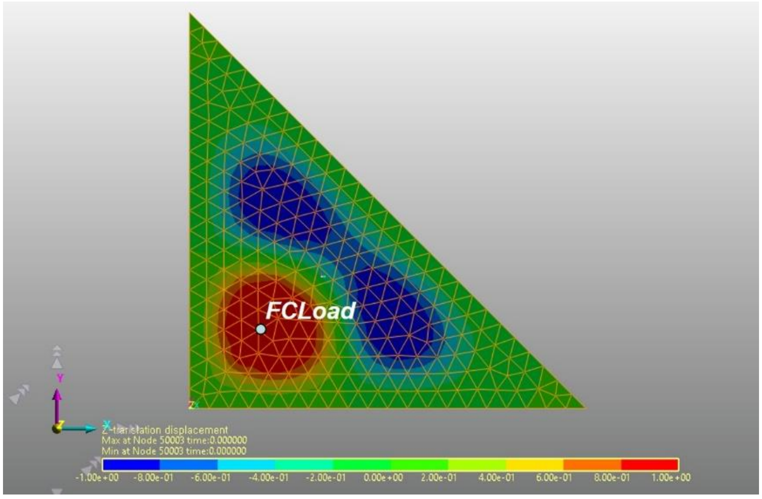


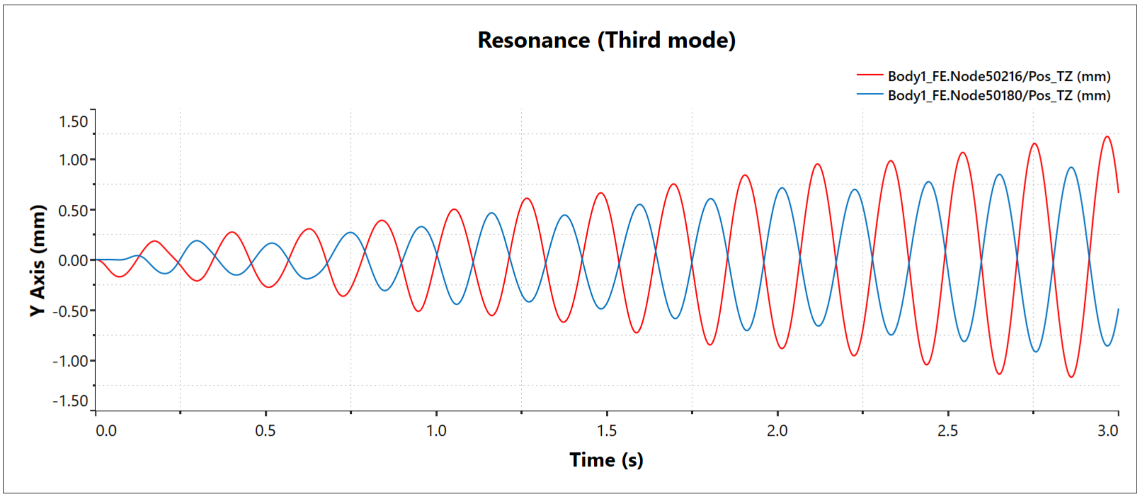
➢ Second mode frequency = 3.87



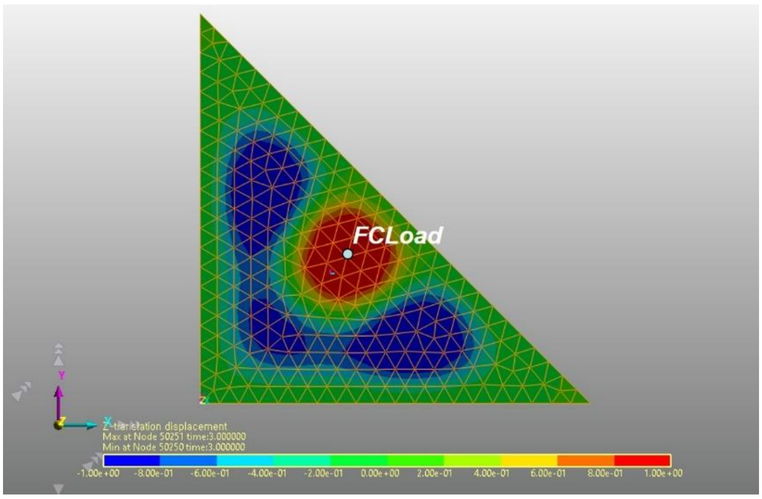


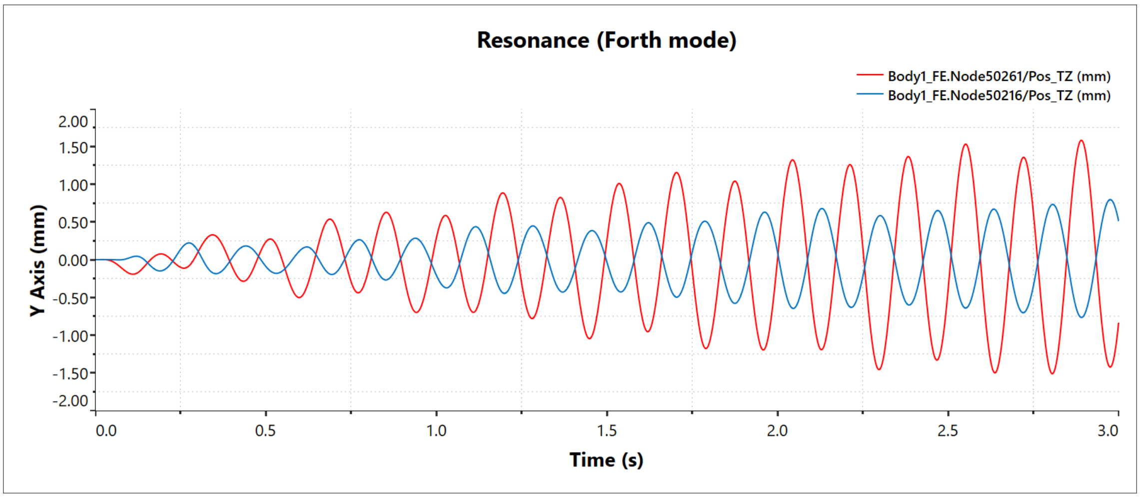
➢ Third mode frequency = 4.67





➢ Forth mode frequency = 5.82



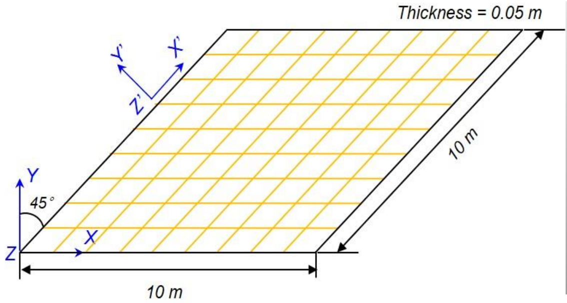


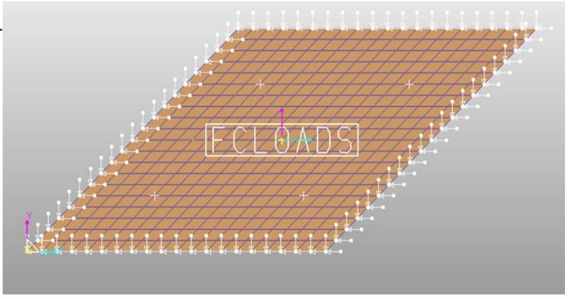
Resonance (Forth mode)
Body1_FE.Node50261/Pos_TZ (mm)
Body1_FE.Node50216/Pos_TZ (mm)
Y Axis (mm)
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time (s)

Rhombic shape of rectangular sheet

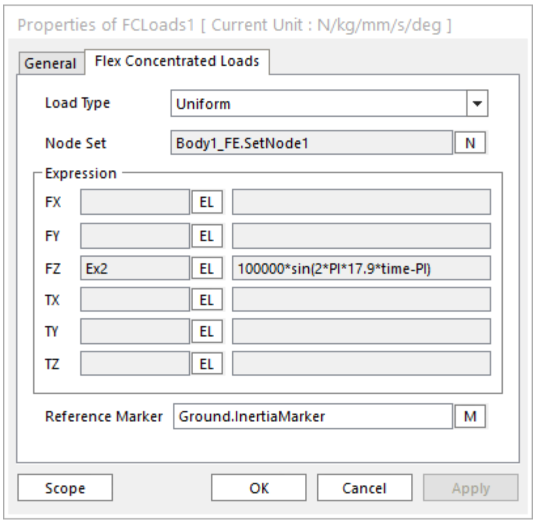
Here, the natural frequency of rhombic plate sheet is verified with the NAFEMS document. (“The Standard NAFEMS Benchmarks”, NAFEMS, 1990.)

- The plate has a rhombic shape with a 45-degree angle.
- All four of edges are fixed with BC boundary condition on the Ground.
- The flexible sheet consists of shell 4 element and the number of elements is set to 400.





- The concentrated load in the z axis is applied at the center as periodical force like below.



Modeling parameter

Given	Symbol	Value	Unit

Width of Sheet	w	10000	mm
Height of Sheet	h	10000	mm
Sheet Thickness	T	50	mm
Young's modulus	E	200000	MPa
Shear modulus	G	76923	Mpa
Poisson's Ratio	nu	0.3	-
Density	ρ	8.0e-6	kg/mm^3

Theoretical Solution

- First critical frequency (λ = 65.93)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

$$= \frac{65.93}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 50^3}{12 \times 8.0\mathrm{e}-6 \times 50 \times (1-0.3^2)}} = 7.938$$

- 2nd critical frequency (λ = 106.6)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

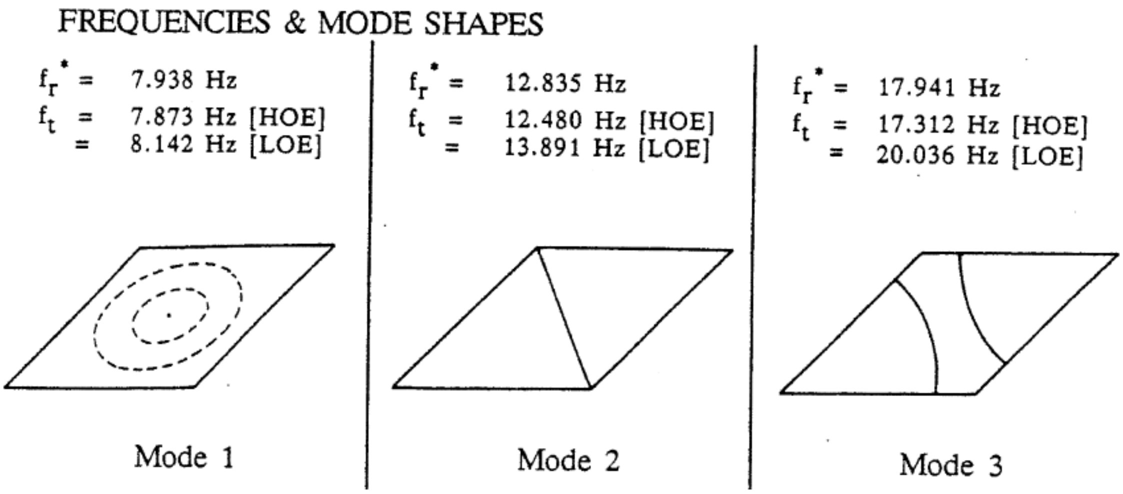
$$= \frac{106.6}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 50^3}{12 \times 8.0\mathrm{e}-6 \times 50 \times (1-0.3^2)}} = 12.835$$

- 3rd critical frequency (λ = 149)

$$f_{n1} = \frac{\lambda}{2\pi w^2}\sqrt{\frac{ET^3}{12\rho T(1-nu^2)}}$$

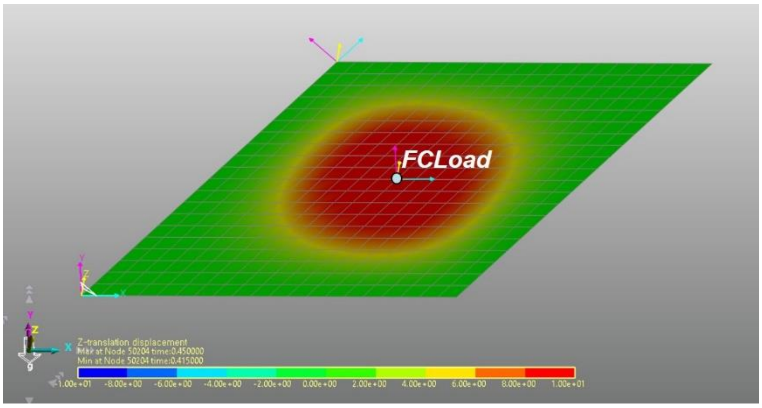
$$= \frac{149}{2\pi \times 10000^2}\sqrt{\frac{200000000 \times 50^3}{12 \times 8.0\mathrm{e}-6 \times 50 \times (1-0.3^2)}} = 17.941$$

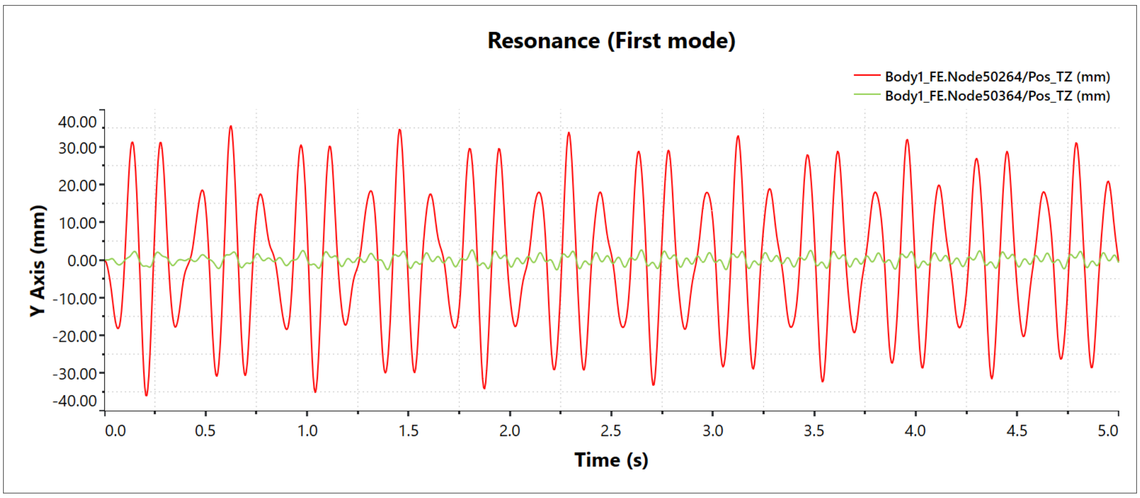
Natural frequencies with mode of 'NAFEMS document'



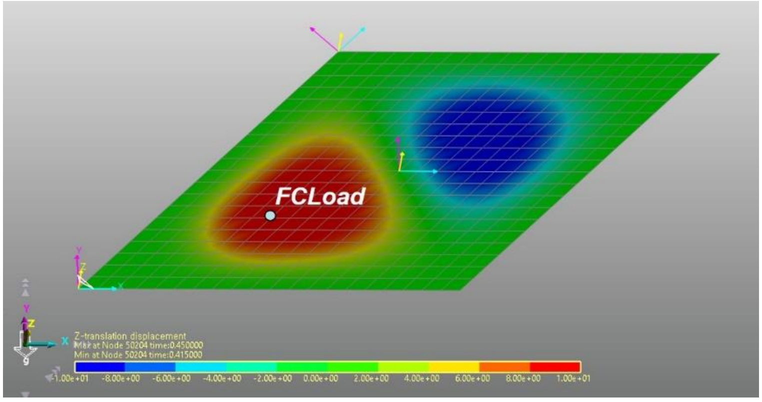
Numerical Solution - RecurDyn

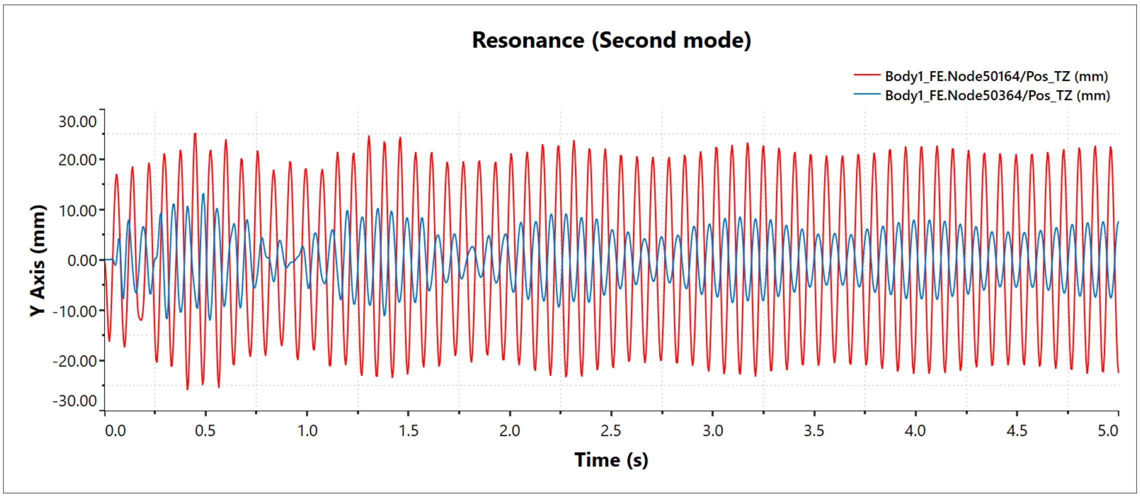
- First mode frequency = 7.938



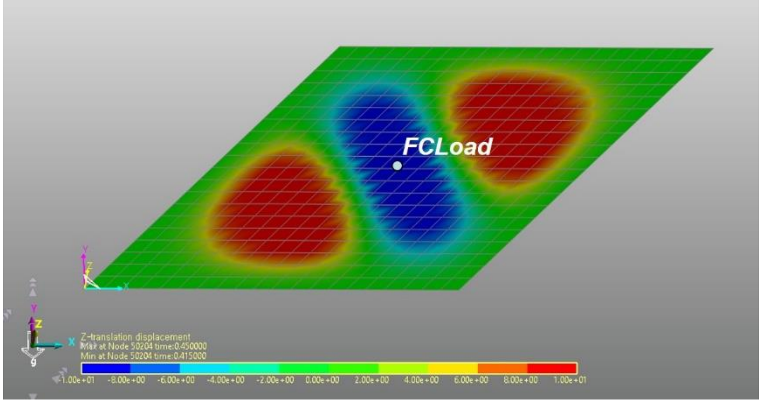


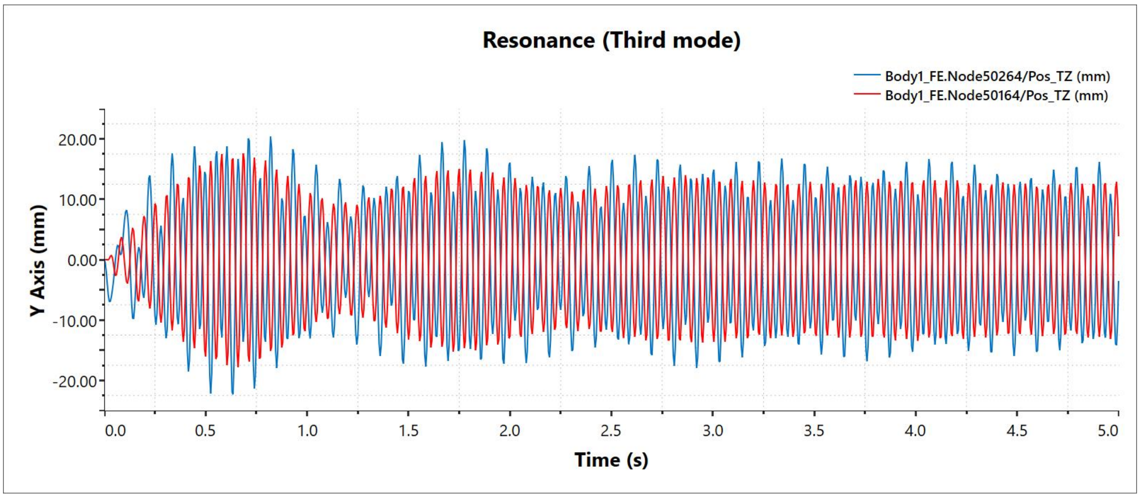
➢ Second mode frequency = 12.835





➢ Third mode frequency = 17.941





Resonance (Third mode)
Body1_FE.Node50264/Pos_TZ (mm)
Body1_FE.Node50164/Pos_TZ (mm)
Y Axis (mm)
20.00
10.00
0.00
-10.00
-20.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Time (s)

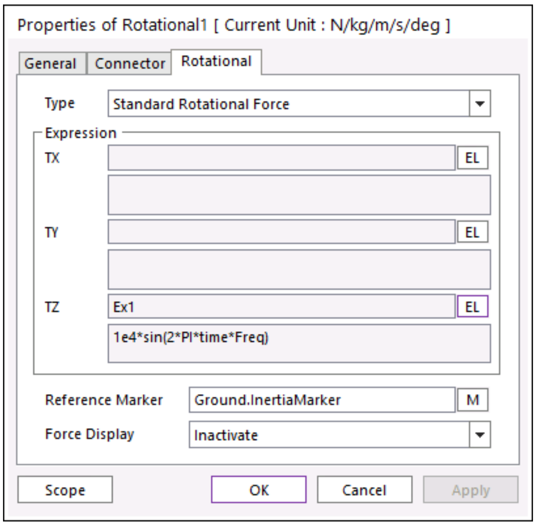
Pin-ended double cross

Here, the natural frequency of 'pin-ended double cross' is verified with the NAFEMS document. ("The Standard NAFEMS Benchmarks", NAFEMS, 1990.)

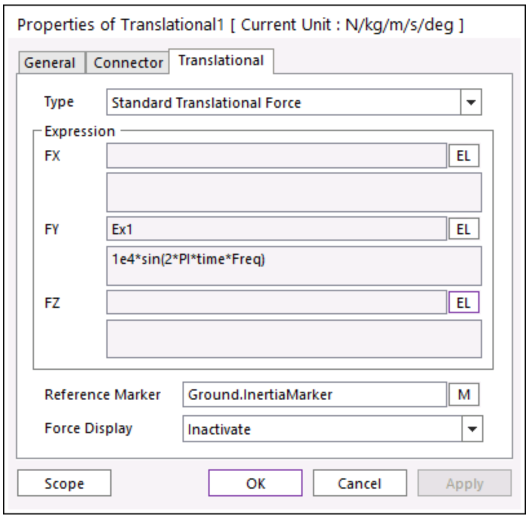
- The geometry, model parameters, boundary conditions, and theorical solutions are presented at the above chapter

- The periodical load is applied to find the resonant frequency

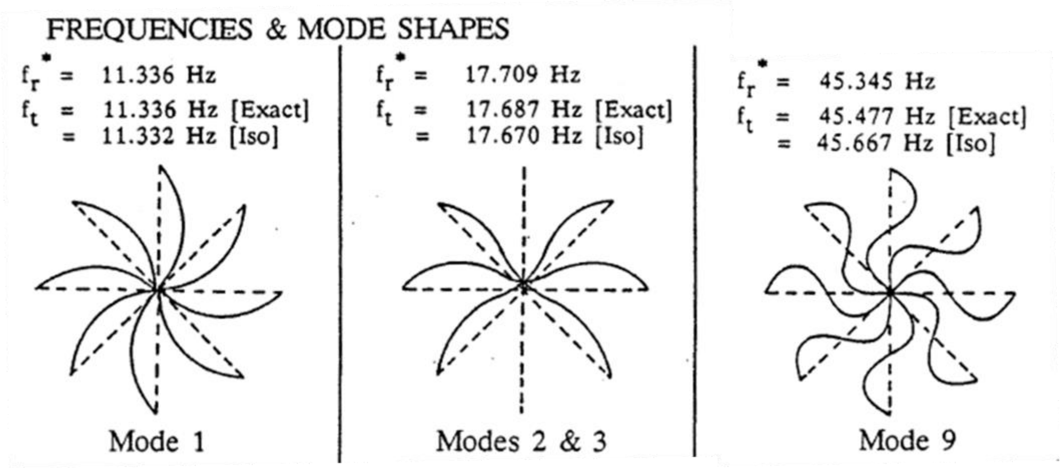
  - Rotational force



  - Translational force

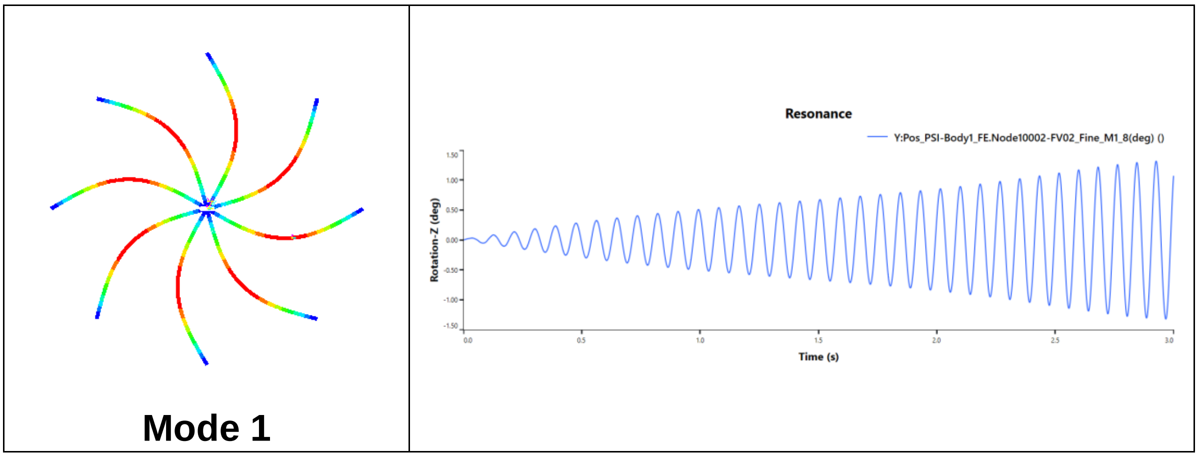


Natural frequencies with mode of ‘NAFEMS document’

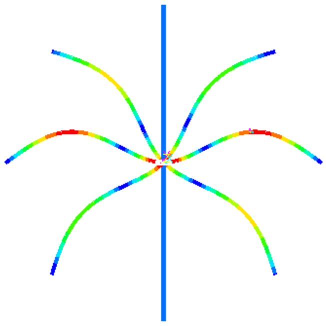


Numerical Solution - RecurDyn

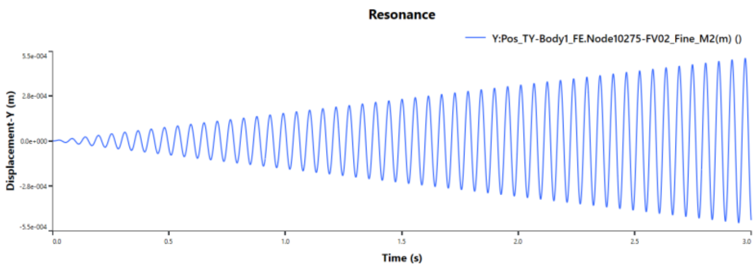
➢ First mode frequency = 11.719



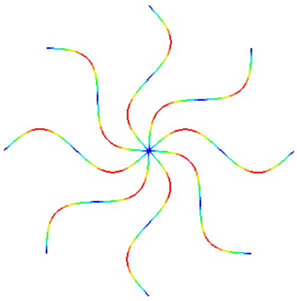
➢ Second mode frequency = 17.65



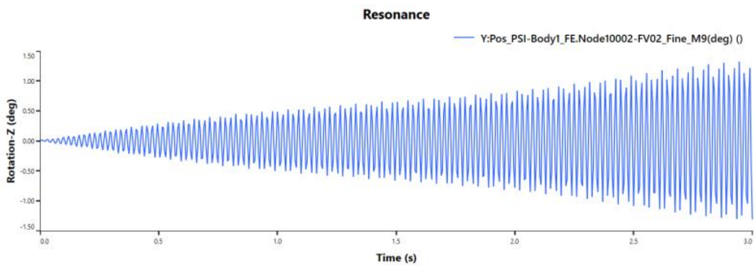
Mode 2&3



➢ Third mode frequency = 45.596



Mode 9



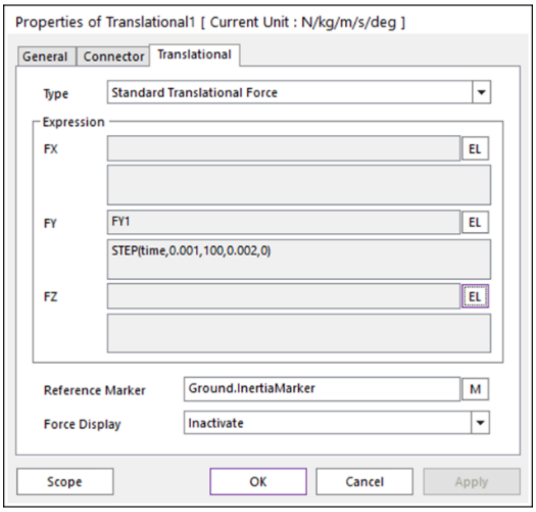
Deep simply-supported beam

The natural frequency of 'deep simply-supported beam' is verified with the NAFEMS document. ("The Standard NAFEMS Benchmarks", NAFEMS, 1990.)

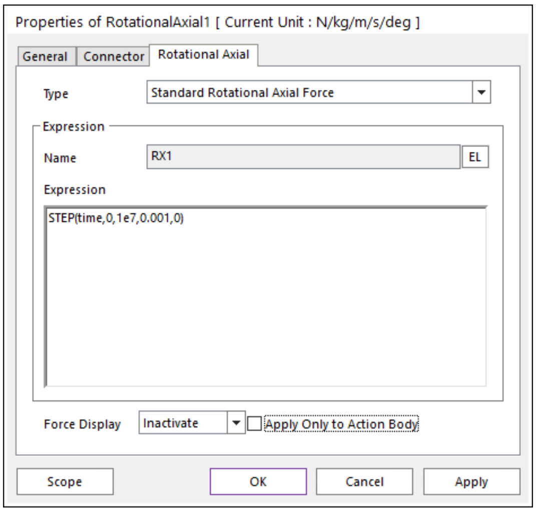
- The geometry, model parameters, boundary conditions, and theorical solutions are presented at the above chapter

- The periodical load is applied to find the resonant frequency

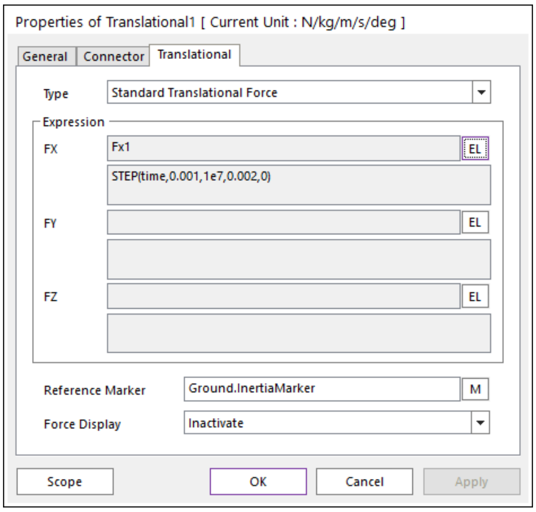
    - Translational force – y-direction



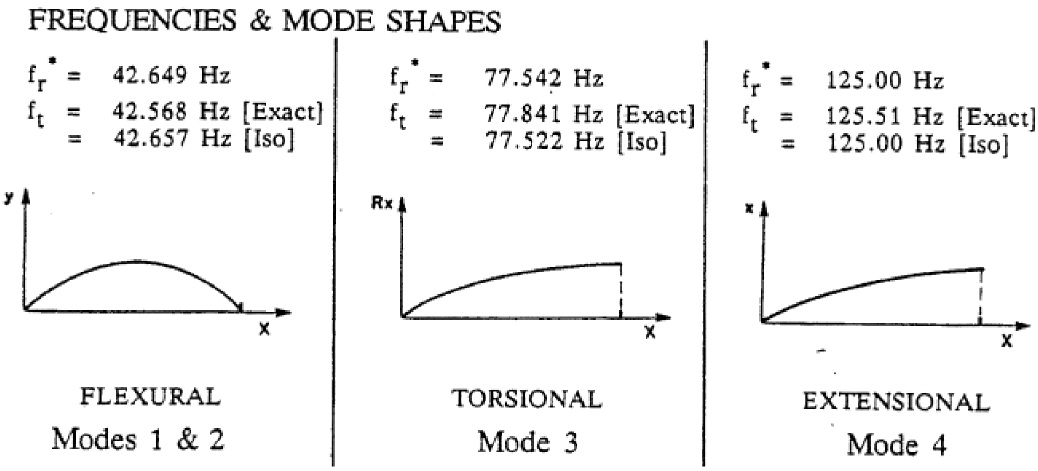
    - Rotational force



- Translational force – x-direction



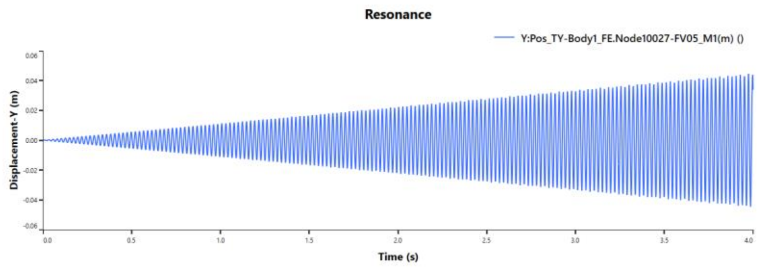
Natural frequencies with mode of 'NAFEMS document'



Numerical Solution - RecurDyn

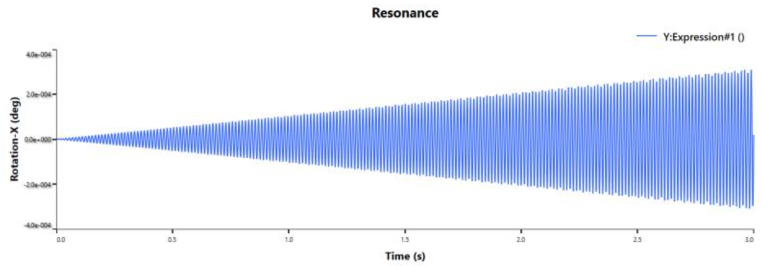
- First mode frequency = 43.15

Mode 1&2



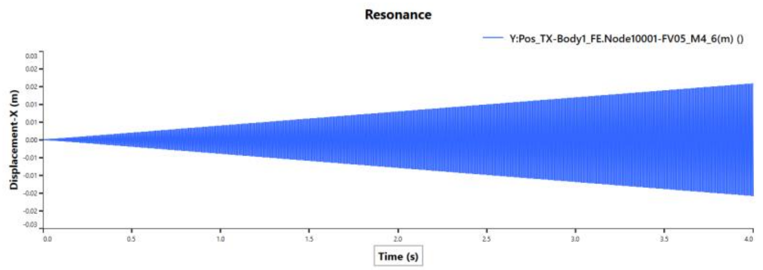
➢ Second mode frequency = 71.26

Mode 3 (DRX)



➢ Third mode frequency = 125.00

Mode 4 (DTX)



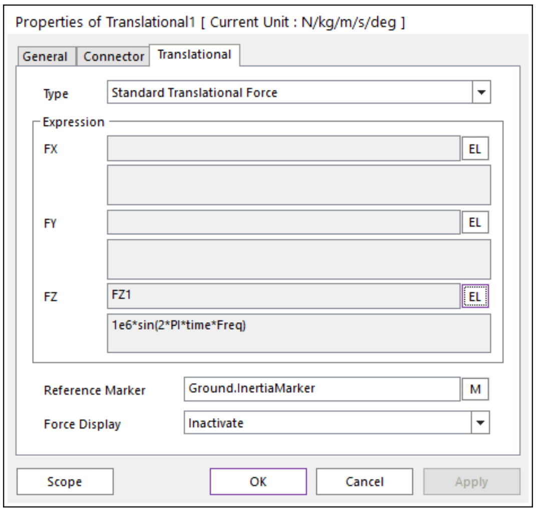
Simply-supported ‘solid’ square

The natural frequency of ‘simply-supported ‘solid’ square’ is verified with the NAFEMS document. (“The Standard NAFEMS Benchmarks”, NAFEMS, 1990.)

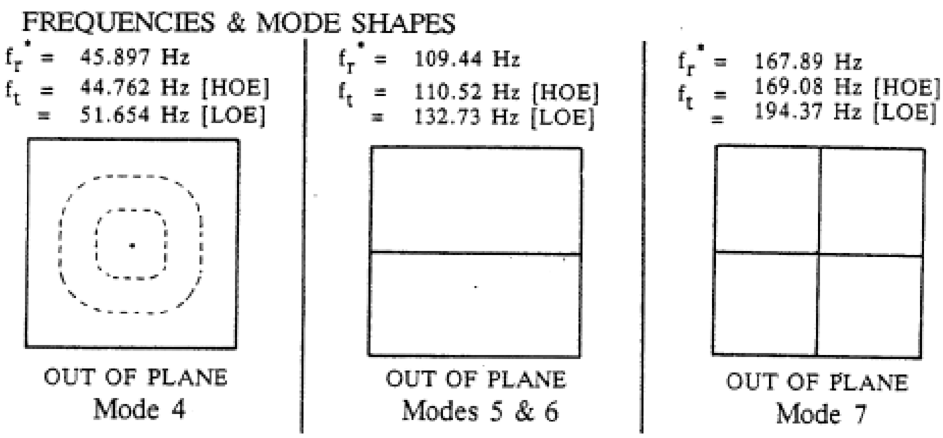
- The geometry, model parameters, boundary conditions, and theorical solutions are presented at the above chapter

- The periodical load is applied to find the resonant frequency

  - Translational force

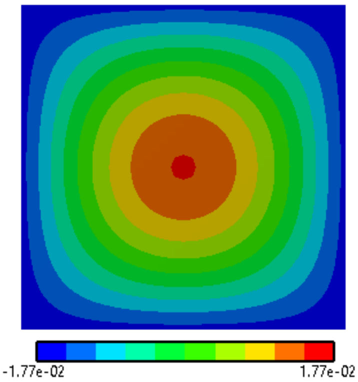


⭘ Natural frequencies with mode of ‘NAFEMS document’

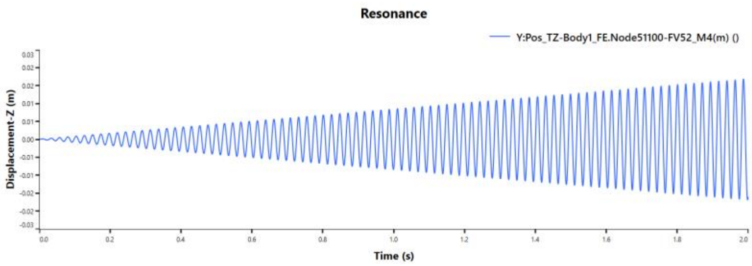


⭘ Numerical Solution - RecurDyn

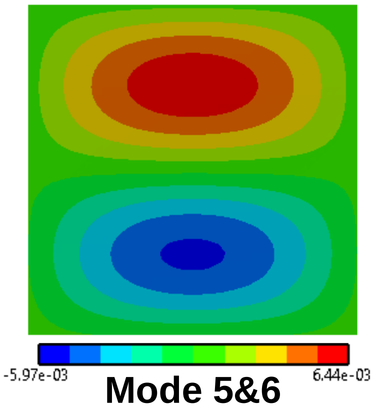
➢ First mode frequency = 43.02



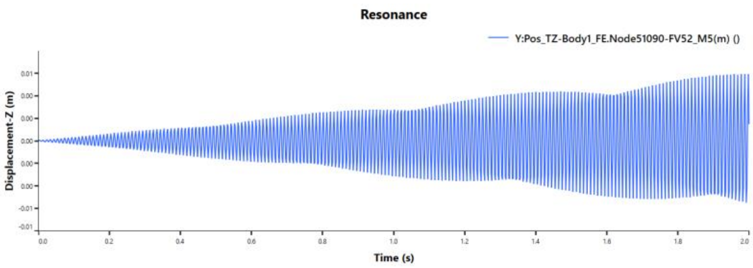
Mode 4



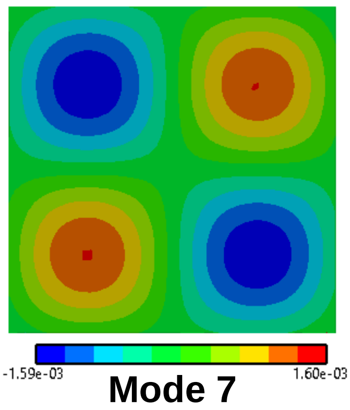
➢ Second mode frequency = 100.4

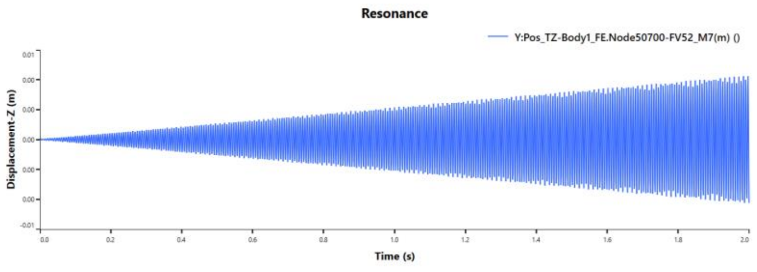


Mode 5&6



➢ Third mode frequency = 154.49


-1.59e-03
1.60e-03
Mode 7


Resonance
Y:Pos_TZ-Body1_FE.Node50700-FV52_M7(m) ()
Displacement-Z (m)
0.01
0.00
-0.01
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Time (s)