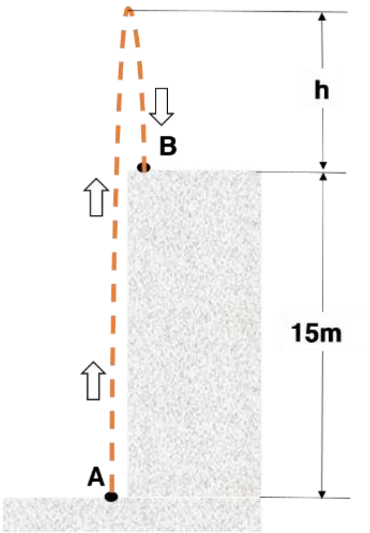
Validation

Rigid Body Dynamics

Dynamics of Rigid Bodies.01



References: Engineering Mechanics: Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.31, P.2.13
Type of Analysis: Kinematics of particles, Linear Motion
Type of Element: Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Initial height	s_0	0	m
Initial velocity	v_0	25	m/s

1. Theoretical Solution

$$a = \frac{dV}{dt}\ ,\quad \int_{V_0}^{V} dV = \int_{t_0}^{t} adt\ ,\quad V = V_0 + at\ \ (a = const)$$

$$V = \frac{dS}{dt}\ ,\quad \int_{S_0}^{S} dS = \int_{t_0}^{t} (V_0 + at)dt\ ,\quad S = S_0 + V_0 t + \frac{1}{2}at^2\ \ (a = const)$$

$$dt = \frac{dV}{a} = \frac{dS}{V}\ ,\quad \int_{V_0}^{V} VdV = \int_{S_0}^{S} adS\ ,\quad V^2 - {V_0}^2 = 2aS\ \ (a = const)$$

2. Time Required to Reach Maximum Height

$$V = V_0 + at$$

$$0 = 25 - 9.807 \cdot \mathrm{t}$$

$$\mathrm{t} = 2.549\ [\mathrm{s}]$$

3. Maximum Height

$$S = S_0 + V_0 t + \frac{1}{2}at^2$$

$$= 0 + 25 \times 2.549 - \frac{1}{2} \times 9.807 \times 2.549^2$$

$$= 31.865\ \mathrm{m}$$

4. Calculation of Height (h)

$$h = S - 15 = 16.865\ m$$

5. Calculation of Velocity (V_B) and Time Required to Drop to Point "B"

$$V_T = 0$$

$${V_B}^2 - {V_T}^2 = 2ah$$

$${V_B}^2 - 0 = 2 \times 9.807 \times 16.865$$

$$V_B = 18.188$$

$$V_B = V_T + at$$

$$18.188 = 0 + 9.807 \times t$$

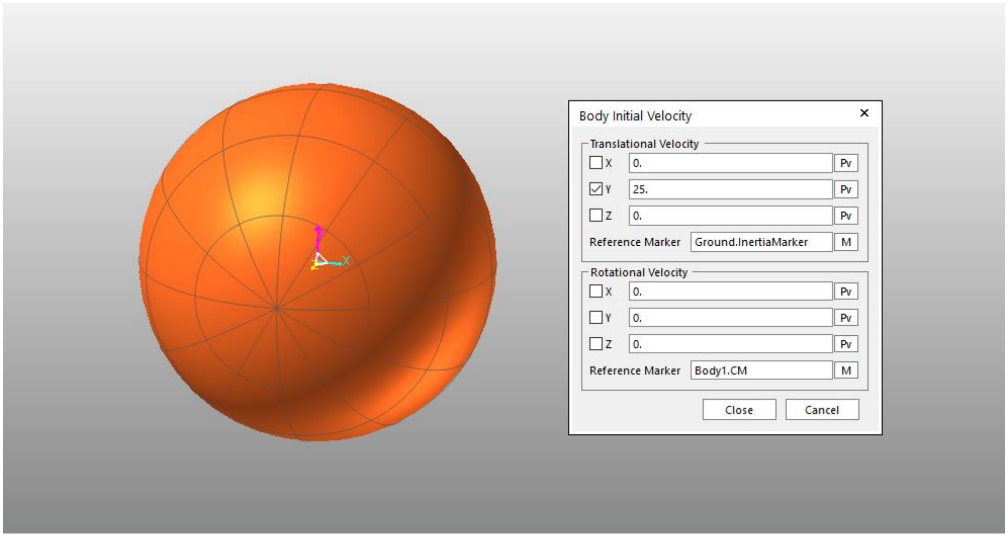
$$\mathrm{t} = 1.855\ \mathrm{s}$$

6. Time Required to Reach Point "B" after Launch

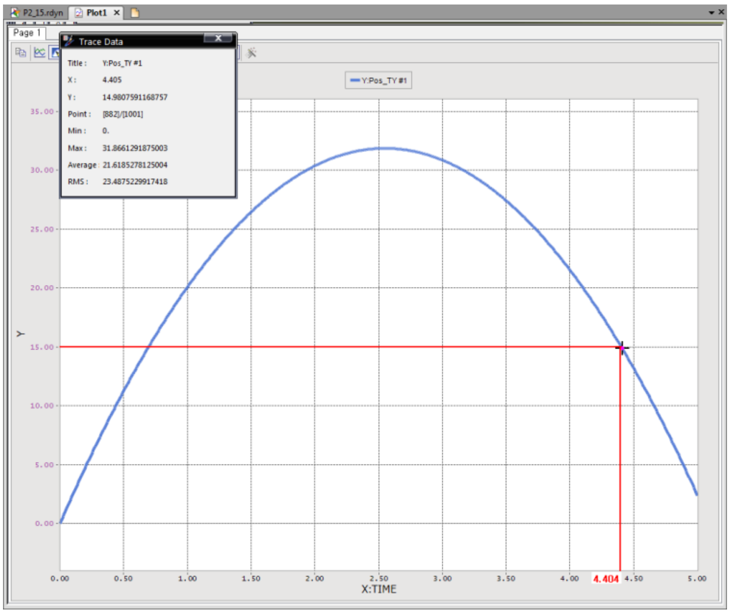
$$t_{A-B} = 4.404\ s$$

⭘ Numerical Solution - RecurDyn

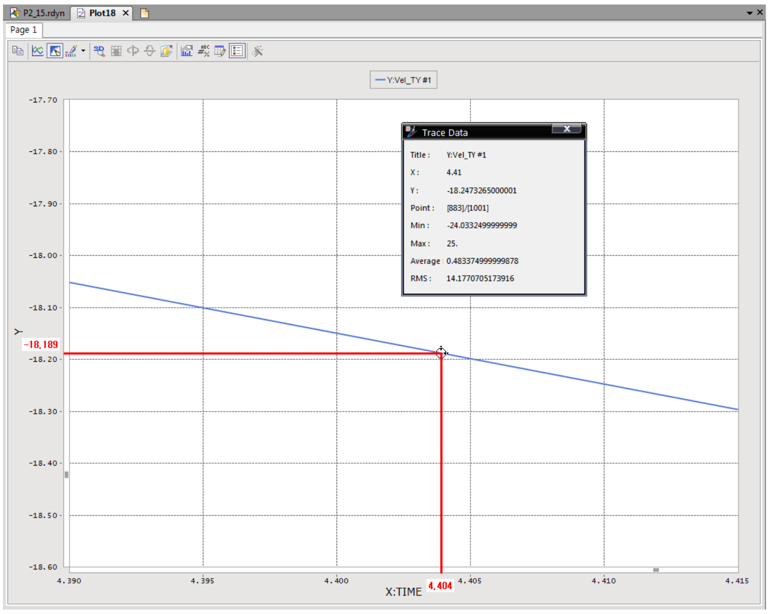
1. Body modeling and Set initial velocity



2. Plot the results
   - Measure the time when the ball reaches a height of 15 *m*



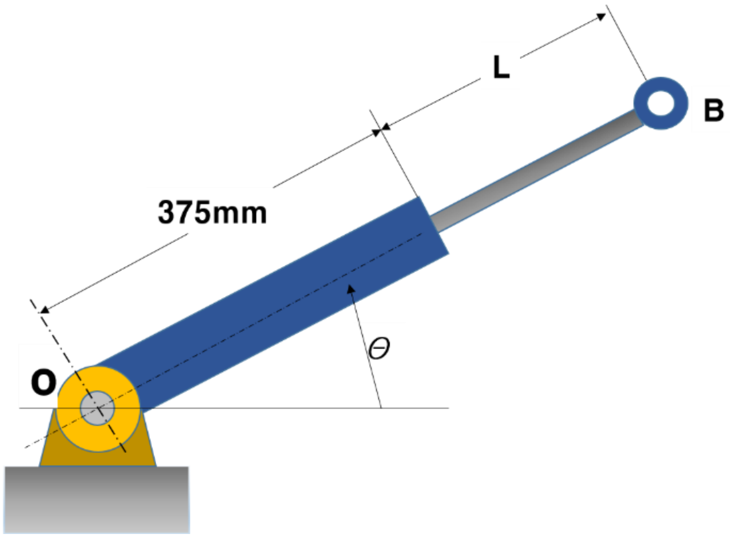
   - Measure the velocity of the ball when the time is 4.404



● Comparison of results

Object Value	Theory	RecurDyn	Error(%)
h $[m]$	16.865	16.866	0.006
$\mathbf{t}$ $[s]$	4.404	4.404	0
v_s $[m/s]$	18.188	18.189	0.005

Dynamics of Rigid Bodies.02



Reference of Problem : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.72, P.2.136
Type of Analysis : Kinematics of Particles, Polar Coordinates
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Initial length	r	500	mm
Decreasing rate	$\dot{r}$	-150	mm/s
Rotation rate	$\dot{\theta}$	60	deg/s

1. Velocity of Point B

$$v_r = \dot{r} = -150\ mm/s$$

$$v_\theta = r\dot{\theta} = 500 \cdot \frac{\pi}{3} = 523.6\ mm/s$$

$$v = \sqrt{({v_r}^2 + {v_\theta}^2)} = 544.7\ mm/s$$

2. Acceleration of the Point B

$$a_r = \ddot{r} - r\dot{\theta}^2 = -548.31\ mm/s^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -314.16\ mm/s^2$$

$$\mathrm{a} = \sqrt{(a_r{}^2 + a_\theta{}^2)} = 631.93\ mm/s^2$$

✓ Note

$$\dot{e_r} = \dot{\theta} e_\theta \quad , \quad \dot{e_\theta} = -\dot{\theta} e_r$$

$$\vec{v} = \dot{\vec{r}} = \dot{r} e_r + r\dot{e_r} = \dot{r} e_r + r\dot{\theta} e_\theta$$

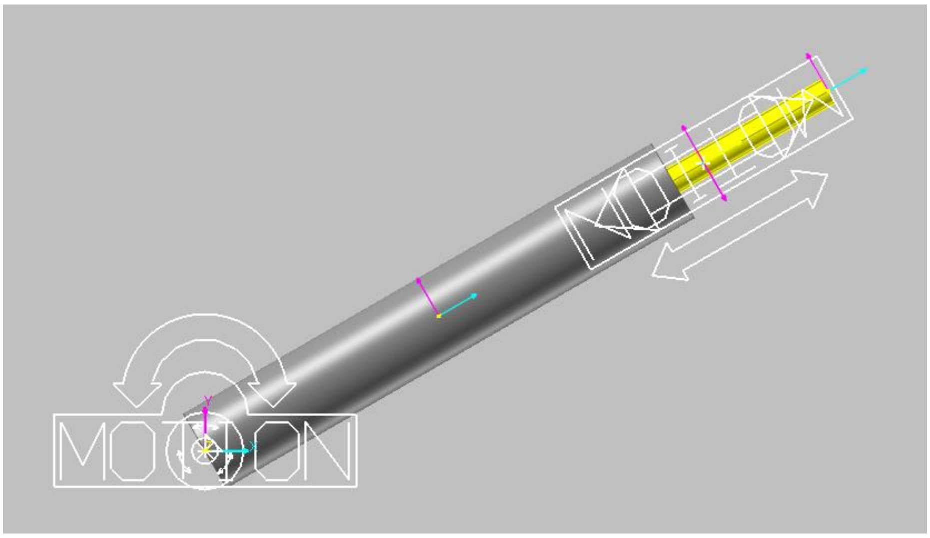
$$\vec{a} = \dot{\vec{v}} = \ddot{r} e_r + \dot{r}\dot{e_r} + \dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta + r\dot{\theta}\dot{e_\theta}$$

$$= \ddot{r} e_r + \dot{r}\dot{\theta} e_\theta + \dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta - r\dot{\theta}^2 e_r$$

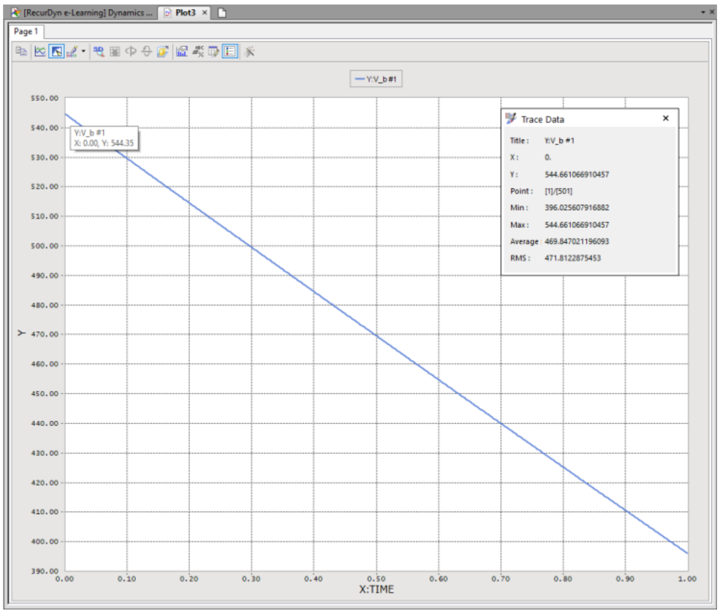
$$= \left(\ddot{r} - r\dot{\theta}^2\right) e_r + (2\dot{r}\dot{\theta} + r\ddot{\theta}) e_\theta$$

⭘ Numerical Solution – RecurDyn

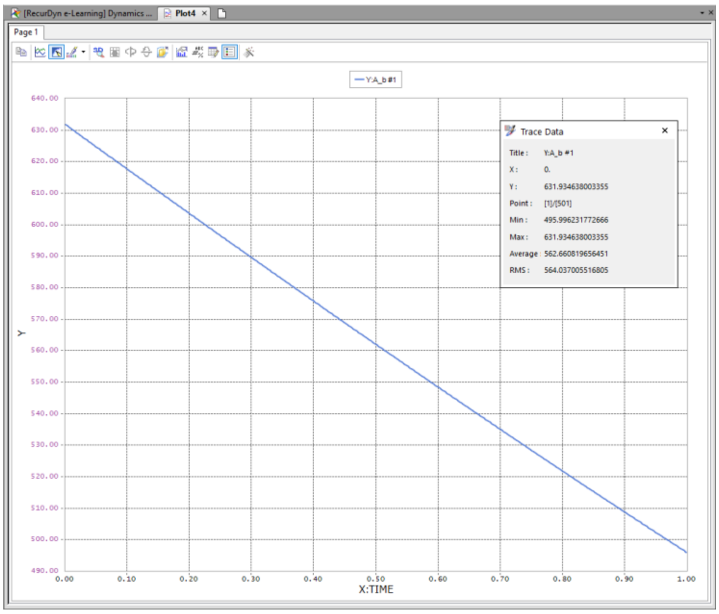
1. Modeling



2. Plot the results
   - Velocity of Point “B”



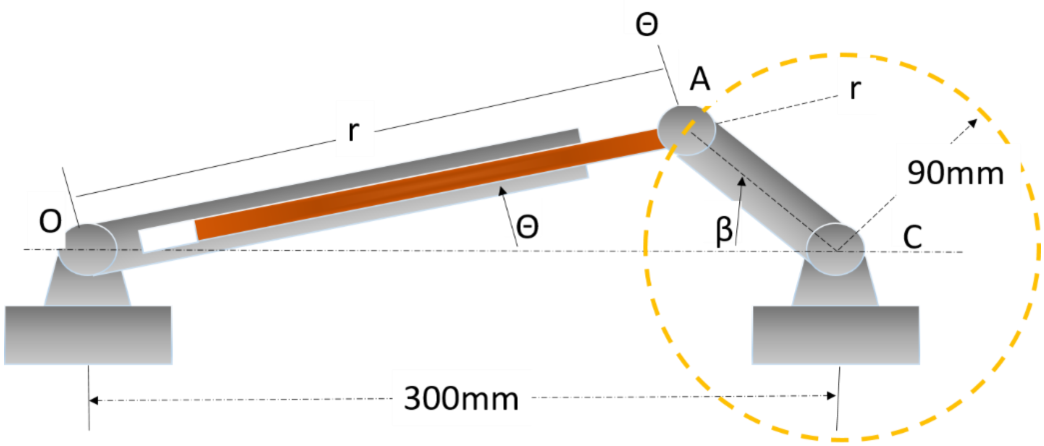
- Acceleration of Point “B”



○ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$v\ [m/s]$	0.5447	0.5447	0
a $[m/s^2]$	0.63193	0.63193	0

Dynamics of Rigid Bodies.03



Reference : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.77, P.2.161
Type of Analysis : Kinematics of Particles, Polar Coordinates
Type of Element : Particle (One part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Initial angle	β	30	deg
Angular velocity	$\dot{\beta}$	60	rad/s

1. Geometrical Constraint

$$r \cdot cos\theta + 90 \cdot cos\beta = 300$$
$$r \cdot cos\theta = 300 - 90 \cdot cos\beta = 222.06$$

$$r \cdot sin\theta = 90 \cdot \sin\beta = 45$$
$$r \cdot cos\theta = 300 - 90 \cdot \cos\beta = 222.06$$
$$\therefore tan\theta = \frac{45}{222.06} \rightarrow \theta = 11.46°$$

$\therefore r = 226.57$

$\therefore \gamma = \beta + \theta = 30 + 11.46 = 41.46°$

2. Velocity and Acceleration of Point A of Link AC

$v = 90 \cdot \dot{\beta} = 5400\, mm/s$

$a = 90 \cdot \dot{\beta}^2 = 324 \cdot 10^3\, mm^2/s$

3. Calculate $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$ with the Velocity and Acceleration

$v_\theta = r\dot{\theta} = v \cdot cos\gamma = 5400 \cdot cos41.46 = 4046.86\, mm/s$

$\dot{\theta} = \frac{4046.86}{226.57} = 17.86\, rad/s$

$v_r = \dot{r} = v \cdot sin\gamma = 5400 \cdot sin41.46 = 3575.32\, mm/s$

$\dot{r} = 3575.32\, mm/s$

$a_r = \ddot{r} - r\dot{\theta}^2 = a \cdot cos\gamma$

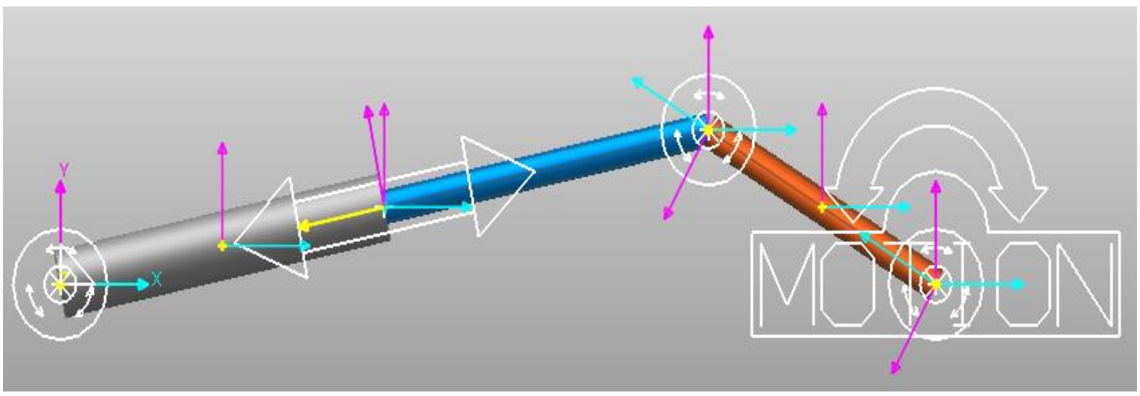
$\ddot{r} = r\dot{\theta}^2 + a \cdot cos\gamma = 226.57 \cdot 17.86^2 + 324 \cdot 10^3 \cdot cos41.46 = 315{,}082.68\, mm/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -a \cdot sin\gamma$

$\ddot{\theta} = \frac{-a \cdot sin\gamma - 2\dot{r}\dot{\theta}}{r} = \frac{(-324 \cdot 10^3 \cdot sin41.46 - 2 \cdot 3575.32 \cdot 17.86)}{226.57} = -1{,}510.48\, rad/s^2$

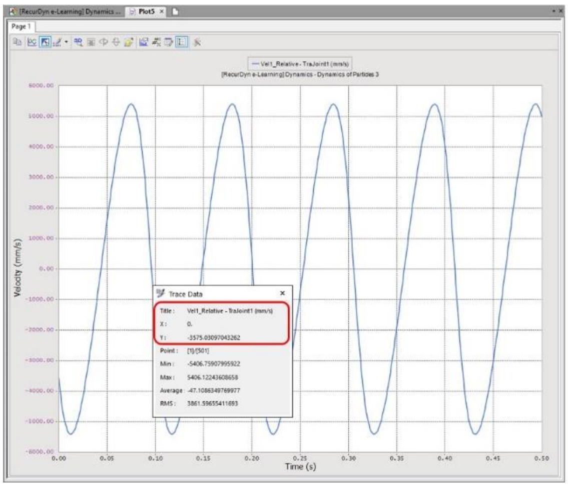
⭘ Numerical Solution - RecurDyn

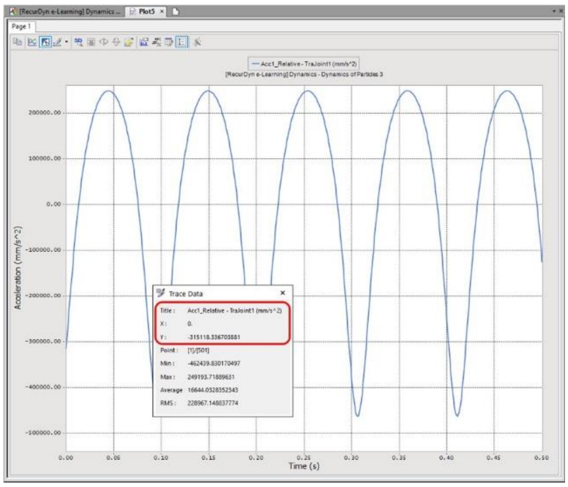
1. Modeling



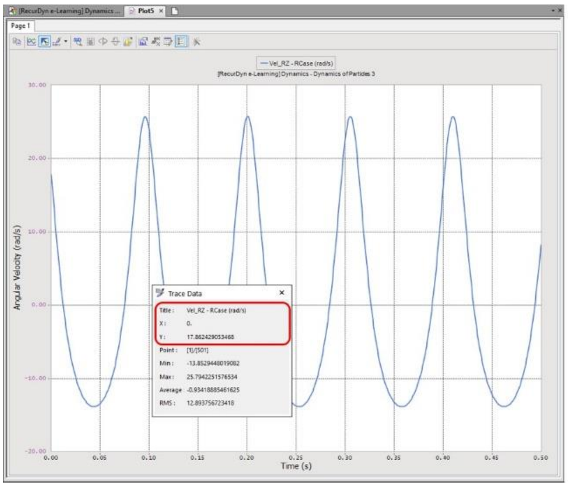
2. Plot the results

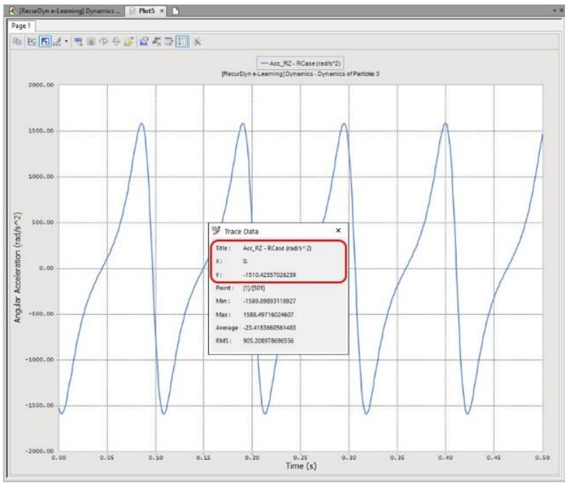
- The relative velocity and acceleration





- The angular velocity of acceleration of the "RCase"

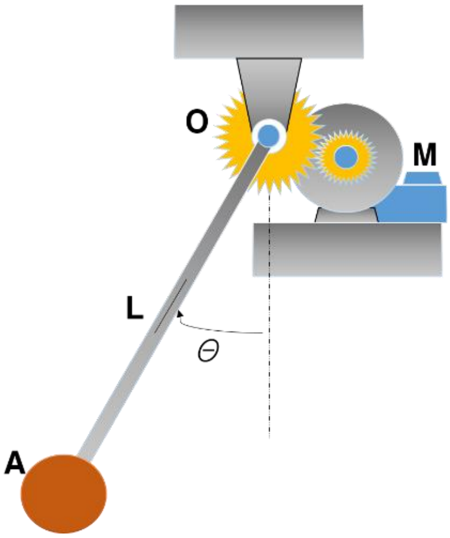




Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\mathbf{r}$	3.58	3.58	0
$\dot{\boldsymbol{r}}$	315	315.12	0.038
$\dot{\boldsymbol{\theta}}$	17.88	17.86	0.112
$\ddot{\boldsymbol{\theta}}$	-1510	-1510.43	0.028

Dynamics of Rigid Bodies.04



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.114, P.2.253
Type of Analysis : Kinematics of particles, Linear Motion
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Initial angle	θ_0	$\pi/3$	rad
"OA" Length	l	0.8	m

Where, $\omega_n = \sqrt{\frac{g}{l}}$

$\theta = \theta_0 \cdot sin\omega_n t$

$\dot{\theta} = \theta_0 \cdot \omega_n \cdot cos\omega_n t$

$\ddot{\theta} = -\theta_0 \cdot {\omega_n}^2 \cdot sin\omega_n t$

A tangential acceleration a_t of the pendulum OA is,

$a_t = l \cdot \ddot{\theta} = -l \cdot \theta_0 \cdot {\omega_n}^2 \cdot sin\omega_n t = -l \cdot \theta_0 \cdot \frac{g}{l} \cdot sin\omega_n t = -\theta_0 \cdot g \cdot sin\omega_n t$

A normal acceleration a_n is,

$a_n = l \cdot \dot{\theta}^2 = l \cdot {\theta_0}^2 \cdot {\omega_n}^2 \cdot cos^2\omega_n t = l \cdot {\theta_0}^2 \cdot \frac{g}{l} \cdot cos^2\omega_n t = {\theta_0}^2 \cdot g \cdot cos^2\omega_n t$

Thus, the acceleration of the pendulum OA is,

$\mathrm{a} = \sqrt{({a_t}^2 + {a_n}^2)} = g\theta_0 \cdot \sqrt{(sin^2\omega_n t + {\theta_0}^2 \cdot cos^4\omega_n t)}$

A Period T of the pendulum OA is,

$T/4$ = 0.4486, where $\mathrm{T} = \frac{2\pi}{\omega_n} = 1.7943$.

Calculate the extreme value,

$\frac{da}{dt} = g\theta_0 \cdot \frac{2\omega_n \cdot sin\omega_n t \cdot cos\omega_n t - 4{\theta_0}^2 \cdot \omega_n \cdot cos^3\omega_n t \cdot sin\omega_n t}{2\sqrt{(sin^2\omega_n t + {\theta_0}^2 \cdot cos^4\omega_n t)}} = 0$

⇨ $g\theta_0 \cdot 2\omega_n \cdot sin\omega_n t \cdot cos\omega_n t \cdot \left(1 - 2{\theta_0}^2 \cdot cos^2\omega_n t\right) = 0$

⇨ $1 - 2{\theta_0}^2 \cdot cos^2\omega_n t = 0$

⇨ $cos^2\omega_n t = \frac{1}{2{\theta_0}^2}$

⇨ $cos\omega_n t = \pm\sqrt{\frac{1}{2{\theta_0}^2}} = \pm 0.67524$

⇨ $\therefore t = 0.23688 \;\; or \;\; 0.66026\, s$

During the 1/4 period, calculate the "a" value to find the maximum value and the minimum value when the time is 0, 0.23688, and 0.4468,

1) t=0

$\theta = 0°$

$a = 10.76\ m/s^2$ *: the maximum value*

2) t=0.23688

$\theta = 0.77241\, rad = 44.26°$

$a = 9.026\ m/s^2$ *: the minimum value*

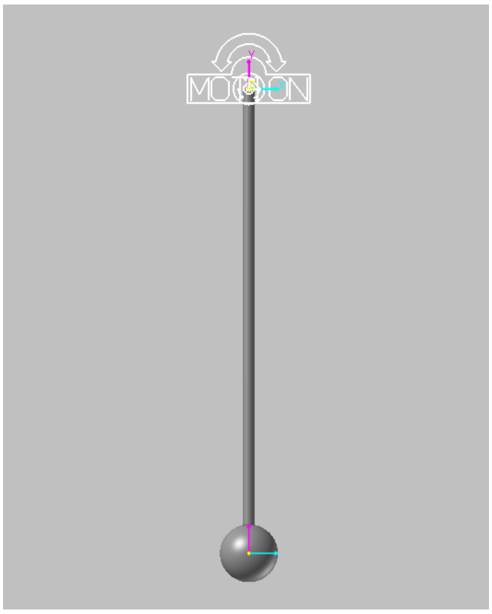
3) t=0.4468

$\theta = 1.04718\, rad = 60.0°$

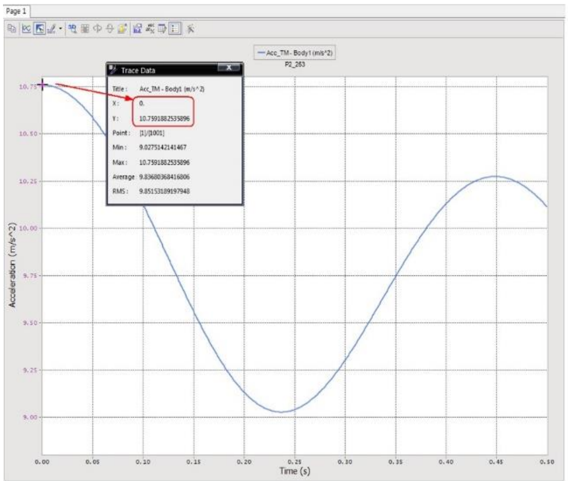
$a = 10.27\ m/s^2$

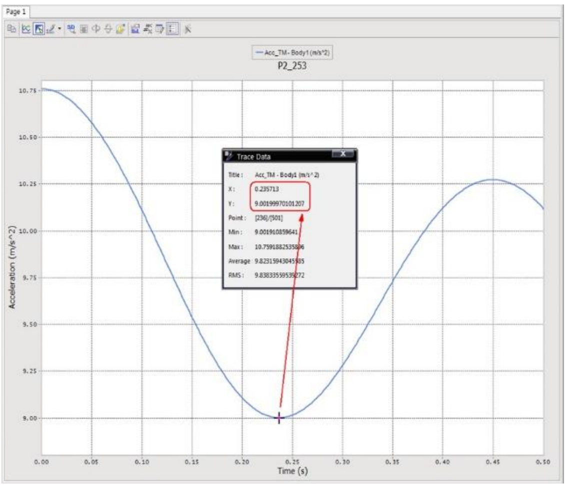
Numerical Solution - RecurDyn

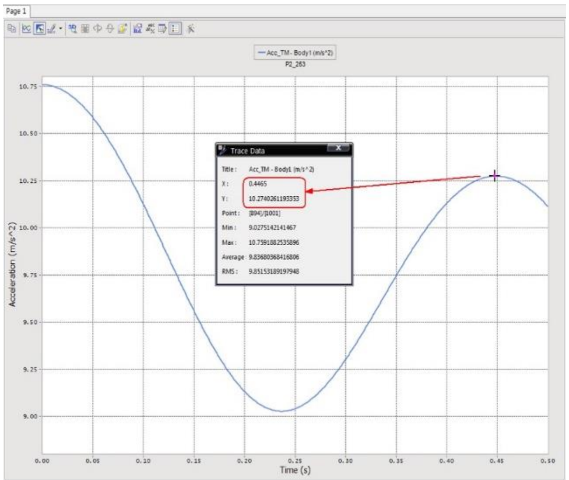
1. Modeling



2. Plot the results
   - Measure the acceleration when t=0, t≈0.23688 and t≈0.4468



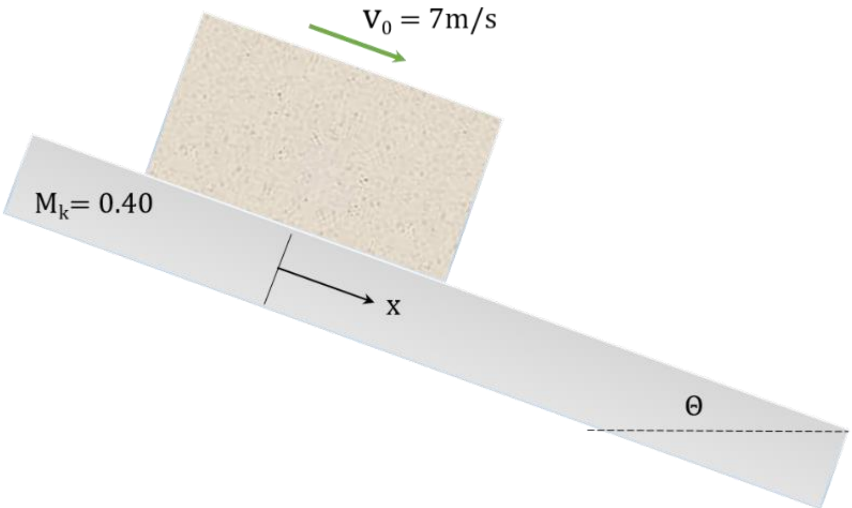




Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$a_{min}\ [m/s^2]$	9.026	9.002	0.266
$t[s], \theta[deg]$	0.23688, 44.26°	0.23688, 44.26°	0
$a_{max}\ [m/s^2]$	10.76	10.76	0
$t[s], \theta[deg]$	0.0, 0°	0.0, 0°	0

Dynamics of Rigid Bodies.05



References: Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.129, P.3.2
Type of Analysis : Kinetics of particles, Linear Motion
Type of Element : Particle (one part)

Theoretical Solution

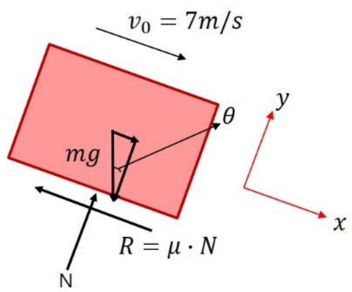
Basic Conditions

Given	Symbol	Value	Unit
Initial velocity	v_0	7	m/s
Initial angle	θ	15, 30	deg
Mass	m	50	kg
Friction coefficient	μ	0.40	-

$$\sum F_y = N - m \cdot g \cdot cos\theta = 0$$

$$N = m \cdot g \cdot cos\theta$$

$$R = \mu \cdot m \cdot g \cdot cos\theta$$



$$\sum F_x = m \cdot g \cdot sin\theta - \mu \cdot m \cdot g \cdot cos\theta = m \cdot a$$

$$\therefore a = g \cdot sin\theta - \mu \cdot g \cdot cos\theta$$

(1) $\theta = 15°$

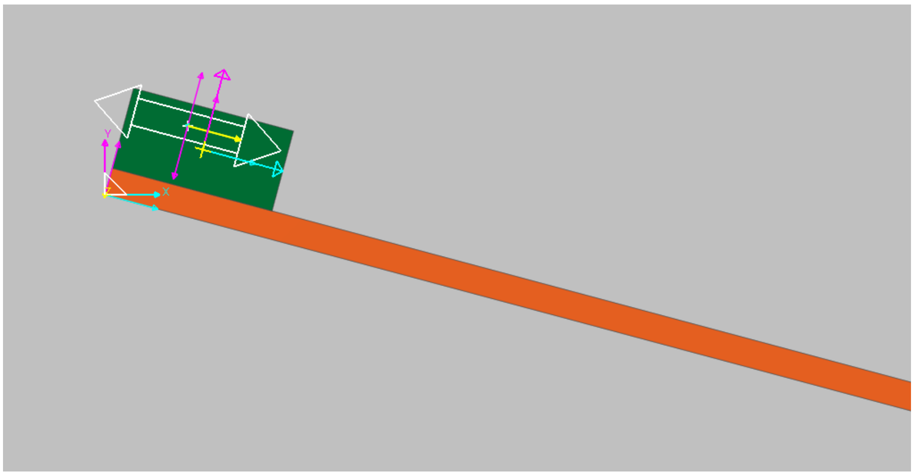
$a_{15} = 9.81 \cdot (sin15 - 0.4 \cdot cos15) = -1.251\, m/s^2$
$v^2 - {v_0}^2 = 2aS$
$0^2 - 7^2 = -2 \cdot 1.251 \cdot S$
$S = 19.584m$
$v = v_0 + at$
$t = -v_0/a = -7/(-1.251) = 5.596\, sec$

(2) $\theta = 30°$

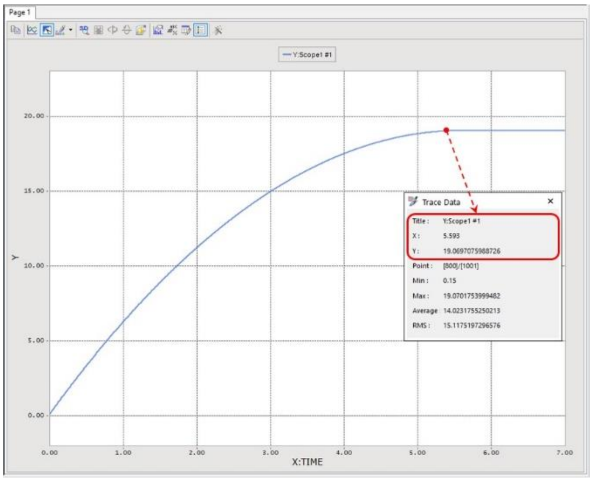
$a_{30} = 9.81 \cdot (sin30 - 0.4 \cdot cos30) = 1.507\, m/s^2$
➔ Sliding occurs continuously because the acceleration has plus valu e

Numerical Solution - RecurDyn

1. Modeling



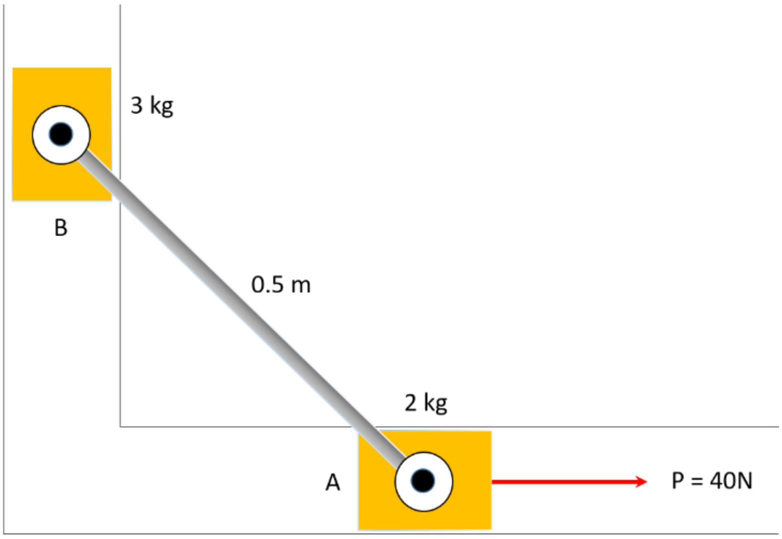
2. Plot the results
   - The displacement on the X axis of the block when the inclination of the slope is 15° (The block slides down the slope as 19.063 m and then it stops when the time is 5.593 ss)



Comparison of results

Object Value		Theory	RecurDyn	Error(%)
$\boldsymbol{\theta = 15°}$	$t\ [s]$	5.59	5.593	0.054
	$x\ [m]$	19.58	19.069	2.61
$\boldsymbol{\theta = 30°}$	$t\ [s]$	-	-	-
	$x\ [m]$	-	-	-

Dynamics of Rigid Bodies.06



References: Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.129, P.3.2
Type of Analysis : Kinetics of particles, Linear Motion
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Units
Initial length	$x_A,\ x_B$	0.4, 0.3	mm
Decreasing rate	v_A, v_B	0.9, 1.2	mm/s
Rotation rate	ω_{AB}	3	deg/s

$$\omega_{AB} = \frac{v_A}{0.3} = \frac{0.9}{0.3} = 3 = \frac{v_B}{0.4}$$

$$v_B = \dot{x_B} = 1.2 \text{ m/s}$$

Geometrical Constraint

$${x_A}^2 + {x_B}^2 = 0.5^2$$

Differentiate this equation, then,

$2x_A\dot{x_A} + 2x_B\dot{x_B} = 0$

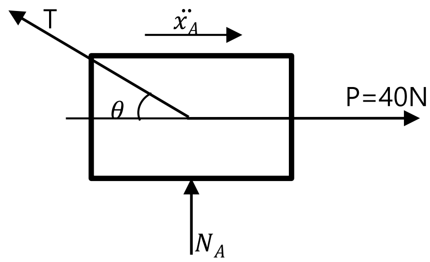
Differentiate one more time, then,

$x_A \cdot \ddot{x_A} + \dot{x_A}^2 + x_B \cdot \ddot{x_B} + \dot{x_B}^2 = 0$

$$\ddot{x_B} = -\frac{\left(x_A \cdot \ddot{x_A} + \dot{x_A}^2 + \dot{x_B}^2\right)}{x_B} = -\frac{\left(\dot{x_A}^2 + \dot{x_B}^2\right)}{x_B} - \frac{x_A \cdot \ddot{x_A}}{x_B} = -\frac{0.9^2 + 1.2^2}{0.3} - \frac{0.4}{0.3}\ddot{x_A}$$

$$\ddot{x_B} = -7.5 - \frac{4}{3}\ddot{x_A} \qquad (1)$$

F.B.D. of the Body A

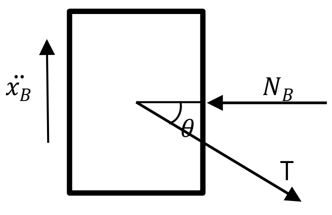


$$\sum F_x = P - T \cdot cos\theta = m_A\ddot{x_A}$$

$$40 - T \cdot \frac{4}{5} = 2 \cdot \ddot{x_A}$$

$$T = \frac{5}{4} \cdot (40 - 2 \cdot \ddot{x_A}) = 50 - \frac{5}{2} \cdot \ddot{x_A}$$

F.B.D. of the Body B



$$\sum F_y = -T \cdot sin\theta = m_B\ddot{x_B}$$

$$-T \cdot \frac{3}{5} = 3 \cdot \ddot{x_B}$$

$$T = -\frac{5}{3} \cdot 3 \cdot \ddot{x_B} = -5 \cdot \ddot{x_B}$$

$T = 50 - \frac{5}{2} \cdot \ddot{x_A} = -5 \cdot \ddot{x_B}$

$\ddot{x_B} = -10 + \frac{1}{2} \cdot \ddot{x_A}$ (2)

From equation (1) and (2),

$\ddot{x_B} = -7.5 - \frac{4}{3}\ddot{x_A} = -10 + \frac{1}{2} \cdot \ddot{x_A}$

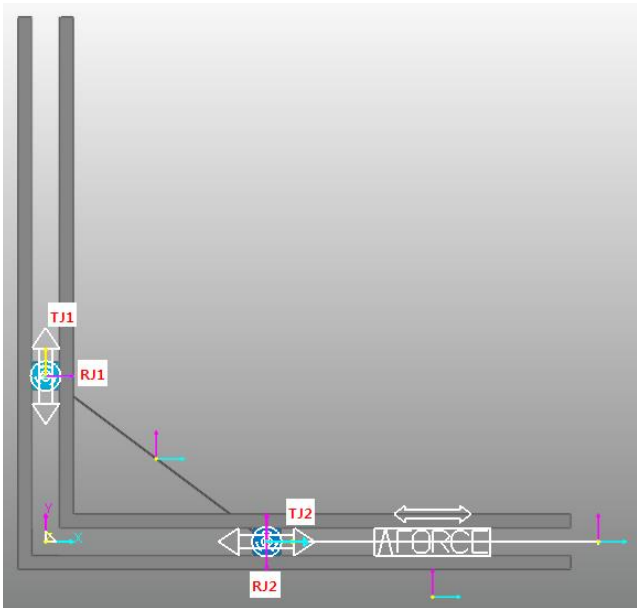
$\therefore \ddot{x_A} = 1.364\ m/s$

$\therefore \ddot{x_B} = -9.318\ m/s$

$\therefore T = 46.59\ N$

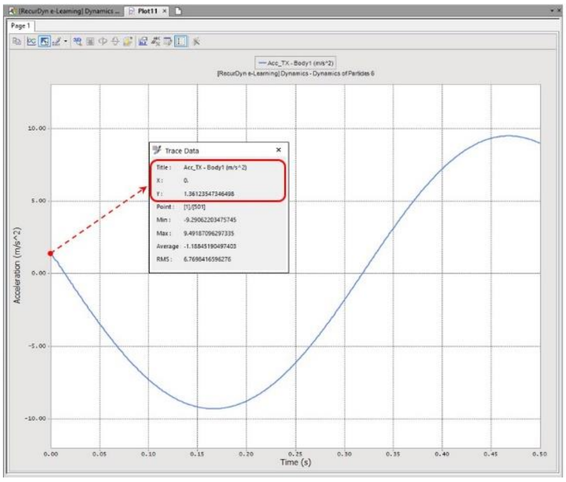
⭘ Numerical Solution - RecurDyn

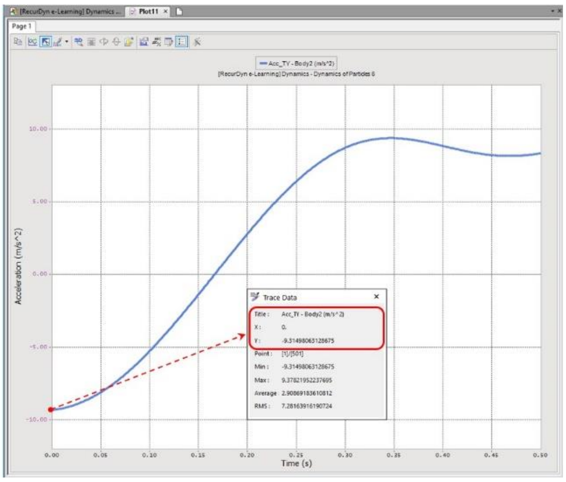
1. Modeling



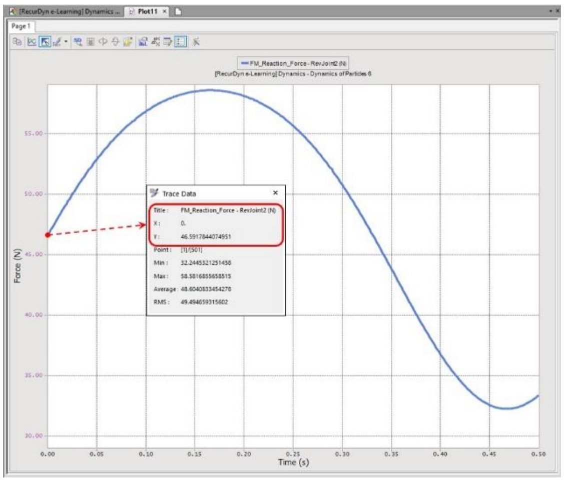
2. Plot the results

- The acceleration on the X axis of "Body A" and acceleration on the Y axis of "Body B"





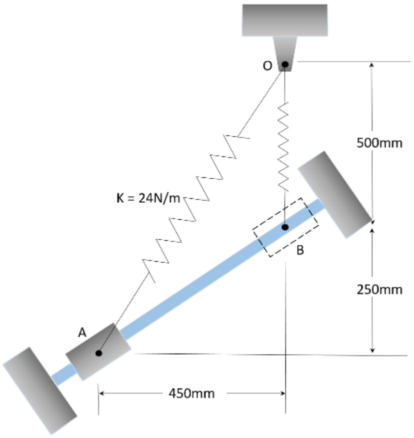
- The total reaction force of “RevJoint2” – “FM_Reaction_Force”



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$a_A\ [m/s^2]$	1.36	1.361	0.074
$a_B\ [m/s^2]$	-9.318	-9.315	0.032
T [N]	46.59	46.592	0.004

Dynamics of Rigid Bodies.07



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.182, P.3.139
Type of Analysis : Kinetics of Particles, Work-energy Equation
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Spring coefficient	k	24	N/m
Initial length	l_0	0.375	m

Law of Energy Conservation

$$mgh_A + \frac{1}{2}mv_A^2 + \frac{1}{2}kx_A^2 = mgh_B + \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$$

$$x_A = 0.4996\ [m]$$

$$h_B = 0.250\ [m]$$

$$x_B = 0.125\ [m]$$

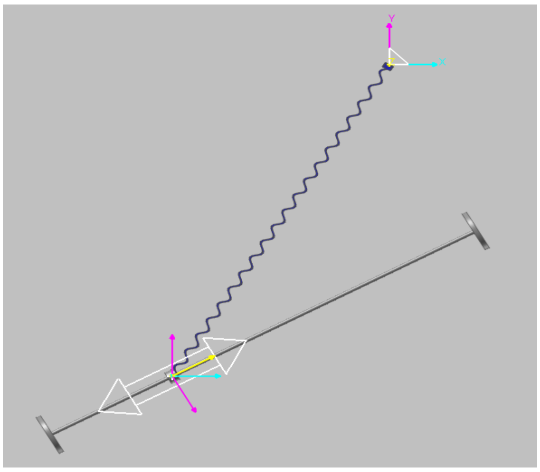
$$0 + 0 + \frac{1}{2}kx_A^2 = mgh_B + \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$$

$$v_B = \sqrt{\frac{k}{m}({x_A}^2 - {x_B}^2) - 2gh_B}$$

$$v_B = 1.1558\, m/s$$

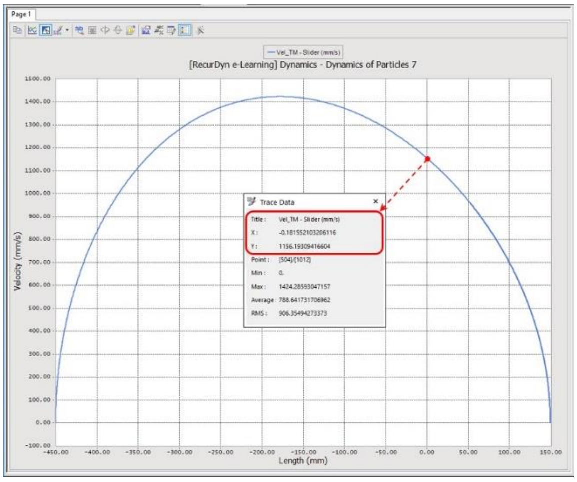
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

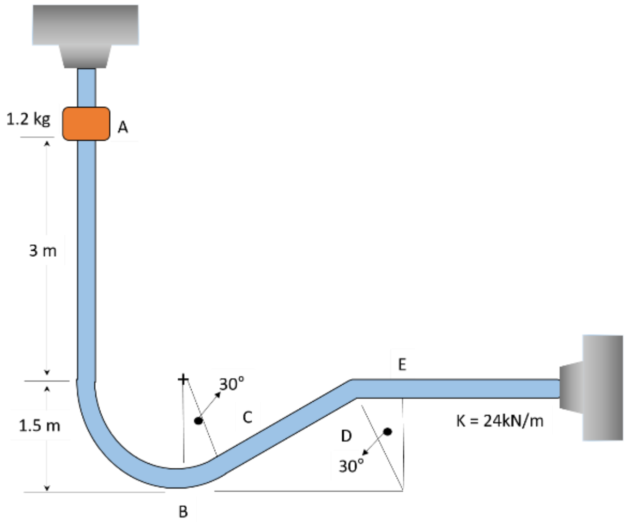
   - Measure the velocity when the displacement on the X axis of the "Slider" is zero



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{v_B}[\boldsymbol{m/s}]$	1.1558	1.1556	0.017

Dynamics of Rigid Bodies.08



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.182, P.3.141
Type of Analysis : Kinetics of Particles, Work-energy Equation
Type of Element : Particle (one part)

⭘ Theoretical Solution

Basic Conditions

Given	Symbol	Value	Units
Initial height	h_A	4.5	m
Initial velocity	v_A	0	m/s

Law of Energy Conservation

Calculate the velocity of the point "B"

The point "B" is the lowest point, thus $h_B = 0\,m,\ v_B = ?$

$$mgh_A + \frac{1}{2}m{v_A}^2 = mgh_B + \frac{1}{2}m{v_B}^2 = mgh_F + \frac{1}{2}k{x_F}^2 + \frac{1}{2}m{v_F}^2$$

A-B : $mgh_A + \frac{1}{2}mv_A{}^2 = mgh_B + \frac{1}{2}mv_B{}^2$

$v_B = \sqrt{2gh_A} = 9.395m/s$

Calculate the maximum deformation of the spring

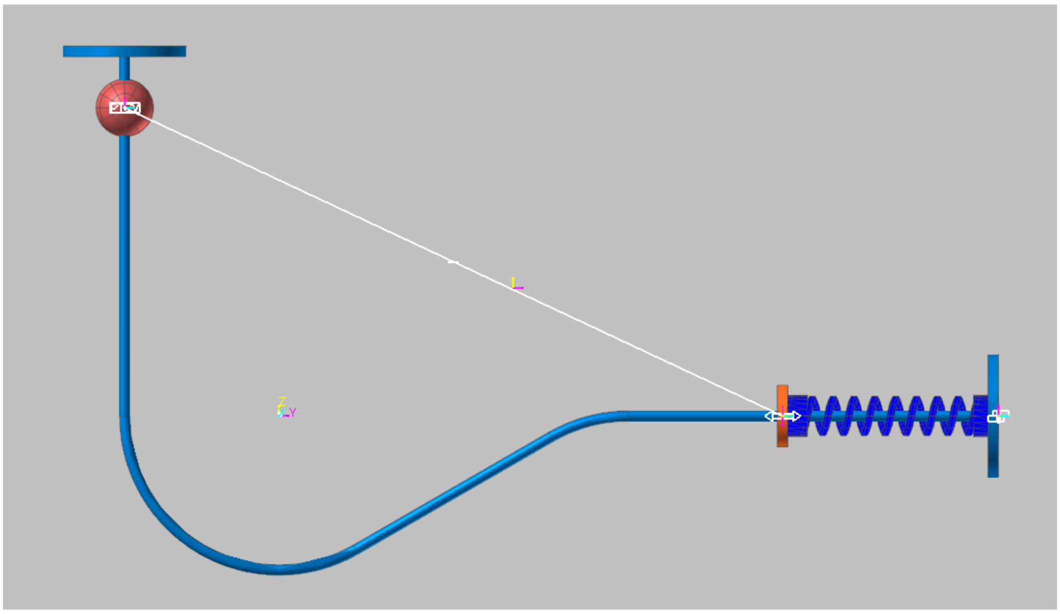
A point of the spring that occurs the maximum deformation is called "F", $h_F = 1.5m,\ \ v_F = 0,\ \ x_F = \delta$

A-F : $mgh_A + \frac{1}{2}mv_A{}^2 = mgh_F + \frac{1}{2}kx_F{}^2 + \frac{1}{2}mv_F{}^2$

$$x_F = \delta = \sqrt{\frac{2mg(h_A - h_F)}{k}} = 54.24\,mm$$

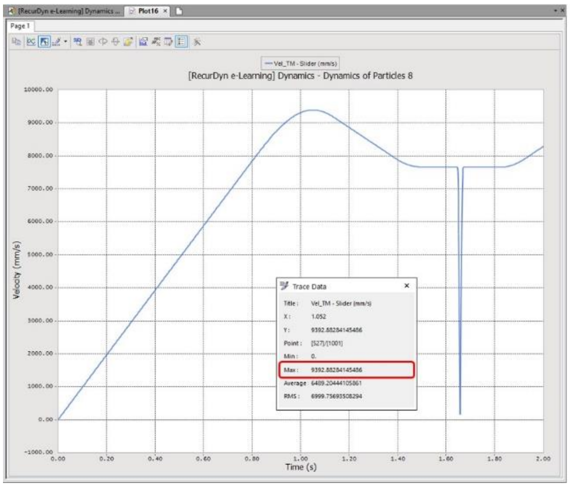
Numerical Solution - RecurDyn

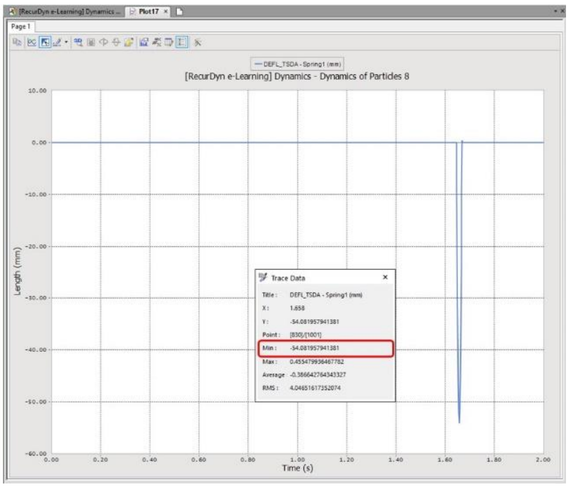
1. Modeling



2. Plot the results

   - Measure the maximum value from the velocity plot of the "Slider - Body A" and the minimum deformation from the deformation plot of "Spring"

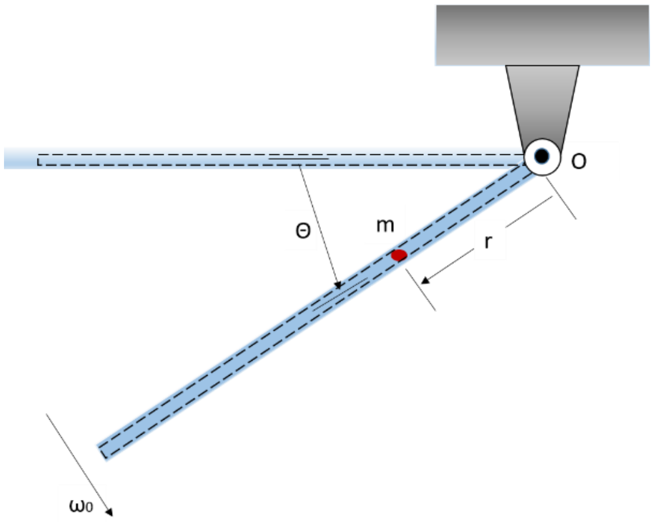




⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{V_B}\ [\boldsymbol{m/s}]$	9.395	9.393	0.213
$\boldsymbol{\delta}\ [\boldsymbol{mm}]$	54.24	54.08	0.295

Dynamics of Rigid Bodies.09



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.263, P.3.350
Type of Analysis : Kinetics of Particles, Work-energy Equation
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

- Calculate the time "t" and the angle "θ" when the particle is separated from the tube with $r = 1[m]$, $\omega_0 = 0.5\ [rad/s] = const$, where "r" is the length of the tube and "ω_0" is the angular velocity of the tube.

Given	Symbol	Value	Unit
Tube length	r	1	m
Angular velocity	ω_0	0.5	rad/s

Newton's Equation of Motion

$$\sum F_r = ma_r$$

$mgsin\theta = m(\ddot{r} - r\dot{\theta}^2)$
$\ddot{r} - r\dot{\theta}^2 = gsin\theta$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$, where $\theta_0 = 0,\ \omega_0 = const$, and $\alpha = 0$

Thus, $\theta = \omega_0 t$
$\ddot{r} - {\omega_0}^2 r = gsin\omega_0 t$

That is the second order linear differential equation,

$$r = r_c + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \left(g/_{2{\omega_0}^2}\right) sin\omega_0 t$$

$$\dot{r} = -C_1\omega_0 e^{-\omega_0 t} + C_2\omega_0 e^{\omega_0 t} - \omega_0\left(g/_{2{\omega_0}^2}\right) cos\omega_0 t$$

Where $t = 0$, and $r = 0,\ \dot{r} = 0$
$C_1 + C_2 = 0$

$$-C_1\omega_0 + C_2\omega_0 - g/_{2\omega_0} = 0$$

$$C_1 = -g/_{4{\omega_0}^2}$$

$$C_2 = g/_{4{\omega_0}^2}$$

$$r = g/_{4{\omega_0}^2}(-e^{-\theta} + e^{\theta} - 2sin\theta)$$

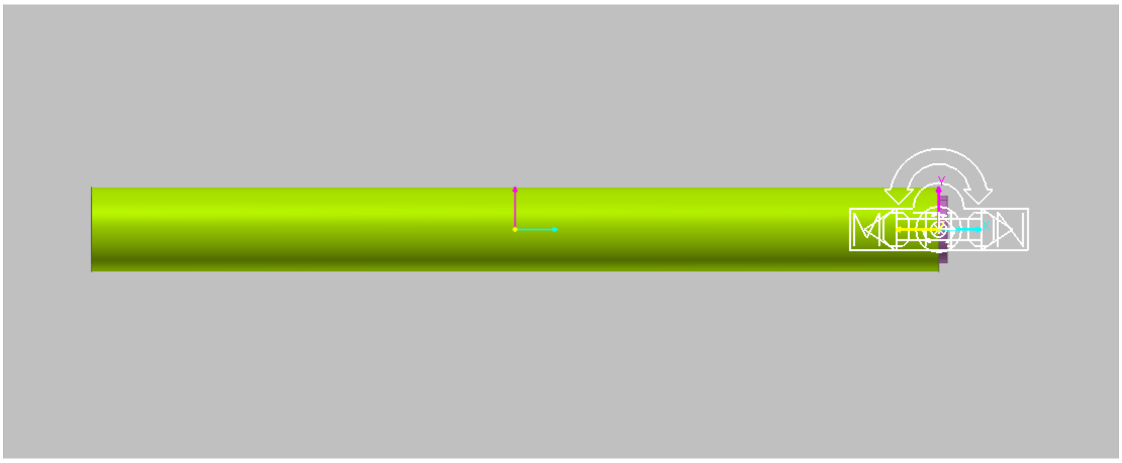
Where $r = 1[m],\ \omega_0 = 0.5\ [rad/s]$
Solve the nonlinear equation using numerical analysis which satisfies the equation, $-e^{-\theta} + e^{\theta} - 2sin\theta - 0.10194 = 0$

It is difficult to solve the nonlinear equation by hand. So the following solution is calculated using numerical analysis on a computer (Newton-Rapson Method, MatLab, Maple, Mathmatica, etc.)

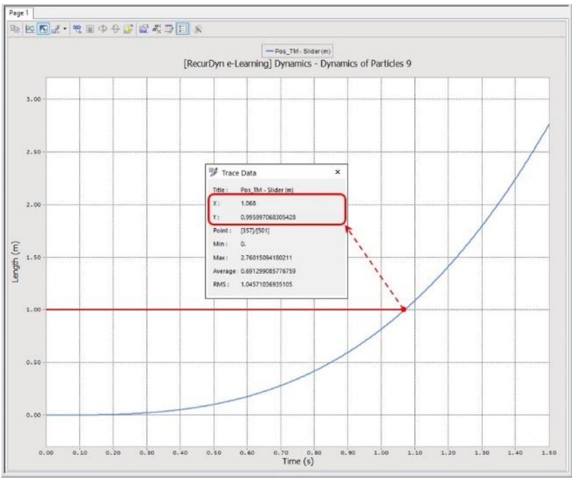
$\theta = 0.5347\ [rad]$
Where $\theta = \omega_0 t$, thus $t = \theta/_{\omega_0} = 1.07\ [\mathrm{sec}]$

⭘ Numerical Solution - RecurDyn

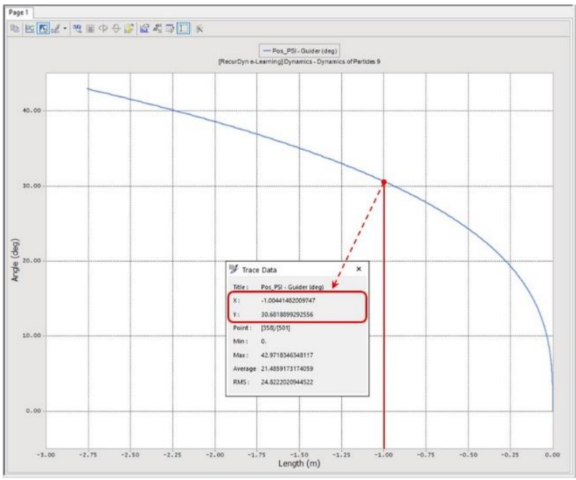
1. Modeling



2. Plot the results
    - Measure the time when the displacement of the "Slider" is "1m"



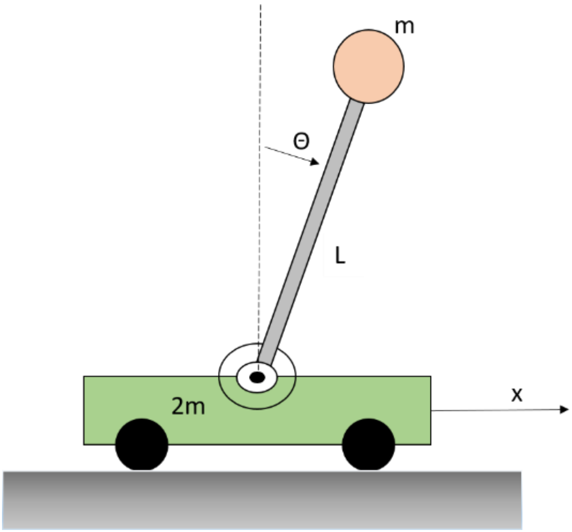
    - A plot of the rotation angle of the "Guider" by the relative displacement of "TraJoint1"



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{t}$ $[\boldsymbol{s}]$	1.07	1.068	0.187
$\boldsymbol{\theta}$ $[\boldsymbol{deg}]$	30.653	30.681	0.009

Dynamics of Rigid Bodies.10



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.287, P.4.31
Type of Analysis : Kinetics of Particles, Work-energy Equation
Type of Element : Particle (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	1	kg
Length	l	0.5	m

Law of Momentum Conservation

$$\int Fdt = \Delta G = G_{A'} + G_{B'} - 0$$

$$0 = 2m \cdot v_x + 2m \cdot (v_x - l\dot{\theta})$$

$$2v_x = l\dot{\theta} \qquad (1)$$

For the entire system,

$$U_{1-2} = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2}\cdot 2m\cdot {v_x}^2 + \frac{1}{2}\cdot 2m\cdot (v_x - l\dot{\theta})^2 - 0$$

$$= m\cdot (2{v_x}^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2)$$

$$\Delta V_g = -2mg\cdot 2l = -4mgl$$

$$U_{1-2} = 0$$

$$m\cdot \left(2{v_x}^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2\right) - 4mgl = 0$$

$$2{v_x}^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2 - 4gl = 0$$

Form equation (1), $2v_x = l\dot{\theta}$

$$2{v_x}^2 - 4{v_x}^2 + 4{v_x}^2 - 4gl = 0$$

$$v_x = \sqrt{2gl}$$

$$\dot{\theta} = 2\frac{v_x}{l} = 2\sqrt{\frac{2g}{l}} \qquad (2)$$

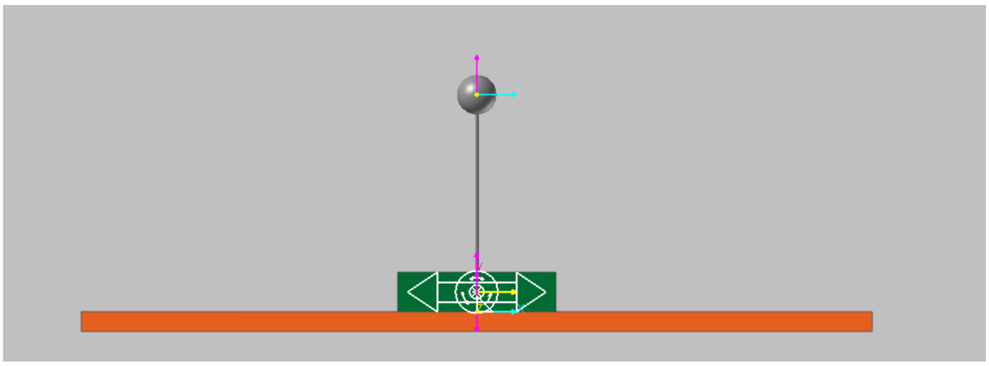
$$m = 1\text{kg}, \qquad l = 0.5m, \qquad g = 9.81m/s^2$$

$$v_x = \sqrt{2gl} = \sqrt{2\cdot 9.81\cdot 0.5} = 3.13\ m/s$$

$$\dot{\theta} = 2\frac{v_x}{l} = 2\sqrt{\frac{2g}{l}} = 2\sqrt{\frac{2\cdot 9.81}{0.5}} = 12.53\ rad/s$$

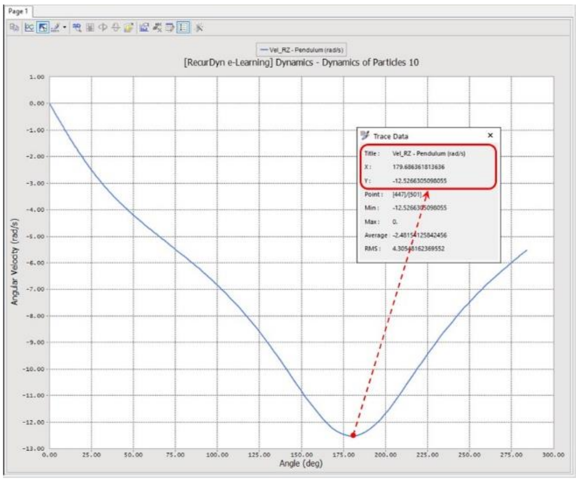
Numerical Solution - RecurDyn

1. Modeling

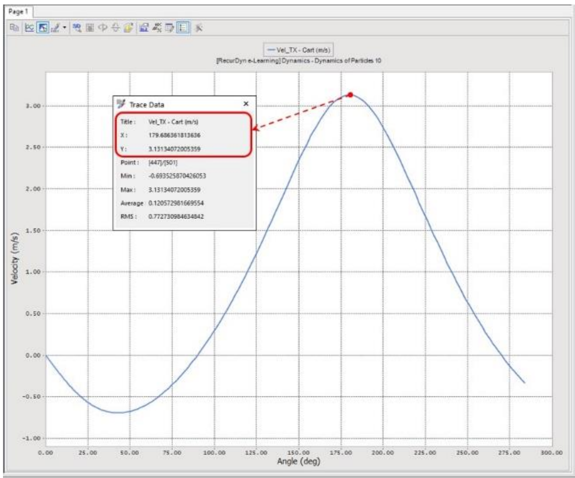


2. Plot the results

- The angular velocity of the “Pendulum”
  (X axis : rotation angle, Y axis : rotation velocity)



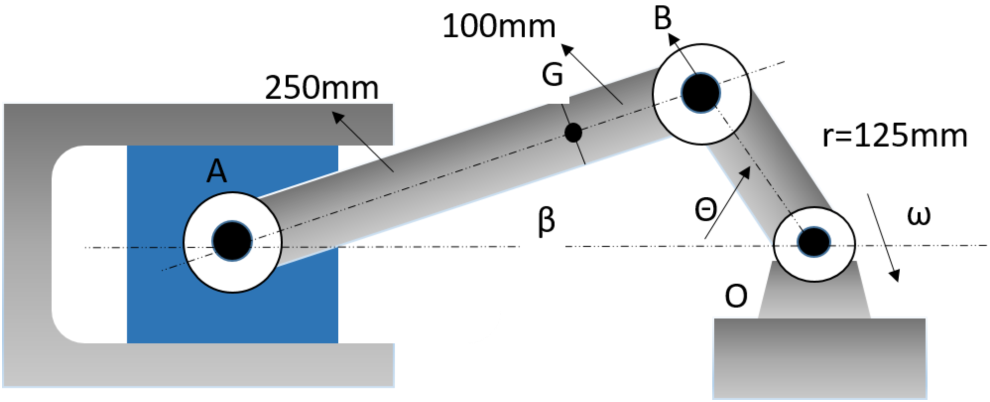
- The velocity of the “Cart”
  (X axis : rotation angle of the “Pendulum”, Y axis : Velocity on X axis)



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\dot{\theta}$ $[rad/s]$	12.53	12.53	0
v_x $[m/s]$	3.13	3.13	0

Dynamics of Rigid Bodies.11



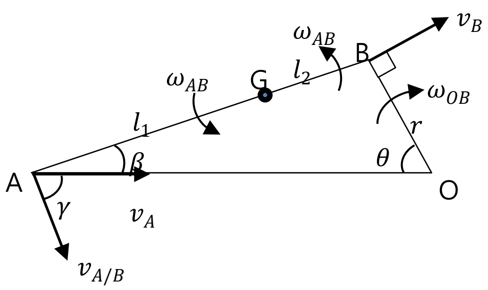
Reference : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.353, E.5.9
Type of Analysis : Plane Kinematics of Rigid Bodies, Relative Velocity
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OB}	$50\,\pi$	rad/s
Length	l_1, l_2	0.25, 0.1	m
"OB" length	r	0.125	m
"AOB" angle	θ	60	deg

Geometrical Constraint



$$\frac{r}{sin\beta} = \frac{l}{sin\theta}$$

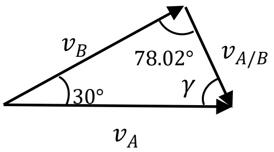
$$\beta = sin^{-1}\left(\frac{r}{l} \cdot sin\theta\right) = sin^{-1}\left(\frac{0.125}{0.35} \cdot sin60\right) = 18.02°$$

$$\therefore \gamma = 71.98°$$

$$v_B = r \cdot \omega_{OB} = 0.125 \times 50\pi = 19.63\ m/s$$

The Equation of Relative Velocity

$$\overrightarrow{v_A} = \overrightarrow{v_B} + \overrightarrow{v_{A/B}}$$



$$\frac{v_A}{sin78.02} = \frac{v_B}{sin\gamma}$$

$$v_A = \frac{v_B}{sin71.98} \cdot sin78.02 = 20.19\ m/s$$

$$\frac{v_{A/B}}{sin30} = \frac{v_B}{sin\gamma}$$

$$v_{A/B} = \frac{v_B}{sin71.98} \cdot sin30 = 10.32\ m/s$$

$$\omega_{AB} = \frac{v_{A/B}}{l} = 29.49\ rad/s$$

$$\overrightarrow{v_G} = \overrightarrow{v_A} + \overrightarrow{v_{G/A}}$$

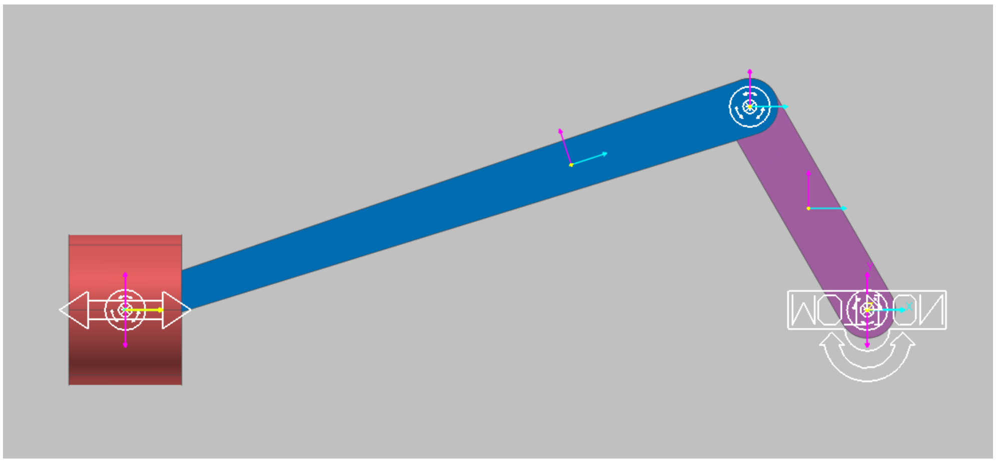
$= 20.19\vec{i} + 29.49\vec{k} \times (0.25cos18.02\vec{i} + 0.25sin18.02\vec{j}) = 20.19\vec{i} + (7.01\vec{j} - 2.28\vec{i}) =$

$17.91\vec{i} + 7.01\vec{j}$

$$v_G = \sqrt{(17.91^2 + 7.01^2)} = 19.23\ m/s$$

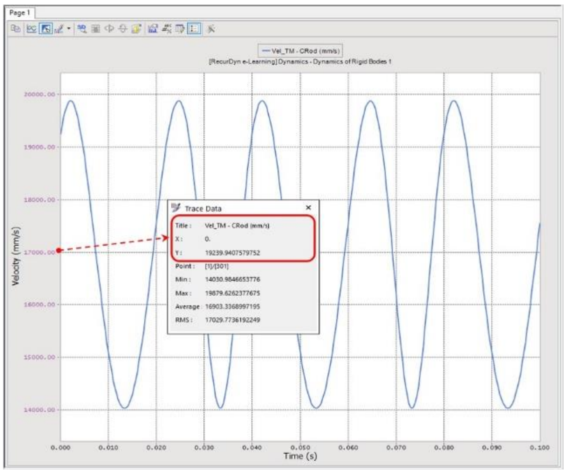
Numerical Solution - RecurDyn

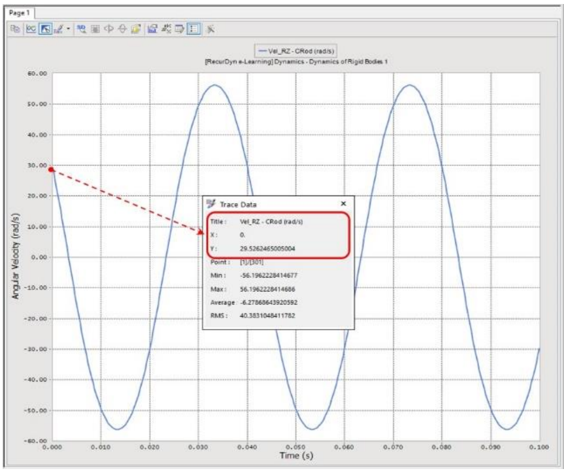
1. Modeling



2. Plot the results

- The velocity and angular velocity of the "CM" of the "CRod"

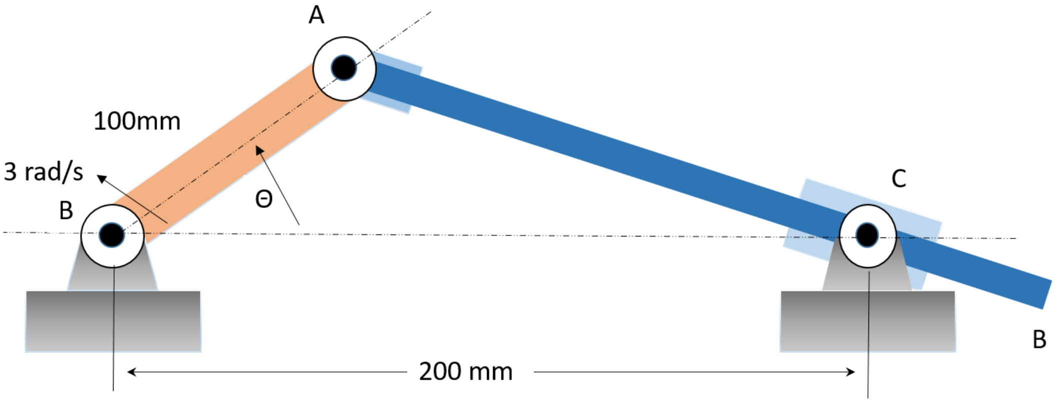




Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$v_G\ [m/s]$	19.23	19.24	0.27
$\omega_{AB}\ [rad/s]$	29.5	29.53	0.346

Dynamics of Rigid Bodies.12



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.346, P.5.54
Type of Analysis : Plane Kinematics of Rigid Bodies, Absolute Motion
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OA}	0.5	rad/s
"ABC" angle	θ	45	deg
"BC" length	r	0.2	m
"AB" length	l	0.1	m

Geometrical Constraint

With the trigonometric function formula,

$$\frac{l}{sin\alpha} = \frac{r}{sin\beta}$$

$$\frac{l}{\sin(\pi - (\theta + \beta))} = \frac{r}{sin\beta}$$

$$\frac{l}{\sin(\theta+\beta)} = \frac{r}{sin\beta}$$

$$r \cdot sin(\theta+\beta) = l \cdot sin\beta \qquad (1)$$

Differentiate both sides of this equation,

$$r \cdot (\dot{\theta}+\dot{\beta}) \cdot cos(\theta+\beta) = l \cdot \dot{\beta} \cdot cos\beta$$

Where, $\omega_{OA} = \dot{\theta}$, $\omega_{AB} = \omega = \dot{\beta}$

$$\left(l \cdot cos\beta - r \cdot cos(\theta+\beta)\right)\dot{\beta} = r \cdot \dot{\theta} \cdot cos(\theta+\beta)$$

$$\omega = \dot{\beta} = \frac{r \cdot \dot{\theta} \cdot cos(\theta+\beta)}{\left(l \cdot cos\beta - r \cdot cos(\theta+\beta)\right)}$$

With the equation (1),

$$r \cdot sin(\theta+\beta) = l \cdot sin\beta$$

$$r \cdot (sin\theta \cdot cos\beta + cos\theta \cdot sin\beta) = l \cdot sin\beta$$

Multiply both sides of this equation by $\frac{1}{cos\beta}$,

$$r \cdot (sin\theta + cos\theta \cdot tan\beta) = l \cdot tan\beta$$

$$(l - r \cdot cos\theta) \cdot tan\beta = r \cdot sin\theta$$

$$tan\beta = \frac{r \cdot sin\theta}{(l - r \cdot cos\theta)}$$

$$\beta = tan^{-1}\frac{r \cdot sin\theta}{l - r \cdot cos\theta}$$

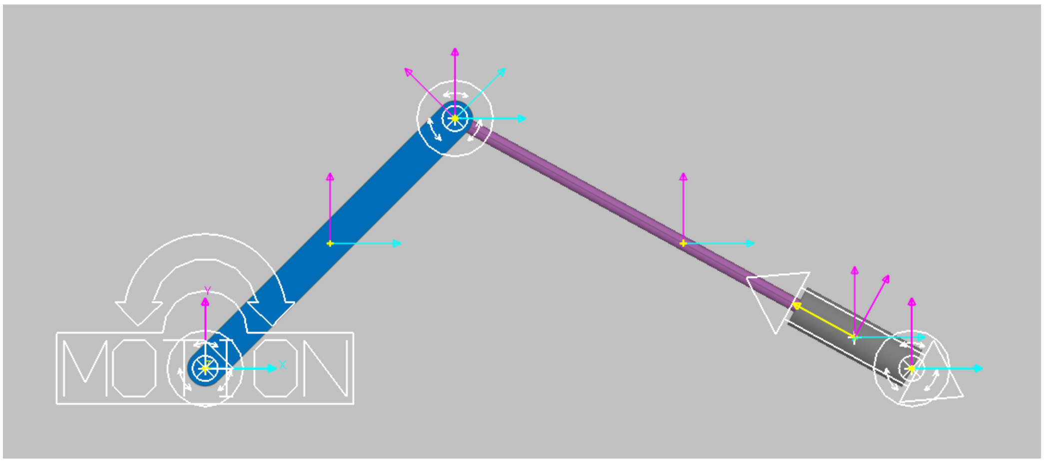
If, $\theta = 45°$, $\omega_{OA} = \dot{\theta} = 3\ \mathrm{rad/s}$, $l = 0.2\ m$, $r = 0.1\ m$,

$$\beta = tan^{-1}\frac{0.1 \cdot sin45}{0.2 - 0.1 \cdot cos45} = 28.68°$$

$$\omega = \dot{\beta} = \frac{0.1 \cdot 3 \cdot cos(45+28.68)}{\left(0.2 \cdot cos28.68 - 0.1 \cdot cos(45+28.68)\right)} = 0.572\ rad/s$$

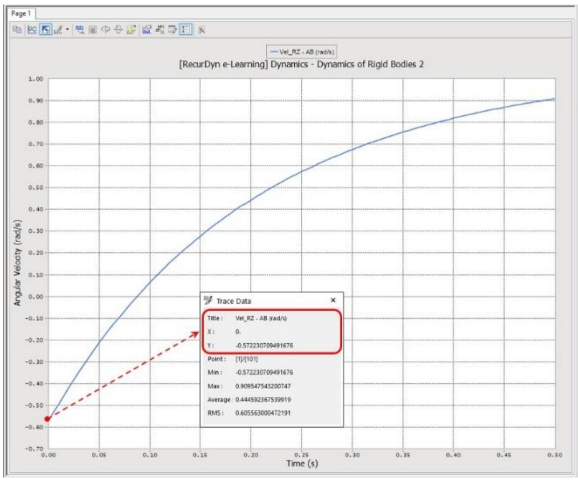
⭘ Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

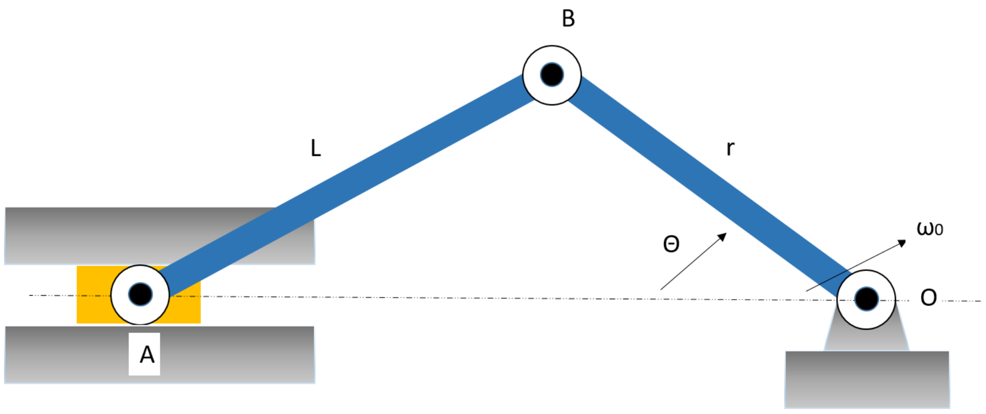
   - The angular velocity on Z axis of the link "AB"



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\omega_{AB}\ [rad/s]$	0.572	0.572	0

Dynamics of Rigid Bodies.13



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.347, P.5.57
Type of Analysis : Plane Kinematics of Rigid Bodies, Absolute Motion
Type of Element : Rigid Body (one part)

⭘ Theoretical Solution

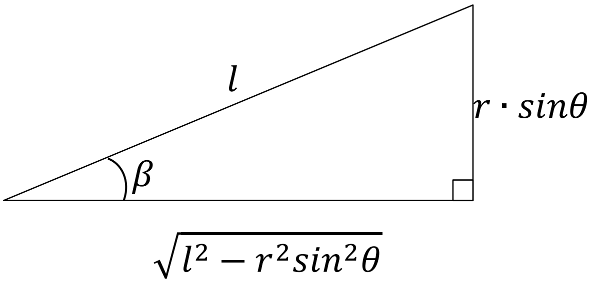
Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_0	3	rad/s
“AB” length	l	0.2	m
“OB” length	r	0.1	m
“AOB” angle	θ	60	deg

Geometrical Constraint

$$l \cdot sin\beta = r \cdot sin\theta \qquad (1)$$

$$sin\beta = \frac{r}{l} sin\theta$$



$$cos\beta = \frac{\sqrt{l^2 - r^2 sin^2\theta}}{l} = \sqrt{1 - (\frac{r}{l} sin\theta)^2}$$

Differentiate both sides of the equation (1),

$$l \cdot \dot{\beta} \cdot cos\beta = r \cdot \dot{\theta} \cdot cos\theta \qquad (2)$$

$$\omega_{AB} = \dot{\beta} = \frac{r \cdot \dot{\theta} \cdot cos\theta}{l \cdot cos\beta} = \frac{r}{l}\omega_0 \frac{cos\theta}{\sqrt{1 - (\frac{r}{l} sin\theta)^2}}$$

Differentiate both sides of the equation (2),

$$l \cdot \ddot{\beta} \cdot cos\beta - l \cdot \dot{\beta}^2 \cdot sin\beta = r \cdot \ddot{\theta} \cdot cos\theta - r \cdot \dot{\theta}^2 \cdot sin\theta$$

Where $\omega_0 = \dot{\theta} = const$, thus $\ddot{\theta} = 0$

$$\alpha_{AB} = \ddot{\beta} = \frac{l \cdot \dot{\beta}^2 \cdot sin\beta - r \cdot \dot{\theta}^2 \cdot sin\theta}{l \cdot cos\beta} = \frac{l \cdot \frac{r^2}{l^2}\omega_0{}^2 \frac{cos^2\theta}{1 - (\frac{r}{l} sin\theta)^2} \frac{r}{l} sin\theta - r \cdot \omega_0{}^2 \cdot sin\theta}{l \cdot \sqrt{1 - (\frac{r}{l} sin\theta)^2}}$$

$$= \frac{r}{l}\omega_0{}^2 \cdot sin\theta \; \frac{\frac{r^2}{l^2}\frac{cos^2\theta}{1 - (\frac{r}{l} sin\theta)^2} - 1}{\sqrt{1 - (\frac{r}{l} sin\theta)^2}} = \frac{r}{l}\omega_0{}^2 \cdot sin\theta \; \frac{\frac{r^2}{l^2} cos^2\theta - 1 + \frac{r^2}{l^2} sin^2\theta}{\left(1 - (\frac{r}{l} sin\theta)^2\right)^{3/2}} =$$

$$= \frac{r}{l}\omega_0{}^2 \cdot sin\theta \; \frac{\frac{r^2}{l^2} - 1}{(1 - (\frac{r}{l} sin\theta)^2)^{3/2}}$$

The angular velocity and the angular acceleration of the link "AB"

Where $l = 0.2\,m$, $r = 0.1\,m$, $\omega_0 = 3\,rad/s$, $\theta = 60°$, thus

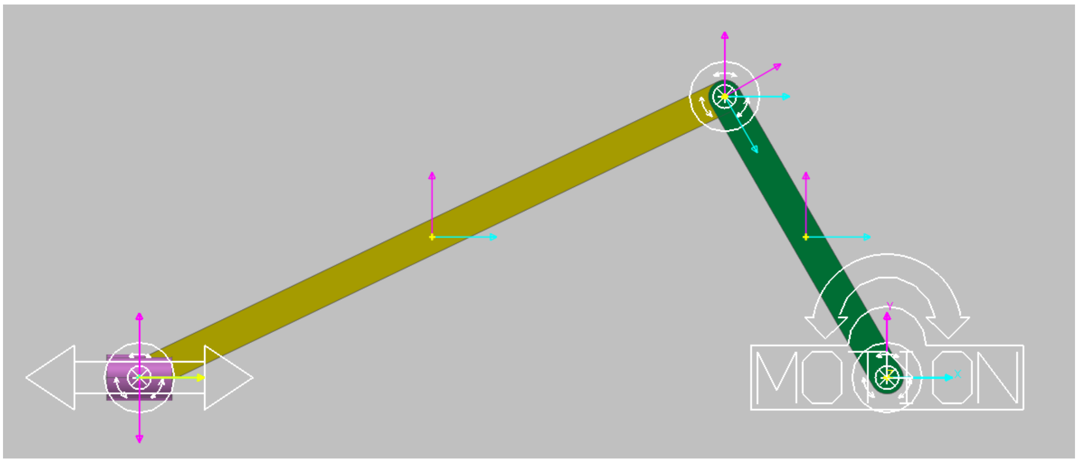
$$\omega_{AB} = \frac{r}{l}\omega_0 \frac{cos\theta}{\sqrt{1-(\frac{r}{l}sin\theta)^2}} = \frac{0.1}{0.2}\cdot 3 \cdot \frac{cos60}{\sqrt{1-(\frac{0.1}{0.2}sin60)^2}} = 0.832\,rad/s$$

$$\alpha_{AB} = \frac{r}{l}{\omega_0}^2 \cdot sin\theta\ \frac{\frac{r^2}{l^2}-1}{\left(1-(\frac{r}{l}sin\theta)^2\right)^{3/2}} = \frac{0.1}{0.2}\cdot 3^2 \cdot sin60\ \frac{\frac{0.1^2}{0.2^2}-1}{\left(1-(\frac{0.1}{0.2}\cdot sin60)^2\right)^{3/2}}$$

$$= -3.99\ \mathrm{rad}/s^2$$

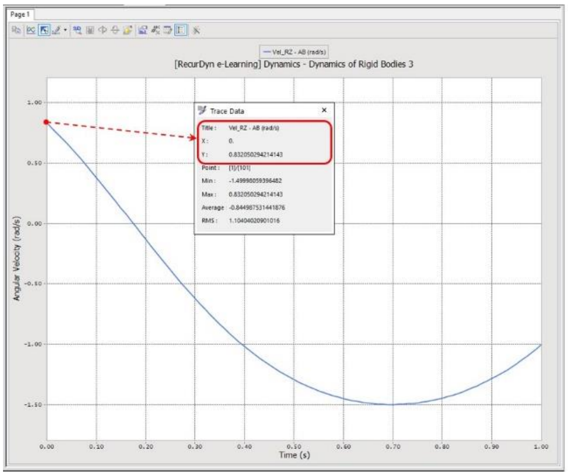
Numerical Solution - RecurDyn

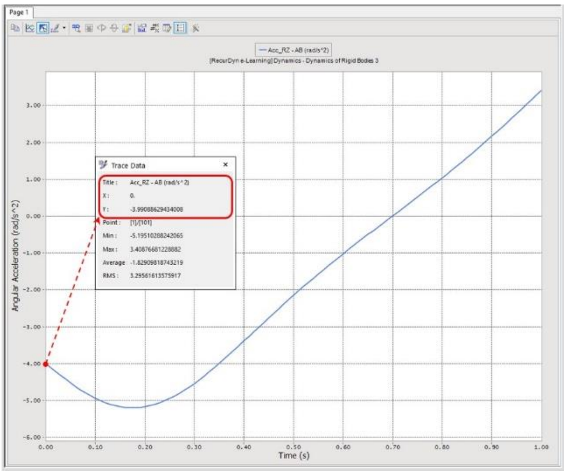
1. Modeling



2. Plot the results

   - The angular velocity and angular acceleration on Z axis of the link "AB"

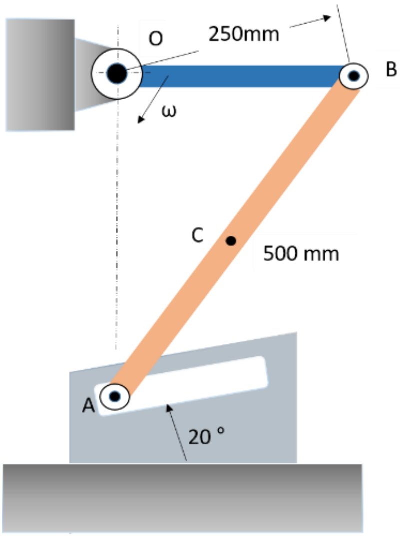




⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
ω_{AB} $[rad/s]$	0.832	0.832	0
α_{AB} $[rad/s^2]$	-3.99	-3.99	0

Dynamics of Rigid Bodies.14

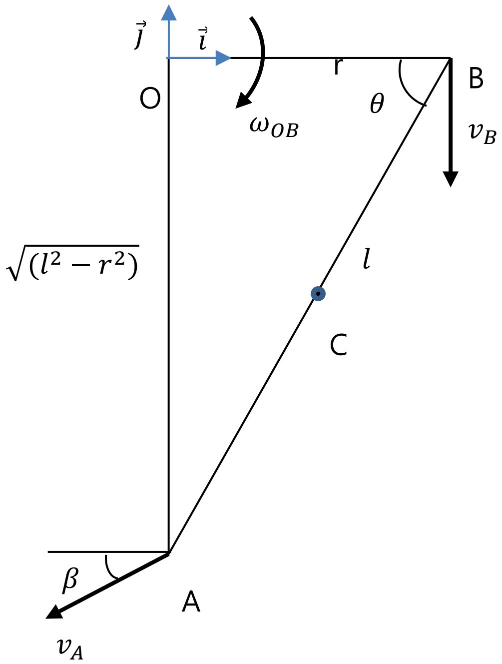


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.359, P.5.80
Type of Analysis : Plane Kinematics of Rigid Bodies, Relative Velocity
Type of Element : Rigid Body (one part)

⭘ Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OB}	0.8	rad/s
“AB” length	l	0.5	m
“OB” length	r	0.25	m
“OA-OB” angle	β	20	deg



The “OB” rotates on the point of “0”, thus

$$\overrightarrow{v_B} = \overrightarrow{\omega_{OB}} \times \overrightarrow{r_{OB}} = -0.8\vec{k} \times 0.25\vec{i} = -0.2\vec{j}$$

$$\overline{OA} = \sqrt{(l^2 - r^2)} = 0.433$$

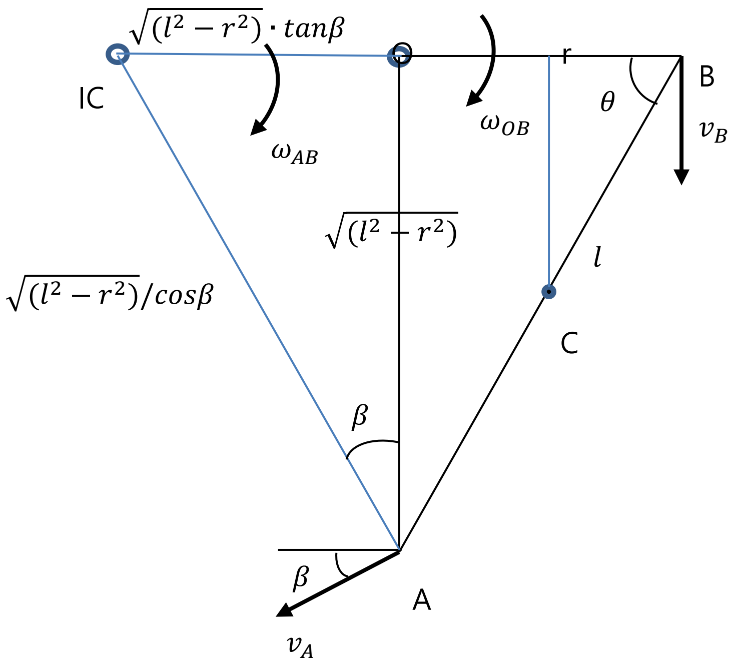
$$\theta = cos^{-1}\left({}^{r}/_{l}\right) = 60°$$

$$\overline{I_{IC}A} = \overline{OA}/cos\beta = 0.461$$

$$\omega_{AB} = {}^{v_B}\!\Big/_{\overline{I_{IC}B}} = {}^{0.2}/_{0.408} = 0.49$$

$$\overrightarrow{v_A} = \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{OA}} = -0.49\vec{k} \times (0.158\vec{i} - 0.433\vec{j}) = -0.0774\vec{j} - 0.2122\vec{i}$$

$$\therefore\ v_A = 0.2259\ [m/s]$$



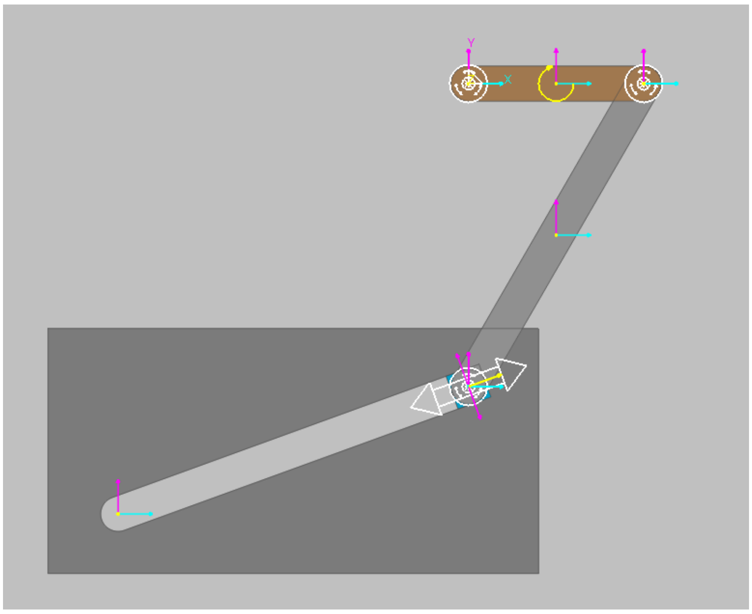
$$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{v_{C/B}} = \overrightarrow{v_B} + \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{BC}} = -0.2\vec{j} - 0.49\vec{k} \times (-0.25 \cdot cos60\vec{i} - 0.25 \cdot sin60\vec{j})$$

$$= -0.1061\vec{i} - 0.1388\vec{j}$$

$$\therefore\ v_C = 0.1747\ [m/s]$$

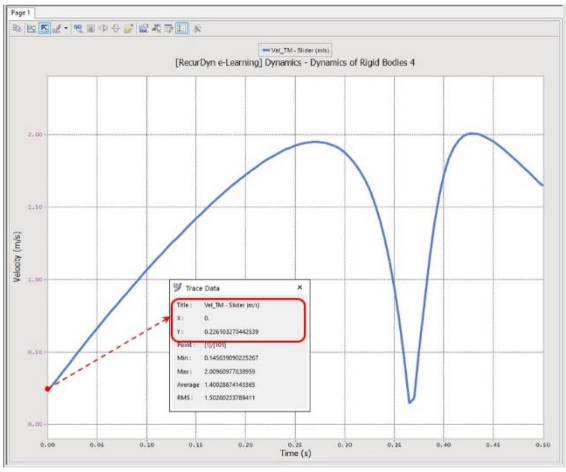
⭘ Numerical Solution - RecurDyn

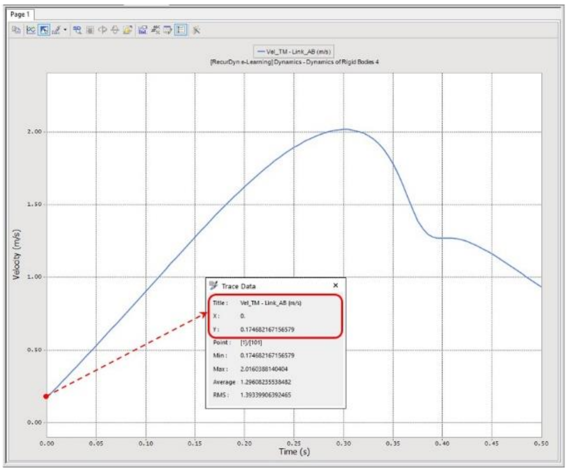
1. Modeling



2. Plot the results

- Measure the time when the displacement of the "Slider" is "1m"

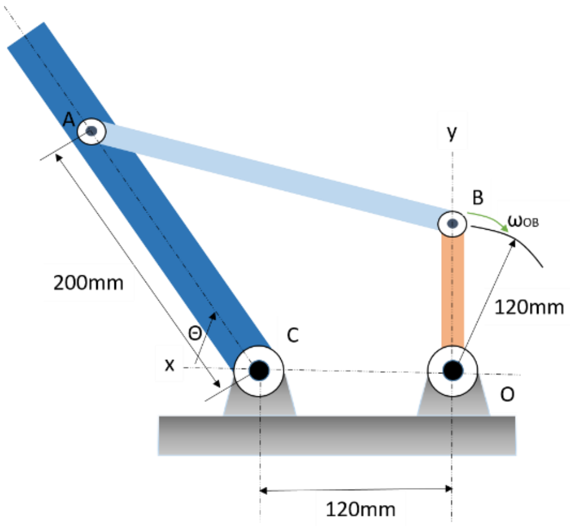




Comparison of results

Object Value	Theory	RecurDyn	Error(%)
v_A $[m/s]$	0.2259	0.2261	0.089
v_C $[m/s]$	0.1747	0.1746	0.057

Dynamics of Rigid Bodies.15

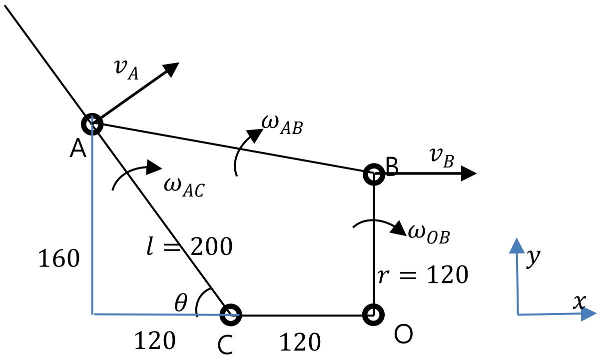


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.361, P.5.89
Type of Analysis : Plane Kinematics of Rigid Bodies, Instantaneous Center of Zero Velocity
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OB}	0.5	rad/s
Angle	$\tan\theta$	$4/3$	-



The Velocities of Each End Point of the Link “AB” (“A” and “B”) and the Relative Velocity of the “A” with Respect to the “B”

$\overrightarrow{v_B} = -\omega_{OB}\vec{k} \times r\vec{j} = -0.5\vec{k} \times 0.12\vec{j} = 0.06\vec{i}$

$\overrightarrow{v_A} = -\omega_{AC}\vec{k} \times (-0.12\vec{i} + 0.16\vec{j}) = 0.16 \cdot \omega_{AC}\vec{i} + 0.12 \cdot \omega_{AC}\vec{j}$

$\overrightarrow{v_{A/B}} = \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{BA}} = -\omega_{AB}\vec{k} \times (-0.24\vec{i} + 0.04\vec{j}) = 0.04 \cdot \omega_{AB}\vec{i} + 0.24 \cdot \omega_{AB}\vec{j}$

Calculate the Accelerations of the Link “AB” and the Link “AC

$\overrightarrow{v_A} = \overrightarrow{v_B} + \overrightarrow{v_{A/B}}$

$0.16 \cdot \omega_{AC}\vec{i} + 0.12 \cdot \omega_{AC}\vec{j} = 0.06\vec{i} + 0.04 \cdot \omega_{AB}\vec{i} + 0.24 \cdot \omega_{AB}\vec{j}$

$\vec{i}\ :\ 0.16 \cdot \omega_{AC} = 0.06 + 0.04 \cdot \omega_{AB}$

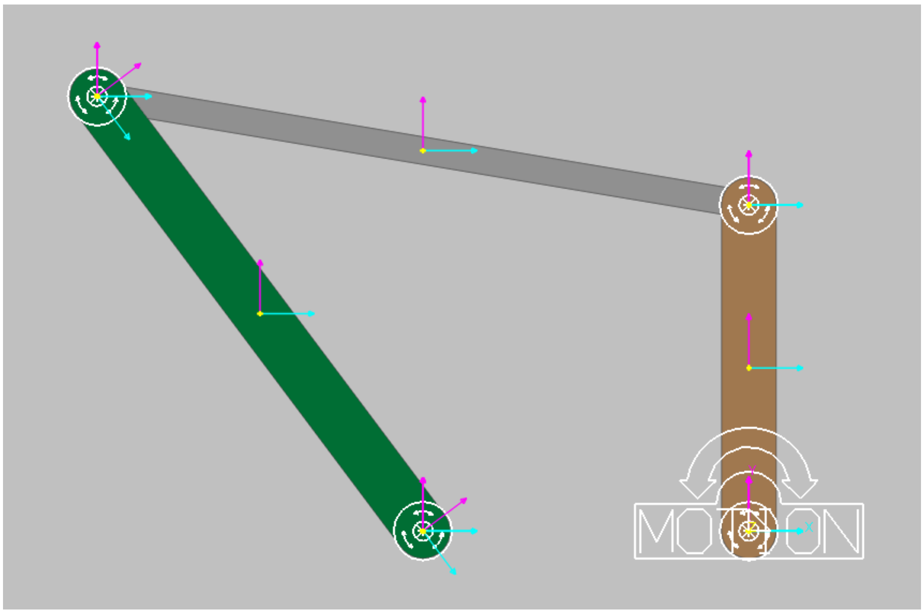
$\vec{j}\ :\ 0.12 \cdot \omega_{AC} = 0.24 \cdot \omega_{AB}$

$\therefore \omega_{AB} = 0.214\ \text{rad/s}$

$\therefore \omega_{AC} = 0.429\ \text{rad/s}$

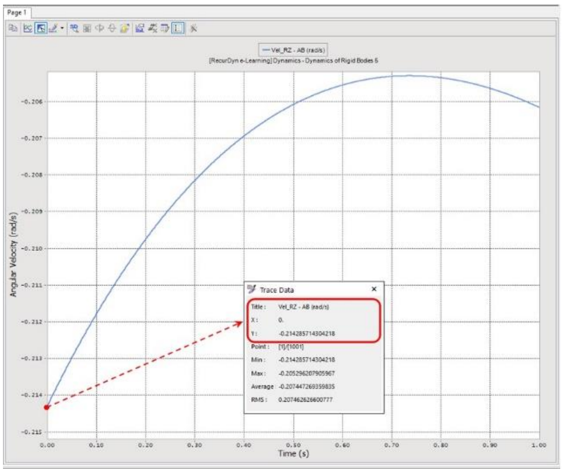
Numerical Solution - RecurDyn

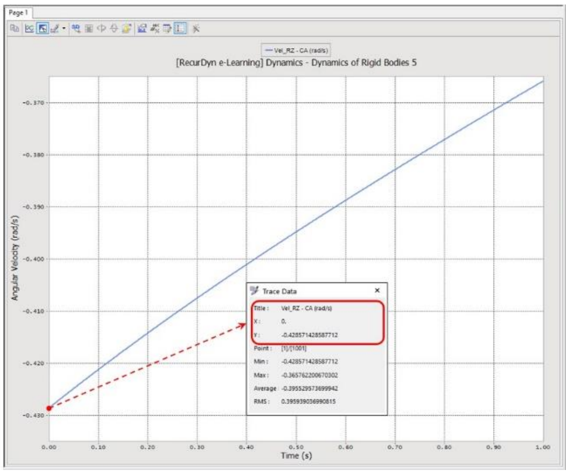
1. Modeling



2. Plot the results

- The angular velocity on Z axis of the “AB” and “C

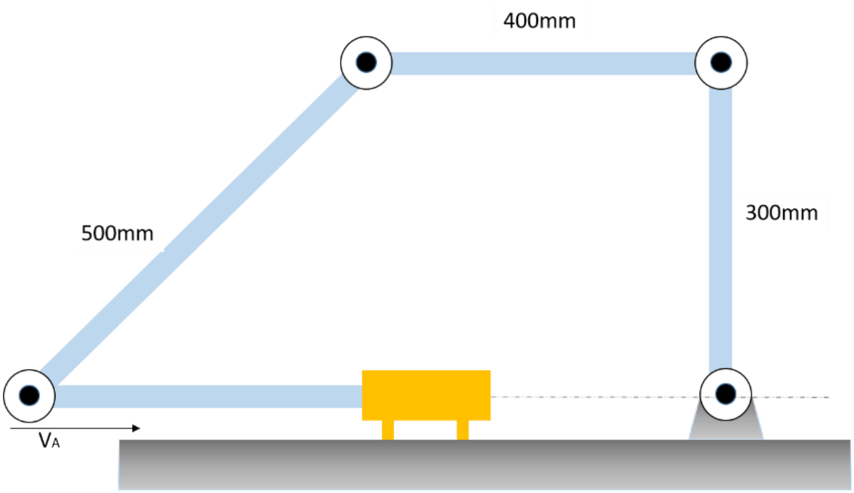




Comparison of results

Object Value	Theory	RecurDyn	Error(%)
ω_{AB} $[rad/s]$	0.214	0.214	0
ω_{AC} $[rad/s]$	0.429	0.429	0

Dynamics of Rigid Bodies.16



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.405, P.5.195
Type of Analysis : Plane Kinematics of Rigid Bodies
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

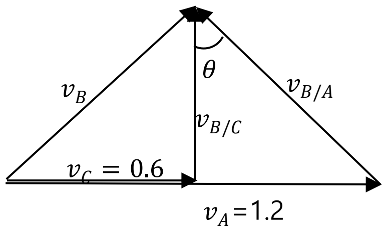
Given	Symbol	Value	Unit
Initial velocity	v_A	1.2	m/s
Angular velocity	ω_{OC}	2	rad/s

The Relative Velocity Equation of the points "A", "B", and "C"

$$\vec{v_B} = \vec{v_A} + \overrightarrow{v_{B/A}} = \vec{v_C} + \overrightarrow{v_{B/C}}$$

$$\vec{v_C} = -\omega_{OC}\vec{k} \times 0.3\vec{j} = 0.6\vec{i}$$

A Graph of the Velocity Relationship



$$v_{B/C} = \frac{v_A - v_C}{\tan\theta} = \frac{0.6}{3/4} = 0.8$$

$$\omega_{BC} = \frac{v_{B/C}}{\overline{BC}} = \frac{0.8}{0.4} = 2\,rad/s$$

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}} = \overrightarrow{v_C} + \overrightarrow{v_{B/C}} = 0.6\vec{\imath} + 0.8\vec{\jmath}$$

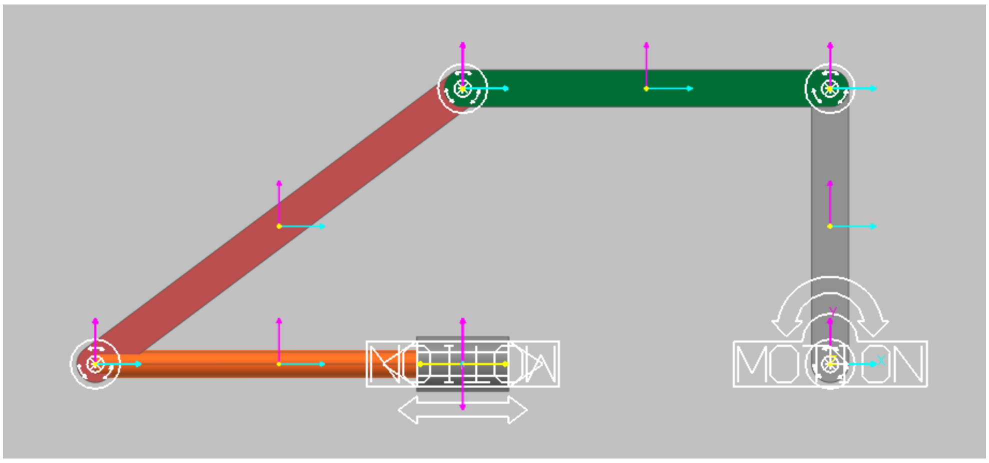
The Angular Acceleration of Link “AB”

$$v_{B/A} = \frac{v_A - v_C}{\sin\theta} = \frac{0.6}{3/5} = 1.0$$

$$\omega_{AB} = \frac{v_{B/A}}{\overline{AB}} = \frac{1.0}{0.5} = 2\,rad/s$$

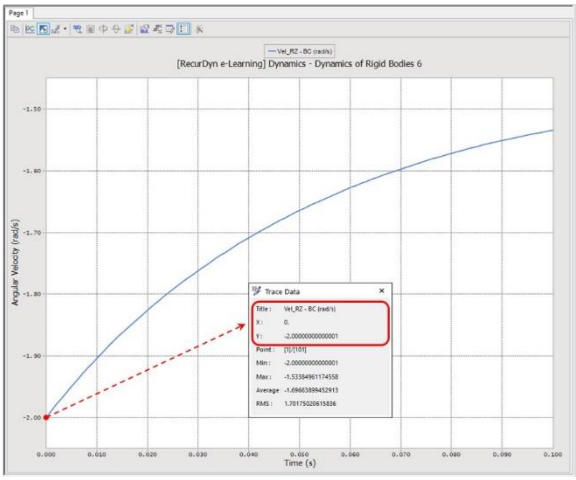
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

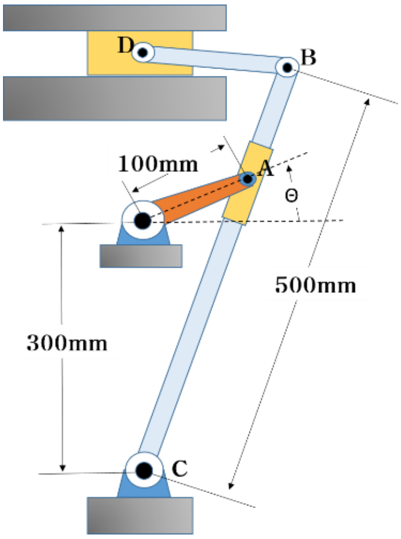
- The angular velocity on Z axis of the “BC”



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
ω_{BC} $[rad/s]$	2	2	0

Dynamics of Rigid Bodies.17



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.405, P.5.198
Type of Analysis : Plane Kinematics of Rigid Bodies, Linear Motion
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OA}	3	rad/s
Angle	θ	30	deg

Geometrical Constraint - Second Law of Cosines

$$\overline{CA}^2 = 0.3^2 + 0.1^2 - 2 \times 0.1 \times 0.3 \times \cos 120 = 0.13$$

$$\overline{CA} = 0.361$$

$$\frac{0.1}{\sin\beta} = \frac{\overline{CA}}{\sin 120}$$

$$\beta = sin^{-1}(\frac{0.1}{\overline{CA}}\sin 120) = sin^{-1}\left(\frac{0.1}{0.361}\sin 120\right) = 13.88°$$

$$\gamma = 180 - 120 - 13.88 = 46.12°$$

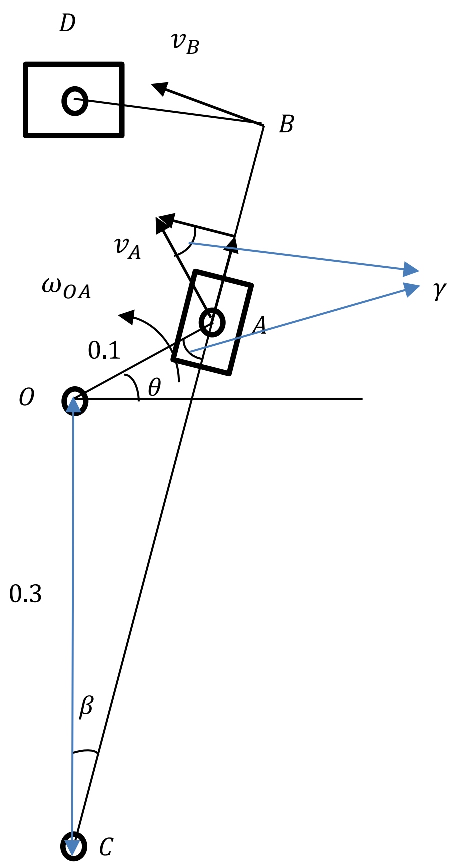
Calculate a Velocity

$$v_A = \omega_{OA} \cdot \overline{OA} = 3 \cdot 0.1 = 0.3\, rad/s$$

$$\omega_{CA} = \frac{v_A \cos\gamma}{\overline{CA}} = \frac{0.3 \cdot \cos 46.12}{0.361} = 0.576\, rad/s$$

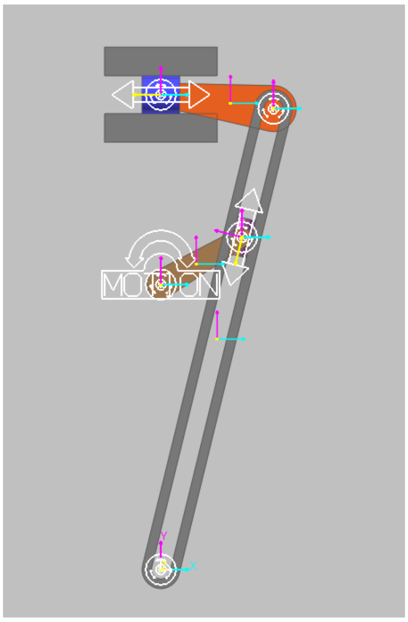
Where, $\omega_{CA} = \omega_{BC}$

$$v_B = \omega_{BC} \cdot \overline{BC} = 0.576 \cdot 0.5 = 0.288\, m/s$$

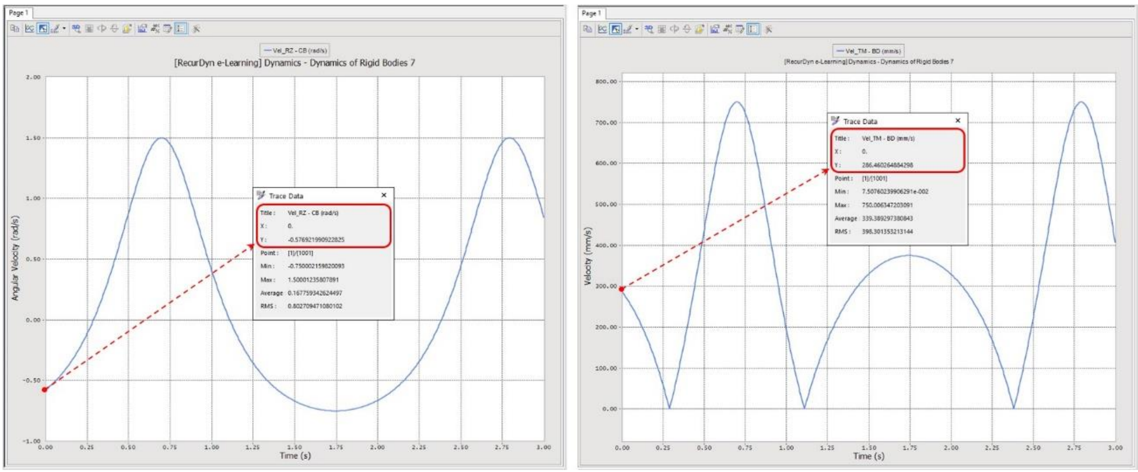


Numerical Solution - RecurDyn

1. Modeling



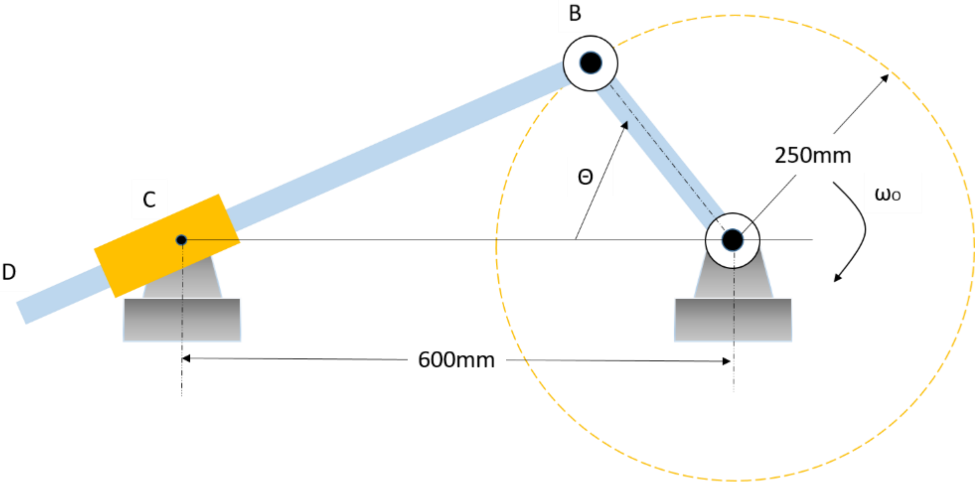
2. Plot the results
   - The acceleration in the Z direction of "CB" and the velocity of the point "B" of "BD"



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\omega_{BC}\ [rad/s]$	0.576	0.577	0.174
$v_B\ [m/s]$	0.288	0.286	0.694

Dynamics of Rigid Bodies.18

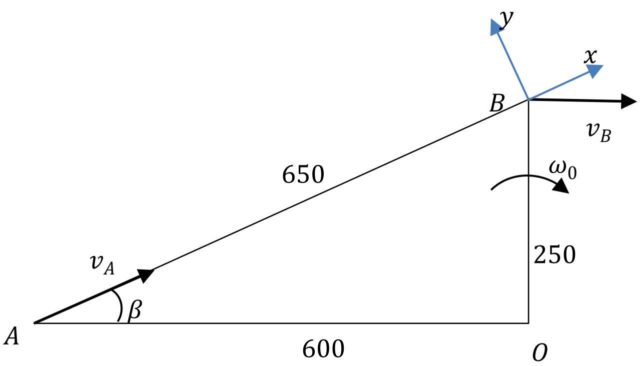


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.408, P.5.204
Type of Analysis : Plane Kinematics of Rigid Bodies
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_0	5	rad/s



$$\overrightarrow{v_B} = \overrightarrow{\omega_0} \times \overrightarrow{r_{OB}} = -5\vec{k} \times (0.25 \cdot \sin 22.62\,\vec{\imath} + 0.25 \cdot \cos 22.62\,\vec{\jmath}) = 1.154\vec{\imath} - 0.481\vec{\imath}$$

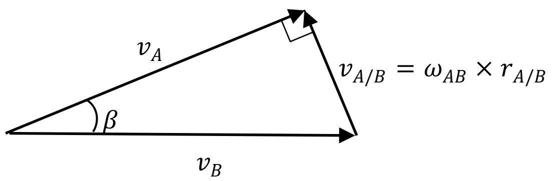
$\beta = tan^{-1}\dfrac{250}{600} = 22.62°$

1. The Equation of Relative Velocity

If a point located on the point "C" of the rod "BD" is called "A",

$\overrightarrow{v_B} = \overrightarrow{\omega_0} \times \overrightarrow{r_{OB}} = -5\vec{k} \times (0.25 \cdot \sin 22.62\,\vec{\imath} + 0.25 \cdot \cos 22.62\,\vec{\jmath}) = 1.154\vec{\imath} - 0.481\vec{\imath}$

$\overrightarrow{v_A} = \overrightarrow{v_B} + \overrightarrow{v_{A/B}} = \overrightarrow{v_B} + \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{A/B}}$



$v_A = v_B \cos\beta = 1.25 \cdot \cos 22.62 = 1.154$

$v_{A/B} = v_B \sin\beta = 1.25 \cdot \sin 22.62 = 0.481 = \omega_{AB} \cdot r_{A/B}$

$r_{A/B} = 0.65$, then the angular velocity is $\omega_{AB} = 0.74\,rad/s$.

These velocities as a vector are,

$\overrightarrow{v_A} = 1.154\vec{\imath}$

$\overrightarrow{v_{A/B}} = 0.481\vec{\jmath}$ (1)

$\overrightarrow{r_{A/B}} = -0.65\vec{\imath}$

If the angular velocity is assumed to be $\overrightarrow{\omega_{AB}} = 0.74\vec{k}$, then

$\overrightarrow{v_{A/B}} = \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{A/B}} = 0.74\vec{k} \times (-0.65\vec{\imath}) = -0.481\vec{\jmath}$ (2)

Since the signs of (1) and (2) are different, $\overrightarrow{\omega_{AB}} = -0.74\vec{k}$

Finally, the angular velocity is $\overrightarrow{\omega_{AB}} = -0.74\vec{k}$ because the equation (1) and the equation (2) have different signs.

2. The Equation of Relative Acceleration

$$\overrightarrow{a_A} = \overrightarrow{a_B} + \dot{\overrightarrow{\omega_{AB}}} \times \overrightarrow{r_{A/B}} + \overrightarrow{\omega_{AB}} \times \left(\overrightarrow{\omega_{AB}} \times \overrightarrow{r_{A/B}}\right) + 2\overrightarrow{\omega_{AB}} \times \overrightarrow{v_{rel}} + \overrightarrow{a_{rel}}$$

$$\overrightarrow{a_B} = \overrightarrow{\omega_0} \times (\overrightarrow{\omega_0} \times \overrightarrow{r_{OB}}) = -5\vec{k} \times \left(-5\vec{k} \times (0.25 \cdot \sin 22.62\,\vec{\imath} + 0.25 \cdot \cos 22.62\,\vec{\jmath})\right)$$

$$= -2.404\vec{\imath} - 5.769\vec{\jmath}$$

$$\dot{\overrightarrow{\omega_{AB}}} \times \overrightarrow{r_{A/B}} = \dot{\omega_{AB}}\vec{k} \times (-0.65\vec{\imath}) = -0.65 \cdot \dot{\omega_{AB}}\vec{\jmath}$$

$$\overrightarrow{\omega_{AB}} \times \left(\overrightarrow{\omega_{AB}} \times \overrightarrow{r_{A/B}}\right) = -0.74\vec{k} \times \left(-0.74\vec{k} \times (-0.65\vec{\imath})\right) = 0.356\vec{\imath}$$

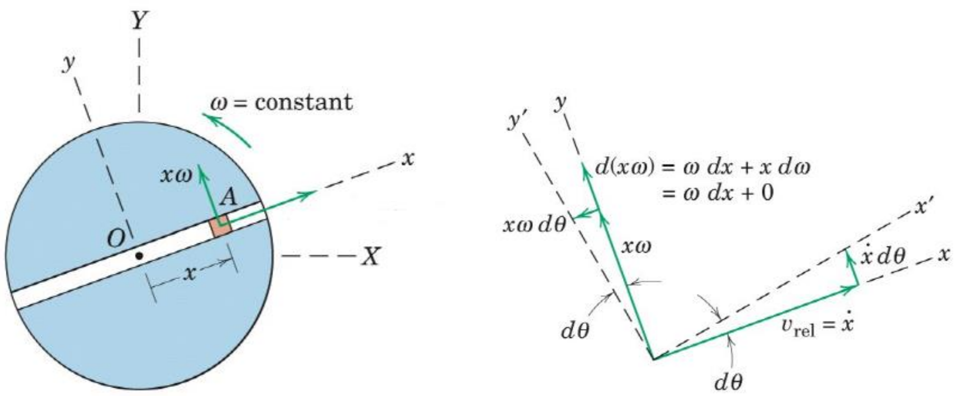
$$\overrightarrow{v_{rel}} = 0$$

$$\overrightarrow{a_{rel}} = 0$$

$$\overrightarrow{a_A} = a_{Ax}\vec{\imath} + a_{Ay}\vec{\jmath} = -2.404\vec{\imath} - 5.769\vec{\jmath} - 0.65 \cdot \dot{\omega_{AB}}\vec{\jmath} + 0.356\vec{\imath}$$

$$\vec{\imath} \;\; : \; a_{Ax} = -2.404 + 0.356 = -2.048$$

$$\vec{\jmath} \;\; : \; a_{Ay} = -5.769 - 0.65 \cdot \dot{\omega_{AB}}$$



Consider Coriolis acceleration of the pivoted collar that rotates about a fixed axis through the point "C",

$$\overrightarrow{a_{Ay}} = 2\overrightarrow{\omega_C} \times \overrightarrow{v_A} = -2 \cdot 0.74\vec{k} \times 1.154\vec{\imath} = -1.708\vec{\jmath}$$

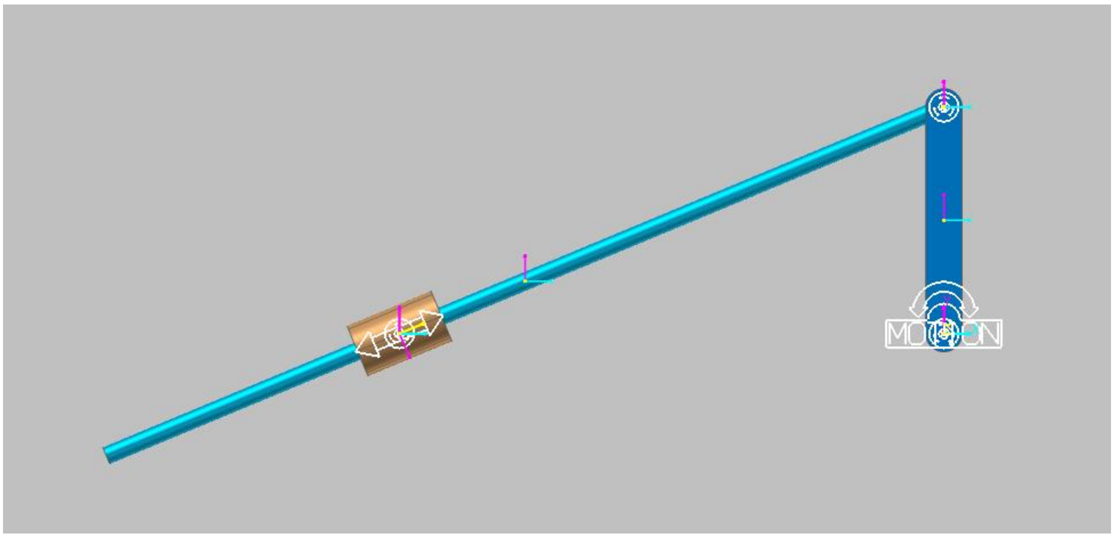
$$a_{Ay} = -5.769 - 0.65 \cdot \dot{\omega_{AB}} = -1.708$$

$$\therefore \dot{\omega_{AB}} = -6.248 \text{ rad/s}$$

$$\therefore \vec{\alpha} = \dot{\overrightarrow{\omega_{AB}}} = -6.248\vec{k}\,\frac{\text{rad}}{\text{s}}$$

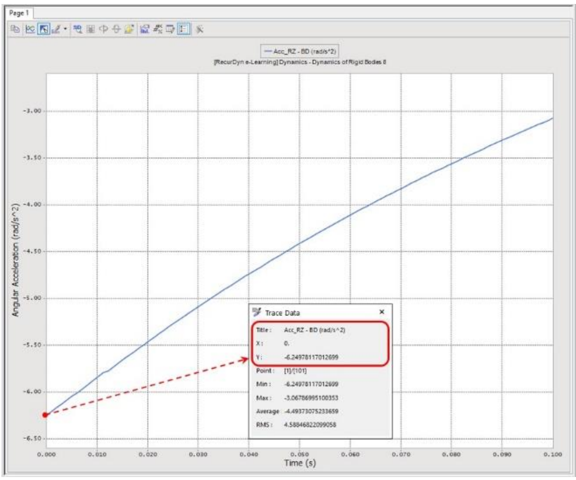
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

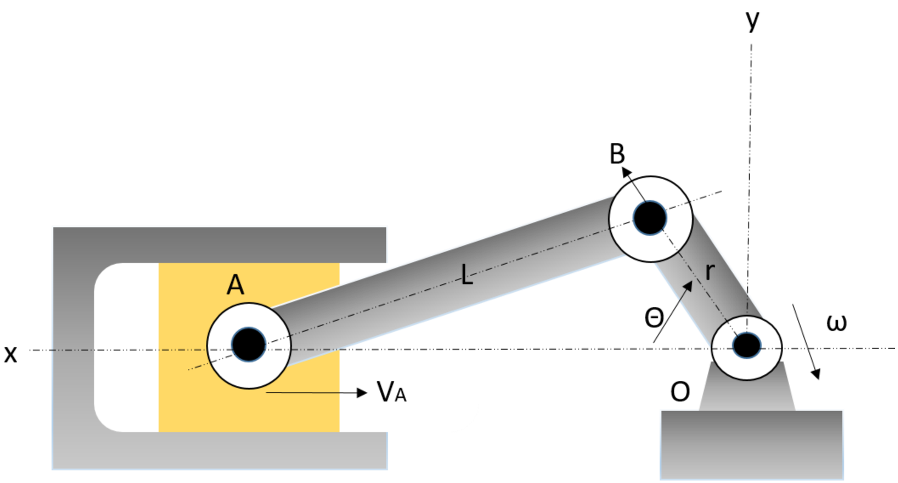
- The angular acceleration about the Z axis of "BD"



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\alpha_{BD}\ [rad/s^2]$	-6.25	-6.24	0.16

Dynamics of Rigid Bodies.19

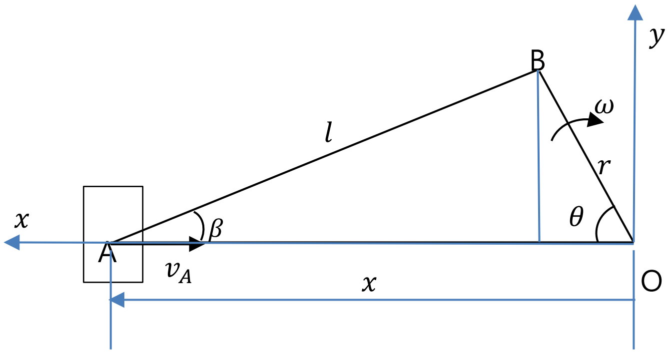


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.408, P.5.212
Type of Analysis : Plane Kinematics of Rigid Bodies
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω	157.08	rad/s
"AB" length	l	0.35	m
"OB" length	r	0.125	m



Geometrical Constraint

$x = l \cdot cos\,\beta + r \cdot cos\,\theta$

$v_A = -\dot{x} = l \cdot \dot{\beta} \cdot sin\,\beta + r \cdot \dot{\theta} \cdot sin\,\theta$

$l \cdot sin\,\beta = r \cdot sin\,\theta$

Calculate a Velocity

Differentiate both sides of the equation,

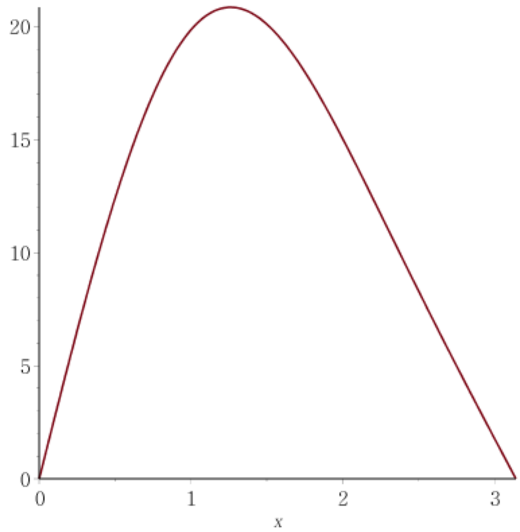
$l \cdot \dot{\beta} \cdot cos\,\beta = r \cdot \dot{\theta} \cdot cos\,\theta$

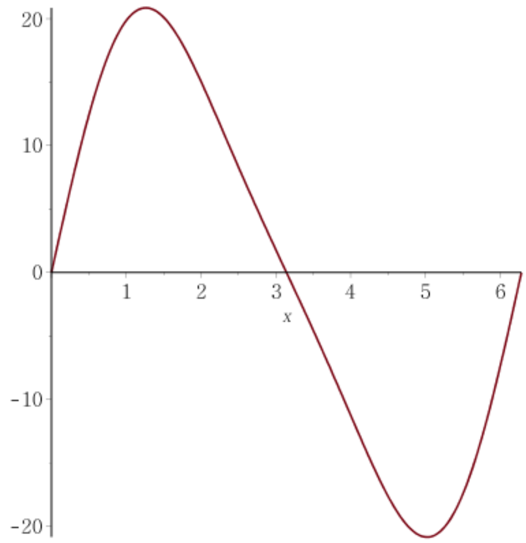
$$\dot{\beta} = \frac{r}{l} \cdot \frac{cos\,\theta}{cos\,\beta} \cdot \dot{\theta} = \frac{r}{l} \cdot \dot{\theta} \cdot \frac{cos\,\theta}{\sqrt{1 - sin^2\beta}} = \frac{r}{l} \cdot \dot{\theta} \cdot \frac{cos\,\theta}{\sqrt{1 - \left(\frac{r}{l}\sin\theta\right)^2}} = \frac{\omega \cdot cos\,\theta}{\sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta}}$$

$$\therefore v_A = l \cdot \frac{\omega \cdot cos\,\theta}{\sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta}} \cdot \frac{r}{l}\sin\theta + r \cdot \omega \cdot sin\,\theta = r \cdot \omega \cdot sin\,\theta \cdot \left[1 + \frac{cos\,\theta}{\sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta}}\right]$$

Where, $l = 0.35\,m$, $r = 0.125\,m$, $\omega = 1500\,rev/m = 1500 \cdot 2\pi/60 = 157.08\,rad/s$

$$v_A = 19.635 \cdot sin\,\theta \cdot \left[1 + \frac{cos\,\theta}{\sqrt{7.84 - sin^2\theta}}\right]$$





$\therefore v_{A_{max}} = 20.865\, m/s \quad at \quad \theta = 1.2566\, rad = 72°$

Calculate the Acceleration

Differentiate the equation to calculate the acceleration,

$$a_A = \frac{dv_A}{dt} = r \cdot \omega \cdot \left[\dot{\theta} \cdot cos\,\theta + \frac{1}{2} \frac{2\dot{\theta}\cos 2\theta \cdot \sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta} - \sin 2\theta \frac{-2\sin\theta \cdot \cos\theta \cdot \dot{\theta}}{2\sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta}}}{\left(\frac{l}{r}\right)^2 - sin^2\theta} \right]$$

$$= r \cdot \omega^2 \cdot \left[cos\,\theta + \frac{\cos 2\theta \cdot \sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta} - \sin 2\theta \frac{-\frac{1}{2}\sin 2\theta}{2\sqrt{\left(\frac{l}{r}\right)^2 - sin^2\theta}}}{\left(\frac{l}{r}\right)^2 - sin^2\theta} \right]$$

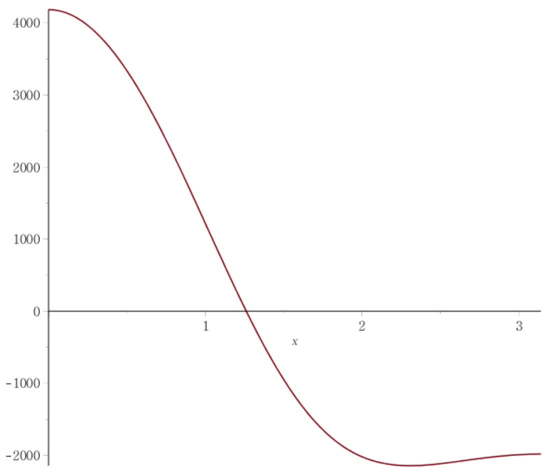
$$= r \cdot \omega^2 \cdot \left[cos\,\theta + \frac{\cos 2\theta \cdot \left[\left(\frac{l}{r}\right)^2 - sin^2\theta\right] + \frac{1}{4} sin^2 2\theta}{\left[\left(\frac{l}{r}\right)^2 - sin^2\theta\right]^{3/2}} \right]$$

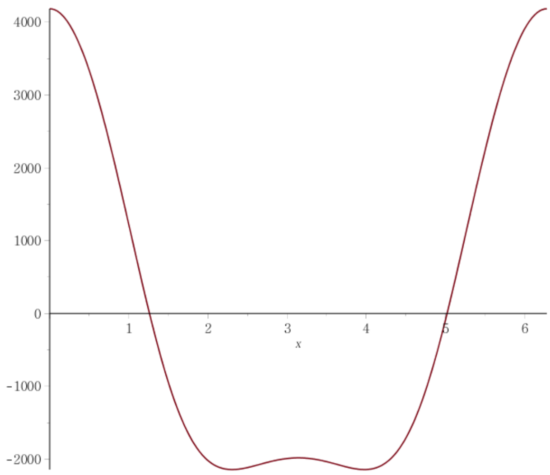
$$= r \cdot \omega^2 \cdot \left[cos\,\theta + \frac{r}{l} \cdot \frac{(cos^2\theta - sin^2\theta) \cdot \left[1 - \left(\frac{r}{l}\right)^2 sin^2\theta\right] + \frac{1}{4}\left(\frac{r}{l}\right)^2 (4 \cdot sin^2\theta \cdot cos^2\theta)}{\left[1 - \left(\frac{r}{l}\right)^2 sin^2\theta\right]^{3/2}} \right]$$

$$\therefore a_A = r \cdot \omega^2 \cdot \left[\cos\theta + \frac{r}{l} \cdot \frac{\left[1 - 2\sin^2\theta + \left(\frac{r}{l}\right)^2 \sin^4\theta\right]}{\left[1 - \left(\frac{r}{l}\right)^2 \sin^2\theta\right]^{3/2}} \right]$$

Where, $l = 0.35\,m$, $r = 0.125\,m$, $\omega = 1500\,rev/m = 1500 \cdot 2\pi/60 = 157.08\,rad/s$

$$a_A = 3084.27 \cdot \left[\cos\theta + 0.357 \cdot \frac{[1 - 2\sin^2\theta + 0.128 \cdot \sin^4\theta]}{[1 - 0.128 \cdot \sin^2\theta]^{3/2}} \right]$$





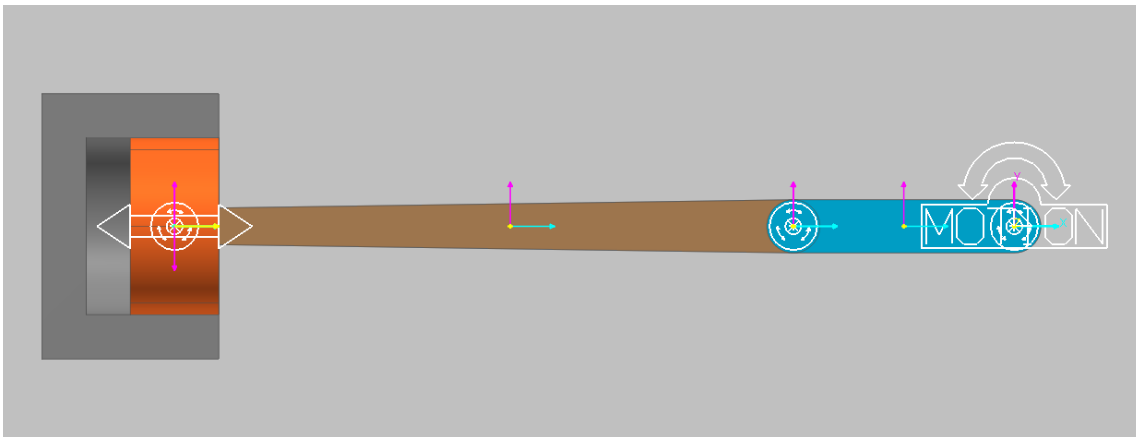
$a_{A_{max}} = 4172.97\,m/s^2 \quad at \;\; \theta = 0°$

$a_A = -1983.19\,m/s^2 \quad at \;\; \theta = 180°$

$a_A = 0\,m/s^2 \;\; at \;\; \theta = 1.26125 rad = 72.26°$

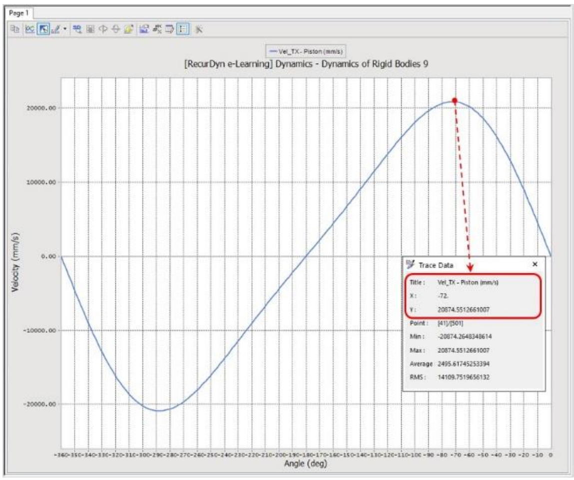
Numerical Solution - RecurDyn

1. Modeling

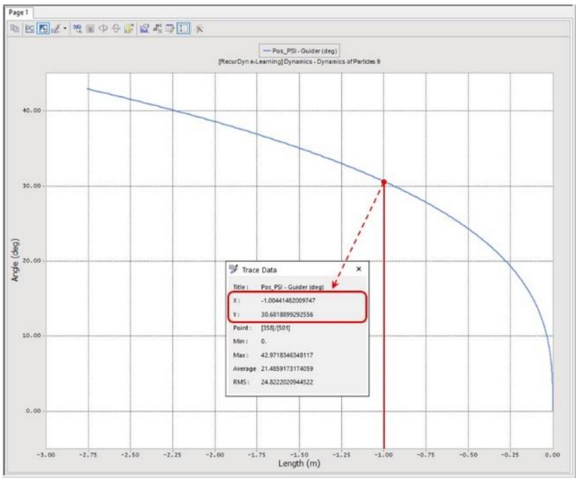


2. Plot the results

- The velocity on X axis of the “Piston”
  (X axis : the angle of the “RevJoint1”,Y axis : The velocity on X axis of the “Piston”)



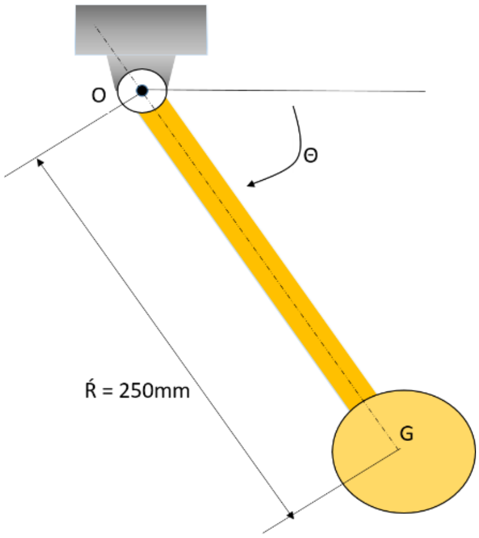
- Calculate of the “θ” value when $a_A = 0\,m/s^2$



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$V_{A,max}$ $[m/s]$	20.865	20.875	0.048
θ $[rad]$	1.2566	1.2566	0

Dynamics of Rigid Bodies.20



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.433, E.6.4
Type of Analysis : Plane Kinematics of Rigid Bodies, Rotation about a Fixed Axis
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	7.5	kg
"OG" length	r_O	0.295	m
Initial angle	θ	60	degree

$$\sum M_O = I_O \alpha = mg\bar{r}\cos\theta$$

$$mr_O{}^2\alpha = 7.5 \times 0.295^2 \times \alpha = 7.5 \times 9.81 \times 0.25 \times \cos\theta$$

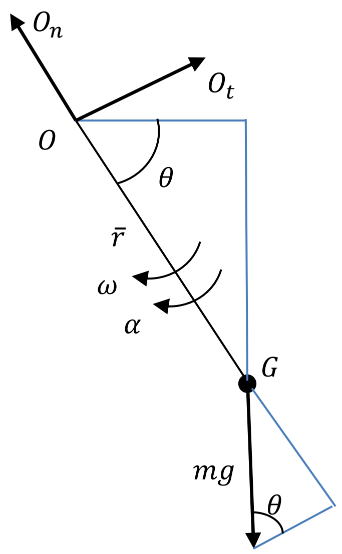
$$\alpha = 28.18\cos\theta$$

$$\therefore \alpha = 14.091 \text{ rad}/s^2$$

$$\int_0^{\omega} \omega d\omega = \int_0^{\theta} \alpha d\theta = \int_0^{\theta} 28.18 \cdot \cos\theta \, d\theta$$

$$\frac{1}{2}\omega^2 = 28.18 \cdot \sin\theta$$

$$\therefore \omega = 6.99 \text{ rad/s}$$



$$\sum F_n = O_n - mg\sin\theta = ma_n = m\bar{r}\omega^2$$

$$O_n - 7.5 \times 9.81 \times \sin 60 = 7.5 \times 0.25 \times 6.99^2$$

$$\therefore O_n = 155.33 \text{ [N]}$$

$$\sum F_t = -O_t + mg\cos\theta = ma_t = m\bar{r}\,\alpha$$

$$-O_t + 7.5 \times 9.81 \times \cos 60 = 7.5 \times 0.25 \times 14.091$$

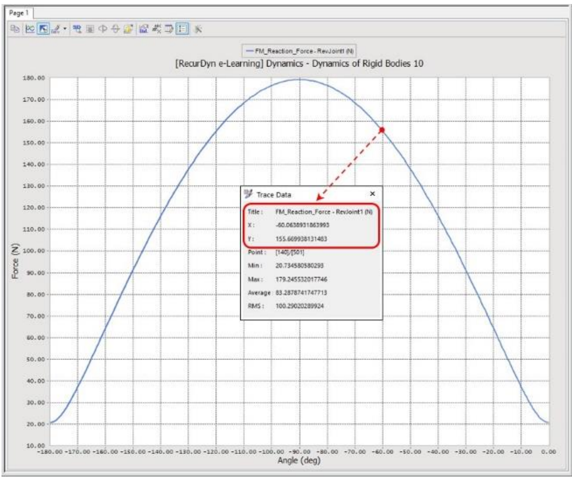
$$\therefore O_t = 10.37 \text{ [N]}$$

$$\therefore \text{O} = \sqrt{{O_t}^2 + {O_n}^2} = 155.68 \text{ [N]}$$

Numerical Solution - RecurDyn

1. Modeling

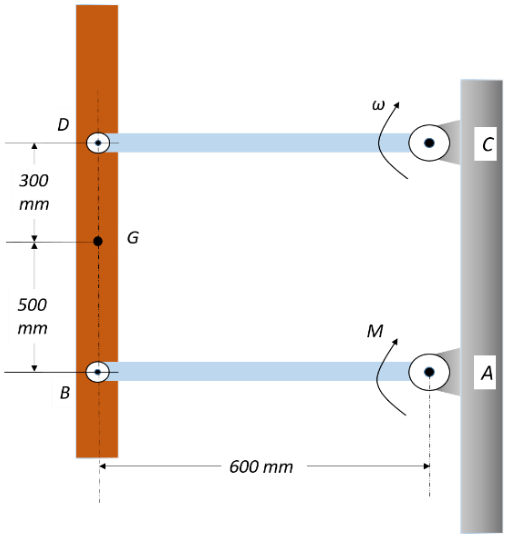
2. Plot the results
   - The total reaction force when the angle is "60.06°"



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
R_O $[N]$	155.65	155.67(60.06°)	0.013

Dynamics of Rigid Bodies.21

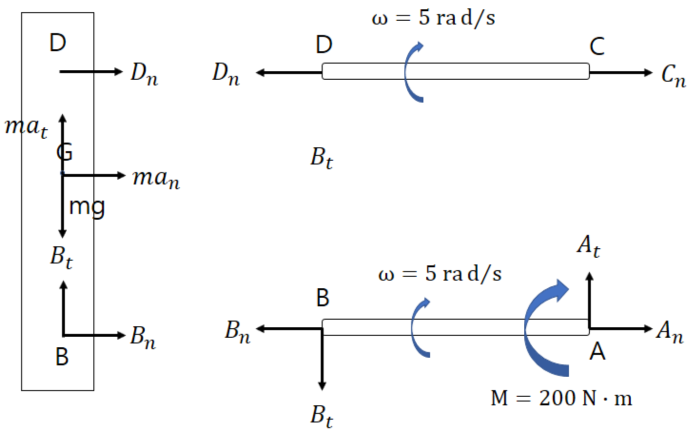


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.428, P.6.25
Type of Analysis : Plane Kinematics of Rigid Bodies, Translational Motion
Type of Element : Rigid Body (three parts)

● Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Momentum	M	200	$N \cdot m$
Angular velocity	ω	5	rad/s



In the case of the link "CD",

$a_n = \mathrm{r}\omega^2 = 0.6 \times 5^2 = 15\ m/s^2$

In the case of the link "BD",

$\sum M_B = D_n \times 0.8 = ma_n\mathrm{d}$

$D_n \times 0.8 = 25 \times 0.6 \times 5^2 \times 0.5 = 187.5$

$\therefore\ D_n = 235.375\ [N]$

It is assumed that the mass of the link "AB" is ignored,

$\sum M_A = M - B_t \times 0.6 = m_{AB}a_t d_1 \approx 0$

$B_t = 200/0.6 = 333.33$

In the case of the link "BD",

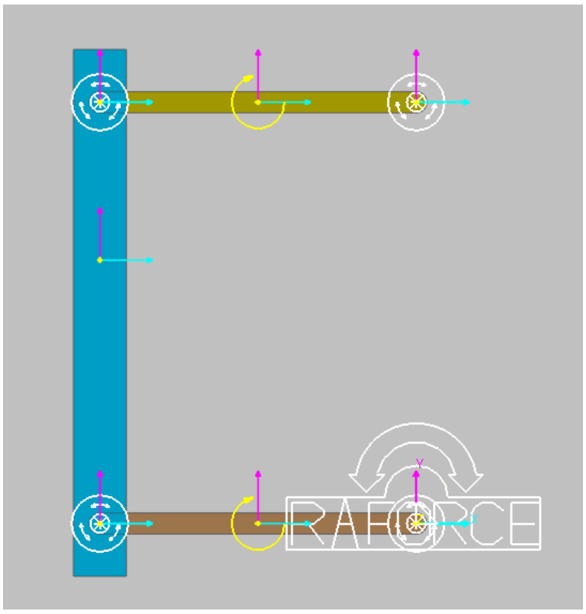
$\sum F_t = ma_t = mr\alpha$

$B_t - mg = mr\alpha$

$\therefore\ \alpha = {(B_t - mg)}/{(mr)} = 5.872\ [\mathrm{r}/s^2]$

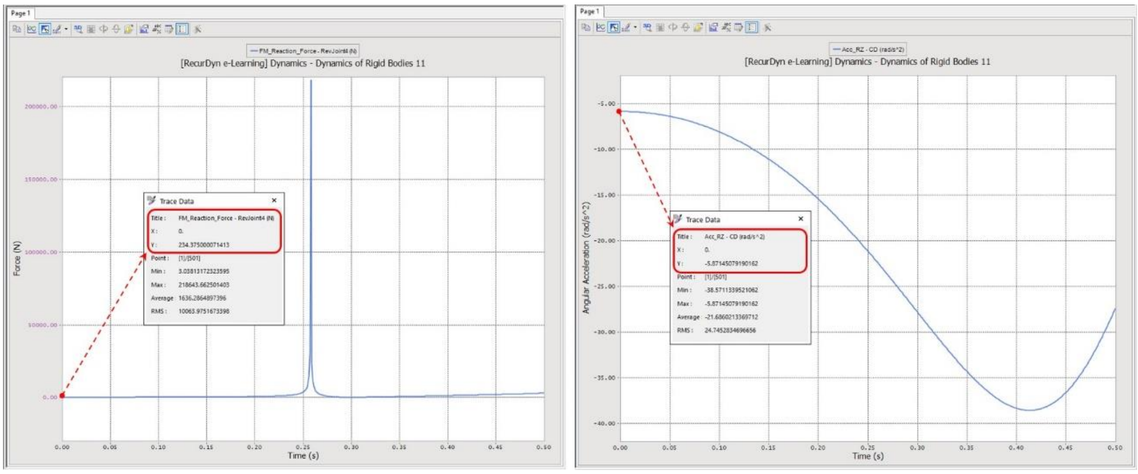
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

   - The size of the reaction force of the Joint "D" and the angular acceleration of the link "CD"

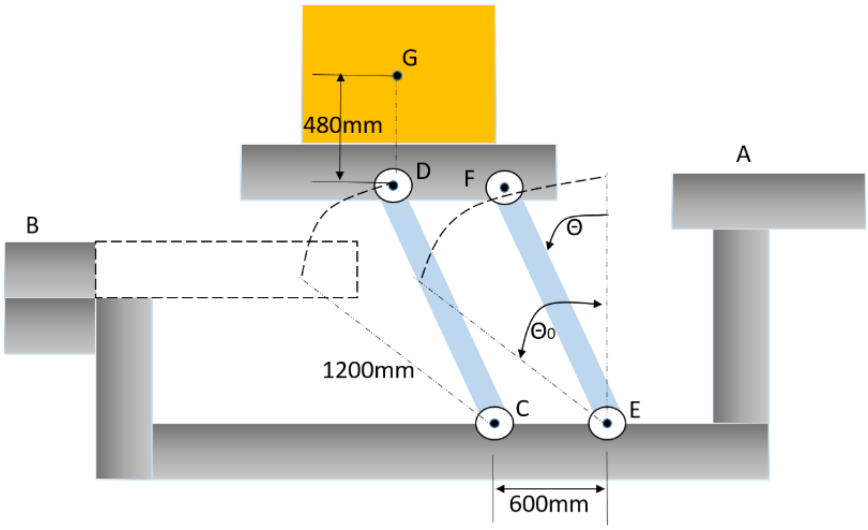


Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{R_D}$ $[m/s^2]$	235.375	234.375	0.425

$\boldsymbol{\alpha}\ [\boldsymbol{rad/s^2}]$	5.871	5.872	0.017

Dynamics of Rigid Bodies.22



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.429, P.6.29
Type of Analysis : Plane Kinematics of Rigid Bodies, Translational Motion
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
"CE" length	l_{CE}	600	mm
"CD" length	l_{CD}	1200	mm

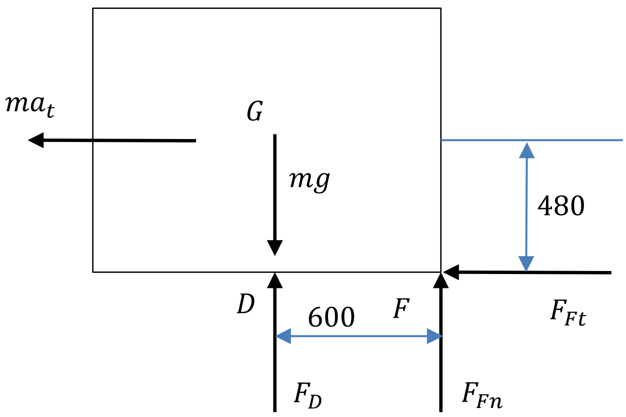
The Driving Constraint

$$\theta = \frac{\pi}{6}(1 - cos\frac{\pi t}{2})$$

$$\dot{\theta} = \frac{\pi}{6} \cdot \frac{\pi}{2} \cdot sin\frac{\pi t}{2} = \frac{\pi^2}{12} \cdot sin\frac{\pi t}{2}$$

$$\ddot{\theta} = \frac{\pi^2}{12} \cdot \frac{\pi}{2} \cdot cos\frac{\pi t}{2} = \frac{\pi^3}{24} \cdot cos\frac{\pi t}{2}$$

(a) $\theta = 0°,\ t = 0$



$$\dot{\theta} = 0$$

$$\ddot{\theta} = \frac{\pi^3}{24}$$

$$a_t = r\ddot{\theta} = 1.2 \times \frac{\pi^3}{24} = 1.55$$

$$\sum M_F = (mg - F_D) \cdot 0.6 = ma_t \cdot 0.48$$

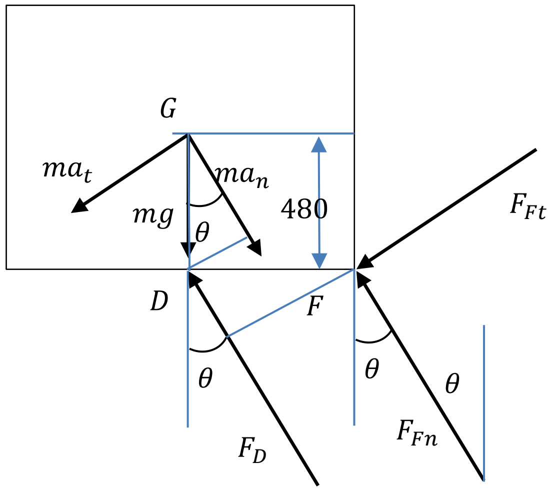
$$\therefore F_D = 1714\ [N]$$

(b) $t = 1\ \mathrm{sec}$

$$\theta = \frac{\pi}{6}\left(1 - cos\frac{\pi}{2}\right) = \frac{\pi}{6}$$

$$\dot{\theta} = \frac{\pi^2}{12} \cdot sin\frac{\pi}{2} = \frac{\pi^2}{12}$$

$$\ddot{\theta} = \frac{\pi^3}{24} \cdot cos\frac{\pi}{2} = 0$$



$a_t = 0$

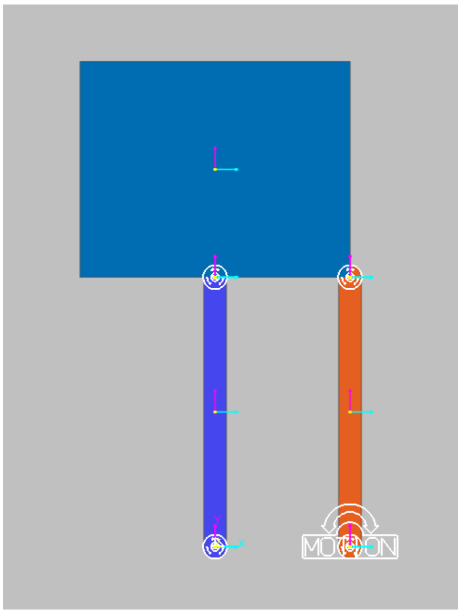
$a_n = r\dot{\theta}^2 = 1.2 \times \left[\frac{\pi^2}{12}\right]^2 = 0.812$

$\sum M_F = mg \cdot 0.6 - F_D \cdot 0.6 \cdot \cos\theta = ma_n \cdot (0.6 \cdot \cos\theta - 0.48 \cdot \sin\theta)$

$\therefore F_D = 2178.13\ [N]$

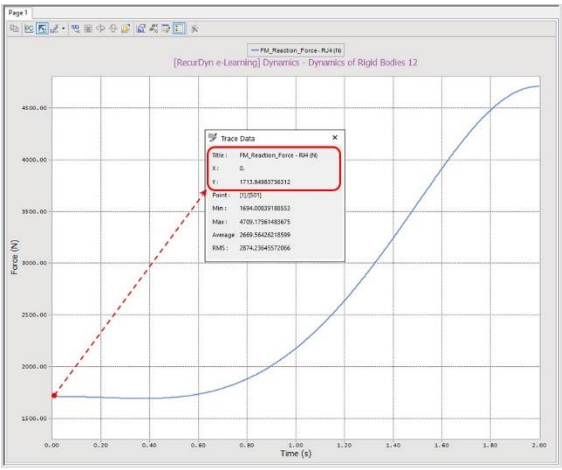
⭘ Numerical Solution - RecurDyn

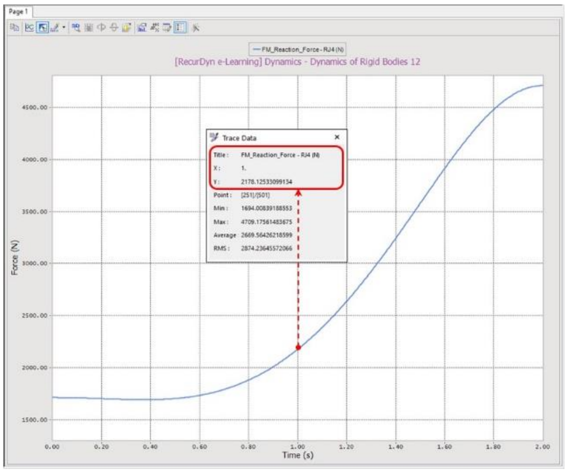
1. Modeling



2. Plot the results

- The reaction force of the revolute joint “*D*” when the time is 0 and 1

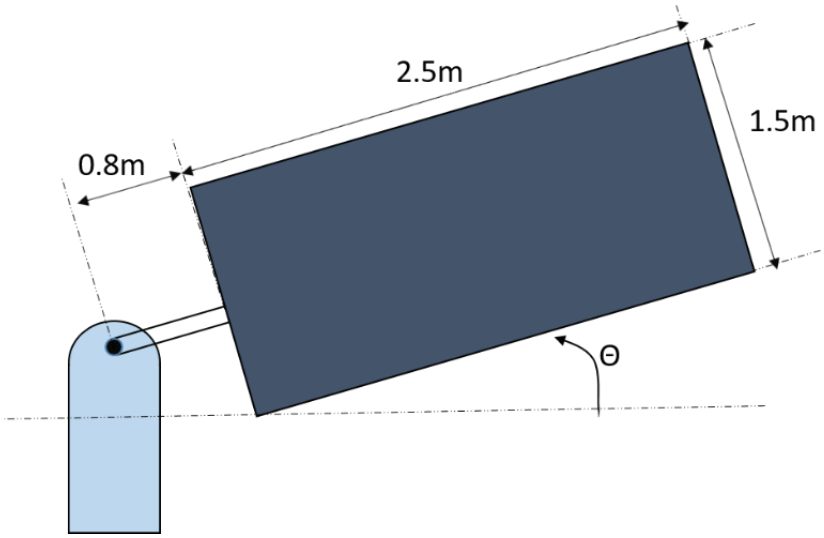




⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$D_{t=0}$ $[N]$	1714	1714	0
$D_{t=1}$ $[N]$	2178	2178	0

Dynamics of Rigid Bodies.23

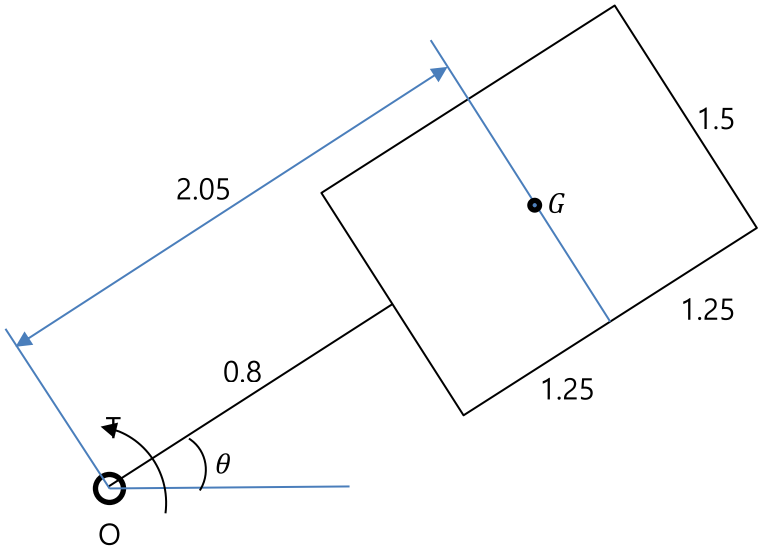


References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.439, P.6.57
Type of Analysis : Plane Kinematics of Rigid Bodies, Rotation about a Fixed Axis
Type of Element : Rigid Body (one part)

⭘ Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	6000	kg
Torque	T	30	$N \cdot m$



$$I_G = \frac{1}{12}6000 \cdot [1.5^2 + 2.5^2] = 4{,}250 \text{ kg} \cdot m^2$$

$$I_O = I_G + md^2 = 4250 + 6000 \cdot 2.05^2 = 29{,}465 \text{ kg} \cdot m^2$$

$$\sum M_O = I_O \alpha$$

$$T = 30 = 29465 \times \alpha$$

$$\alpha = 1.018 \times 10^{-3}\ rad/s^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Calculate the time when the rotation angle, "θ", is $45°$,

$\theta_0 = 0$, $\omega_0 = 0$, $\theta = 45° = \frac{\pi}{4}$

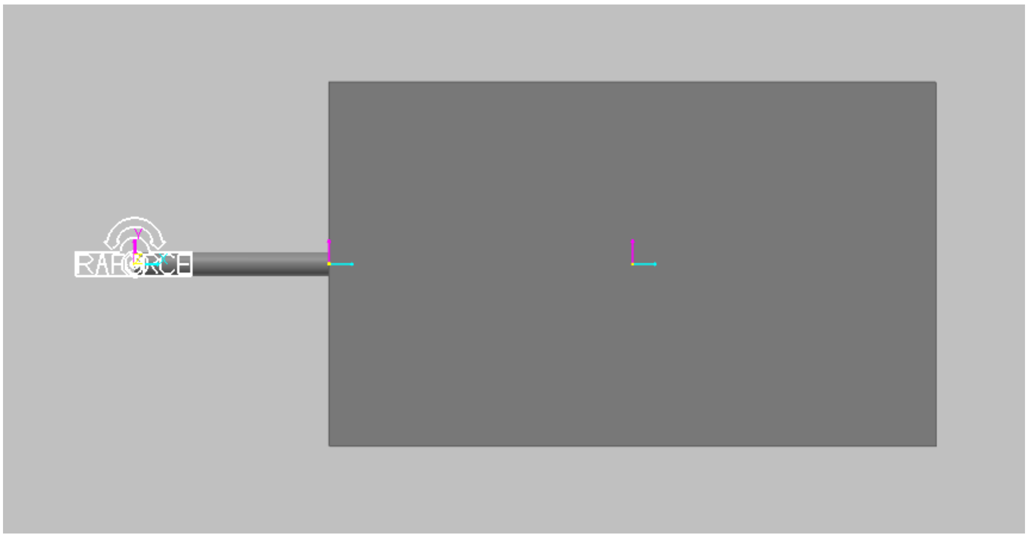
$$\frac{\pi}{4} = \frac{1}{2} \times 1.018 \times 10^{-3} \times {t_1}^2$$

$$t_1 = 39.28\ s$$

$$\therefore t = 78.56\ s$$

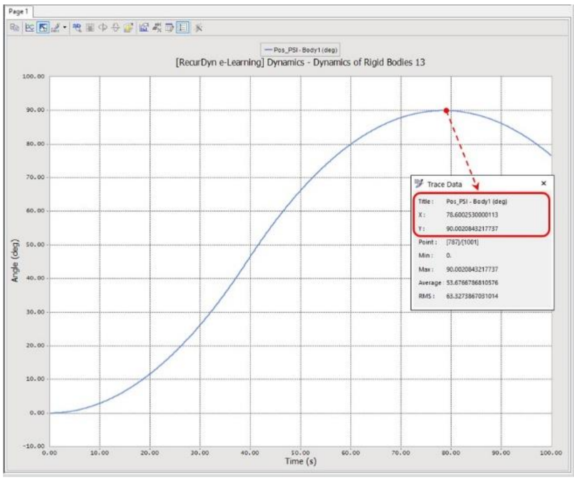
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

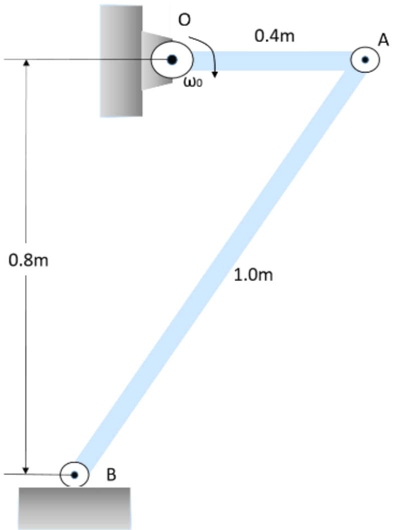
   - Measure the time when the rotation angle is $90°$



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
t $[s]$	78.56	78.6	0.051

Dynamics of Rigid Bodies.24

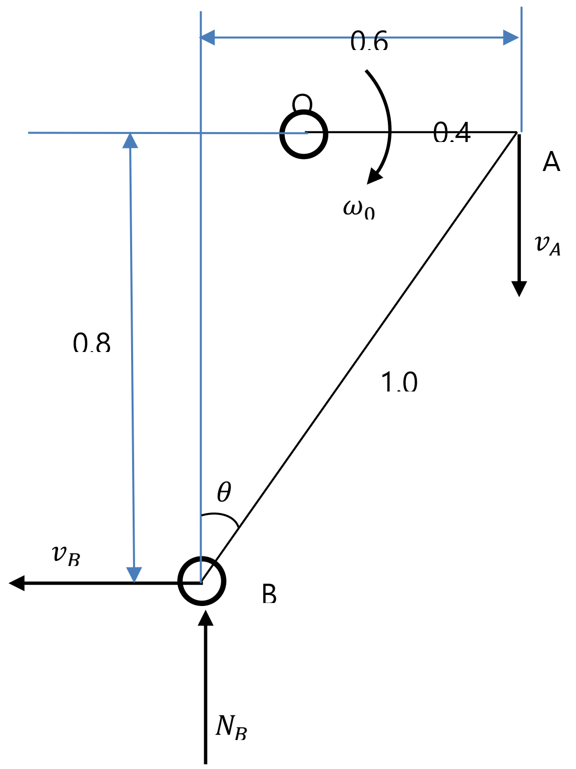


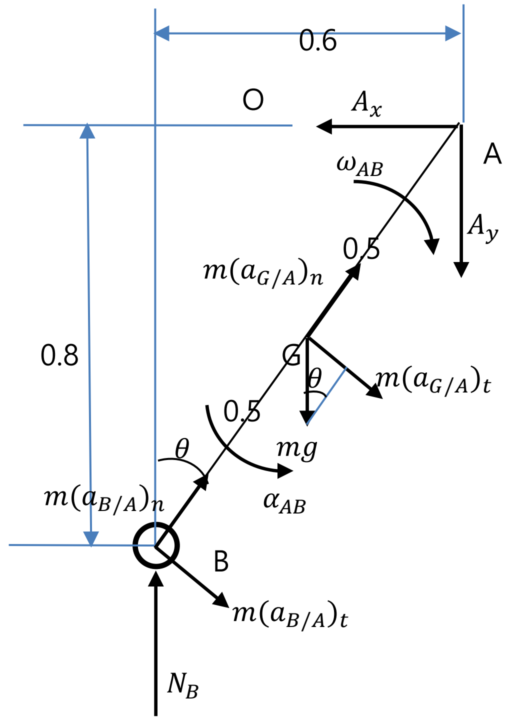
References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.454, P.6.92
Type of Analysis : Plane Kinematics of Rigid Bodies, General Plane Motion
Type of Element : Rigid Body (one part)

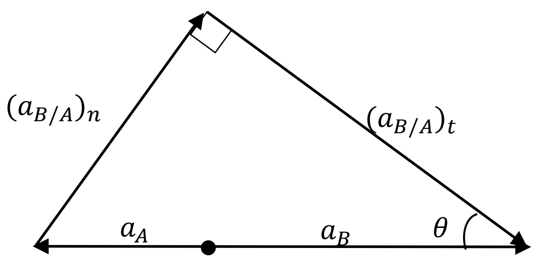
Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	10	kg
Angular velocity	ω_0	4.5	rad/s
Angular velocity	ω_{AB}	3	rad/s







The Equation of Relative Velocity

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$a_A = \overline{OA} \cdot {\omega_0}^2 = 0.4 \times 4.5^2 = 8.1\ m/s^2$$

$$(a_{B/A})_n = \overline{AB} \cdot {\omega_{AB}}^2 = 1.0 \times 3^2 = 9\ m/s^2$$

$$\tan\theta = \frac{(a_{B/A})_n}{(a_{B/A})_t}$$

$$(a_{B/A})_t = \overline{AB} \cdot \alpha_{AB} = \frac{(a_{B/A})_n}{\tan\theta} = \frac{9}{3/4} = 12\ m/s^2$$

$$\alpha_{AB} = \frac{(a_{B/A})_t}{\overline{AB}} = 12\ \mathrm{rad}/s^2$$

$$\sum M_A = \bar{I}\alpha + m\bar{a}d = I_G\alpha + ma_G d$$

$$a_G = a_A + (a_{G/A})_n + (a_{G/A})_t$$

$$(a_{G/A})_t = r\alpha_{AB} = 0.5 \times 12 = 6\ \mathrm{rad}/s^2$$

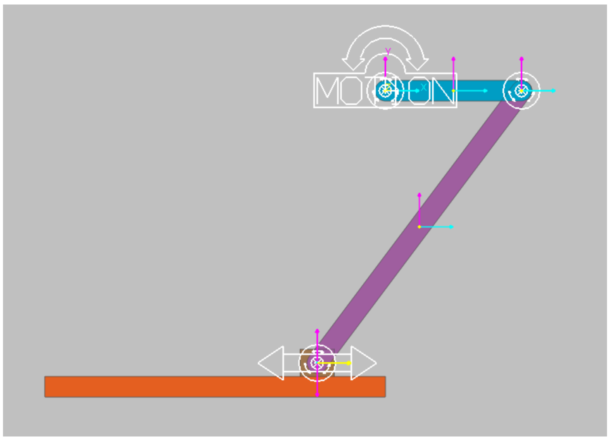
$$\therefore \sum M_A = mg \cdot 0.3 - N_B \cdot 0.6 = \frac{1}{12}ml^2 \cdot \alpha_{AB} + m\left(a_{G/A}\right)_t \cdot 0.5 - ma_A \cdot 0.4$$

$$10 \times 9.81 \times 0.3 - N_B \times 0.6 = \frac{1}{12} \times 10 \times 1.0^2 \times 12 + 10 \times 6 \times 0.5 - 10 \times 8.1 \times 0.4$$

$$\therefore N_B = 36.38\ [N]$$

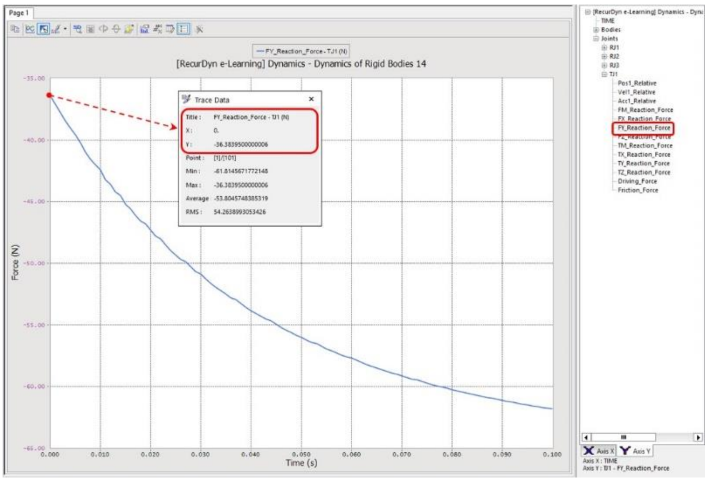
Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

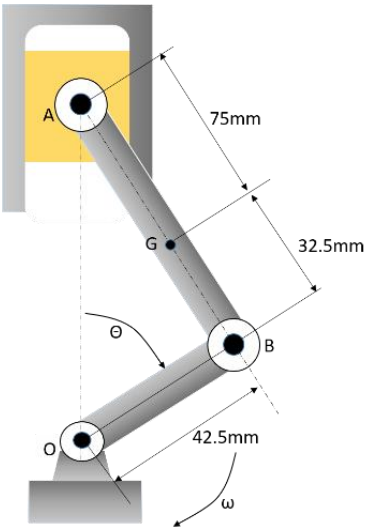
   - The reaction force acting on the end of the roller “B”, which is attached at one end of the link “AB”



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{N_B}$ $[\boldsymbol{N}]$	36.38	36.38	0

Dynamics of Rigid Bodies.25



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.457, P.6.105
Type of Analysis : Plane Kinematics of Rigid Bodies, General Plane Motion
Type of Element : Rigid Body (one part)

Theoretical Solution

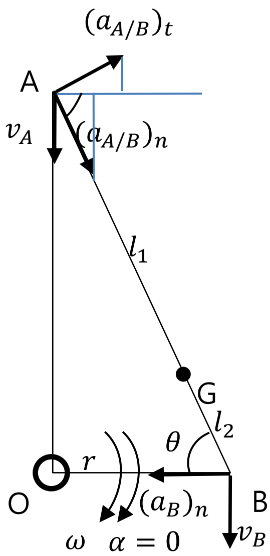
Basic Conditions

Given	Symbol	Value	Unit
Angular velocity	ω_{OB}	100π	rad/s
Angular acceleration	α_{OB}	0	rad/s^2

$m_{AB} = 0.6\,kg,\ m_P = 0.82\,kg$

$l_1 = 0.075\,m,\ l_2 = 0.0325\,m$

$r = 0.0425\,m,\ r_G = 0.028\,m$



Calculate the velocity and the acceleration

$v_B = \omega_{OB} \times 0.0425 = 13.35\ \text{m/s}$

$v_A = v_B = 13.35 \text{ m/s}$

$\therefore \omega_{AB} = 0 \; rad/s$

$a_B = (a_B)_n = r_{OB} \cdot {\omega_{OB}}^2 = 0.0425 \times (100\pi)^2 = 4.195 \times 10^3 \; m/s^2$

$\theta = cos^{-1}\dfrac{r_{OB}}{l_1 + l_2} = cos^{-1}\dfrac{0.0425}{0.075 + 0.0325} = 66.71°$

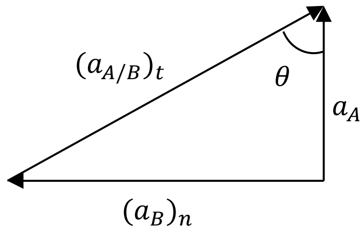
The Equation of Relative Velocity

$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$

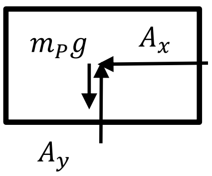
$(a_{A/B})_t = \dfrac{(a_B)_n}{\sin\theta} = \dfrac{4.195 \times 10^3}{\sin 66.71} = 4.567 \times 10^3 \; m/s^2$

$\alpha_{AB} = \dfrac{(a_{B/A})_t}{\overline{AB}} = 42483.7 \text{ rad}/s^2$

$a_A = \dfrac{(a_B)_n}{\tan\theta} = \dfrac{4.195 \times 10^3}{\tan 66.71} = 1.806 \times 10^3 \; m/s^2$



(1) Piston



$\sum F_x = m_P a_{px}$

$A_x = 0$

$\sum F_y = m_P a_{py}$

$A_y - m_P g = m_P a_A$

$m_P g \cong 0$: Ignore

$A_y = 0.82 \times 1.806 \times 10^3 = 1480.7 \; [N]$

$m_P g \neq 0$: Consider

$A_y = 0.82 \times (9.81 + 1.806 \times 10^3) = 1488.8 \; [N]$

(2) The link "AB"

$$\sum M_B = I_B\alpha + \bar{\rho} \times m a_B$$

$I_B = I_G + m\bar{\rho}^2 = m{r_G}^2 + m\bar{\rho}^2 = m({r_G}^2 + \bar{\rho}^2) = 0.6 \cdot (0.028^2 + 0.0325^2) = 0.00110415$

$a_B = 4.195 \times 10^3 \ m/s^2$

$A_x \cdot l \cdot \sin\theta - A_y \cdot r_{OB} - m_{AB}g \cdot \bar{\rho} \cdot \cos\theta = I_B \ \cdot \alpha_{AB} - m_{AB} \cdot a_B \cdot \bar{\rho} \cdot \sin\theta$

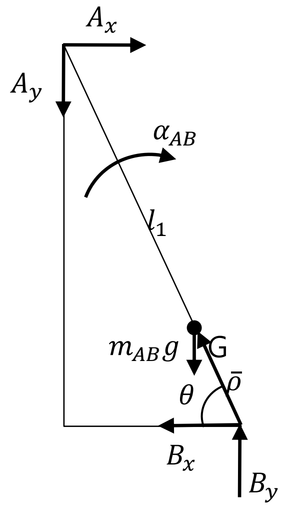
$m_{AB}g \cong 0$: Ignore

$A_x \times 0.1075 \times \sin 66.71 - 1488.8 \times 0.0425$

$= 0.0011 \ \times 42483.7 - 0.6 \times 4195 \times 0.0325 \times \sin 66.71$

$A_x = 353.1 \ [N]$

$\therefore A = \sqrt{{A_x}^2 + {A_y}^2} = 1530.1 \ [N]$



$m_{AB}g \neq 0$: Consider

$A_x = 353.9 \ [N]$

$\therefore A = \sqrt{{A_x}^2 + {A_y}^2} = 1530.3 \ [N]$

$$\sum M_B = \bar{I}\alpha + m\bar{a}d$$

$a_G = a_B + (a_{G/A})_n + (a_{G/A})_t$

$I_G = m{r_G}^2 = 0.6 \times 0.028^2 = 0.0004704$

$m_{AB}g \cong 0$: Ignore

$A_x \times 0.1075 \times \sin 66.71 - 1488.8 \times 0.0425$

$= 0.0004704 \ \times 42483.7 + 0.6 \times (-4195 \times 0.0325 \times \sin 66.71$

$+ 0.0325^2 \times 42483.7)$

$A_x = 354.9 \ [N]$

$\therefore A = \sqrt{{A_x}^2 + {A_y}^2} = 1530.5 \ [N]$

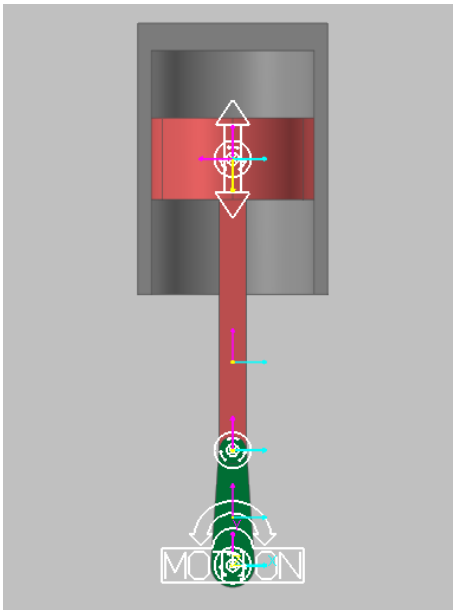
$m_{AB}g \neq 0$: Consider

$$A_x = 355.69\ [N]$$

$$\therefore A = \sqrt{{A_x}^2 + {A_y}^2} = 1530.7\ [N]$$

● Numerical Solution - RecurDyn

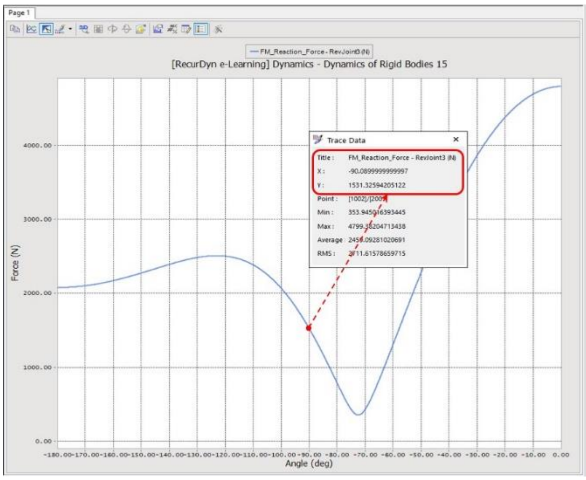
1. Modeling



2. Plot the results

- The reaction force of the joint when $\theta = 90°$

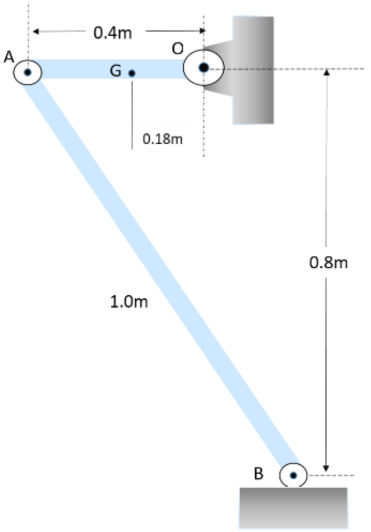
(X axis : the angle of Revolute Joint "O", Y axis : the reaction force of Revolute Joint "A")



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{R_A}$ $[\boldsymbol{N}]$	1530.5	1531.3	0.052

Dynamics of Rigid Bodies.26



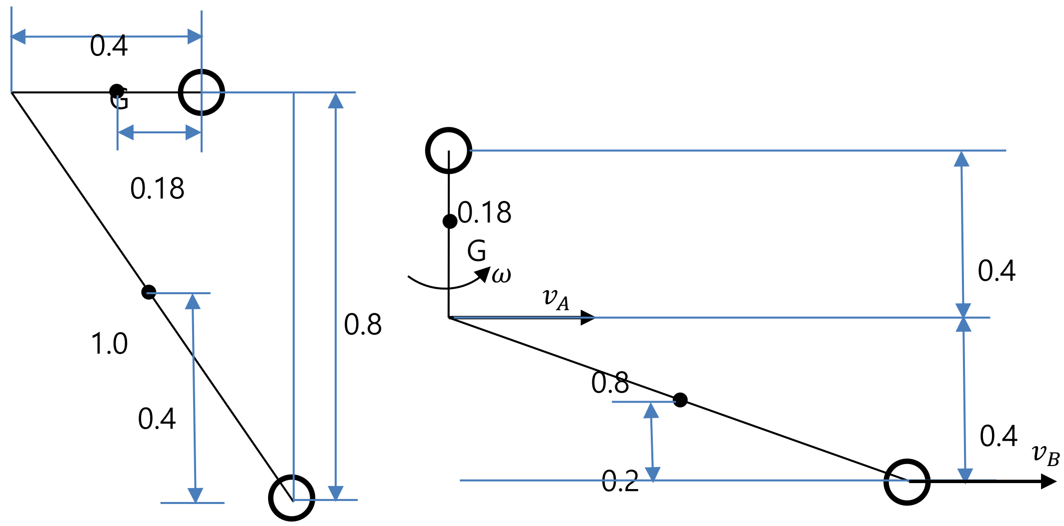
References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.474, P.6.139

Type of Analysis : Plane Kinematics of Rigid Bodies, Work-energy Equation

Type of Element : Rigid Body (one part)

⭘ Theoretical Solution

Basic Conditions



Given	Symbol	Value	Unit
Mass	$m_{OA,}m_{AB}$	8, 12	kg
Gyration radius	r_G	0.22	m

$v_A = 0.4\omega$ m/s

$v_A = v_B = v$

$\omega_{AB} = 0$

Work-energy Equation

The link "OA"

$\Delta V_g = -m_{OA}gh_1 = -8 \times 9.81 \times 0.18 = -14.13$

$$\Delta \mathrm{T} = \frac{1}{2}I_O\omega^2 = \frac{1}{2} \times m_{OA}{r_O}^2 \times \left[\frac{v_A}{0.4}\right]^2 = \frac{1}{2} \times 8 \times 0.22^2 \times \frac{{v_A}^2}{0.4^2} = 1.21 \cdot v^2$$

The link "AB"

$\Delta V_g = -m_{AB}gh_2 = -12 \times 9.81 \times 0.2 = -23.54$

$$\Delta \mathrm{T} = \frac{1}{2}m_{AB}{v_B}^2 = \frac{1}{2} \times 12 \times {v_B}^2 = \frac{1}{2} \times 12 \times {v_B}^2 = 6 \cdot v^2$$

The total system

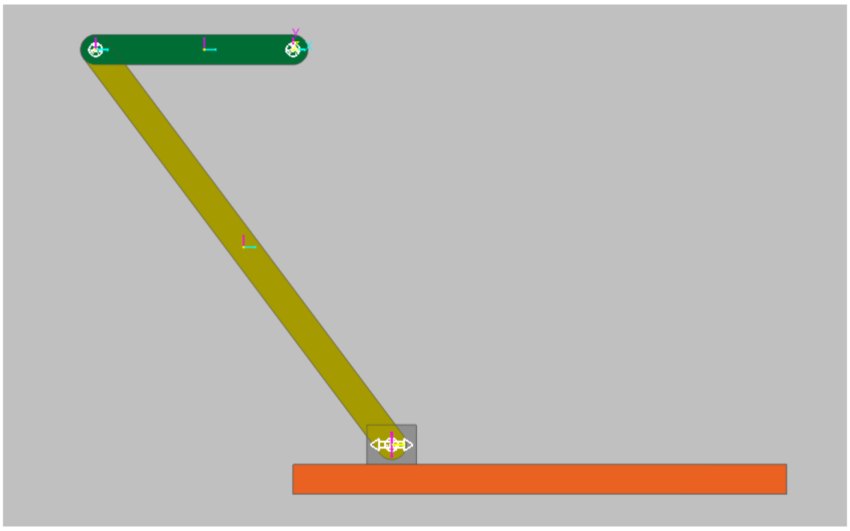
$U_{1-2} = \Delta \mathrm{T} + \Delta V_g$

$0 = -14.13 + 1.21 \cdot v^2 - 23.54 + 6 \cdot v^2$

$\therefore v = 2.29\ m/s$

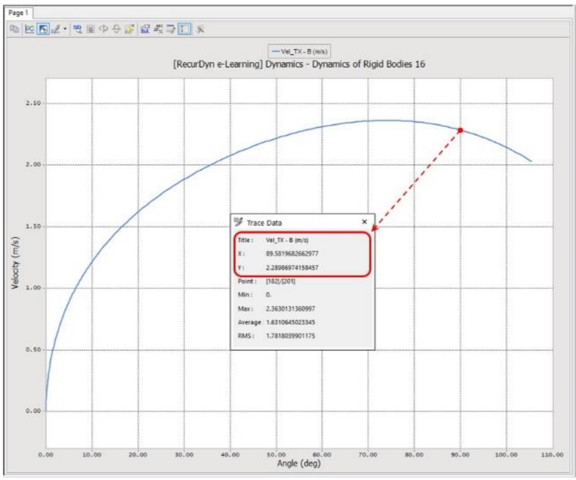
○ Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

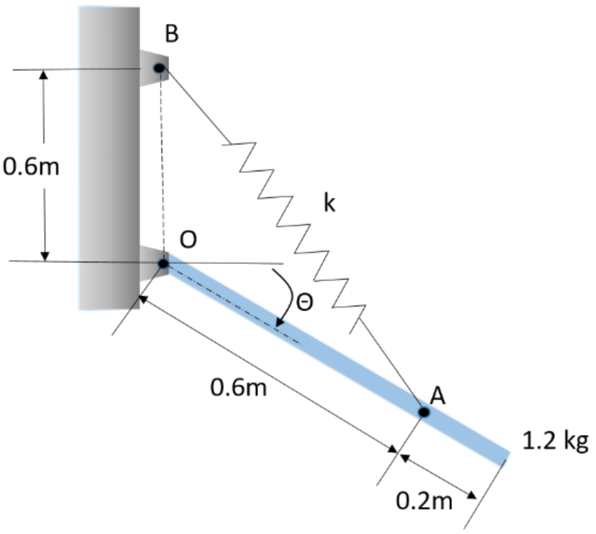
- The velocity of end B (X axis : the rotation angle of Revolute Joint “O”, Y axis : the velocity on X axis of “B”)



⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
v_B $[m/s]$	2.29	2.29	0

Dynamics of Rigid Bodies.27



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.509, P.6.221
Type of Analysis : Plane Kinematics of Rigid Bodies
Type of Element : Rigid Body (one part)

Theoretical Solution

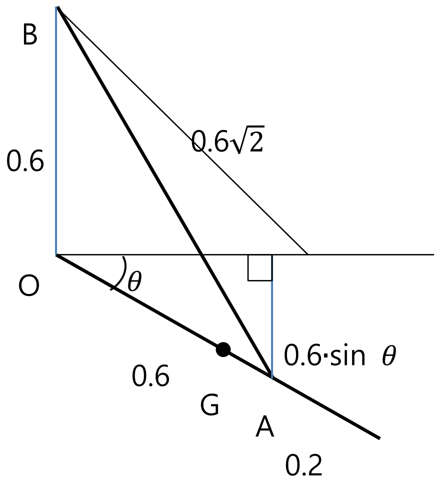
Basic Conditions

Given	Symbol	Value	Unit
Mass	m	1.2	kg
Spring coefficient	k	100	N/m

$l_1 = 0.6\text{ m}, l_2 = 0.2\text{ m}$

$\theta = 0° \rightarrow v_0 = 0, \quad \delta_0 = 0$

The length of the spring without deformation, $l_0 = 0.6\sqrt{2}\text{ m}$



Work-energy Equation

The length of the spring with deformation when the angle is θ,

$l^2 = 0.6^2 + 0.6^2 - 2 \times 0.6 \times 0.6 \times \cos(90 + \theta) = 2 \times 0.6^2 \times (1 + \sin\theta)$

Thus, the equation of deformation of the spring is, $\delta = l - l_0 = \sqrt{2 \times 0.6^2 \times (1 + \sin\theta)} - 0.6\sqrt{2} = 0.6\sqrt{2}(\sqrt{1 + \sin\theta} - 1)$

$U_{1-2} = \Delta T + \Delta V_g = 0$

$\theta_{max} \rightarrow v = 0$

$$mgh_1 + \frac{1}{2}k{x_1}^2 + \frac{1}{2}I_O{\omega_1}^2 = mgh_2 + \frac{1}{2}k{x_2}^2 + \frac{1}{2}I_O{\omega_2}^2$$

$x_1 = 0\,, \quad \omega_1 = 0\,, \quad h_2 = 0\,, \quad \omega_2 = 0$

$$1.2 \times 9.81 \times 0.4 \sin\theta = \frac{1}{2} \times 100 \times \left[0.6\sqrt{2}(\sqrt{1 + \sin\theta} - 1)\right]^2$$

$4.7088 \cdot \sin\theta = 36 \cdot (2 + \sin\theta - 2\sqrt{1 + \sin\theta})$

$31.2912 \cdot \sin\theta - 72\sqrt{1 + \sin\theta} + 72 = 0$

If $\sin\theta = X$, then,

$31.2912 \cdot X - 72\sqrt{1 + X} + 72 = 0$

$31.2912 \cdot X + 72 = 72\sqrt{1 + X}$

Squares both sides of the equation,

$979.14 \cdot X^2 + 4505.93 \cdot X + 5184 = 5184(1 + X)$

$979.14 \cdot X^2 - 678.07 \cdot X = 0$

$X \cdot (979.14 \cdot X - 678.07) = 0$

$X = 0 \quad or \quad X = 0.6925$

$\therefore \theta = 0° \quad or \quad \theta = 43.83°$

$\therefore \theta_{max} = 43.83°$

When θ is an arbitrary angle,

$U_{1-2} = \Delta \mathrm{T} + \Delta V_g = 0$

$mgh_1 + \frac{1}{2}k{x_1}^2 + \frac{1}{2}I_O{\omega_1}^2 = mgh_2 + \frac{1}{2}k{x_2}^2 + \frac{1}{2}I_O{\omega_2}^2$

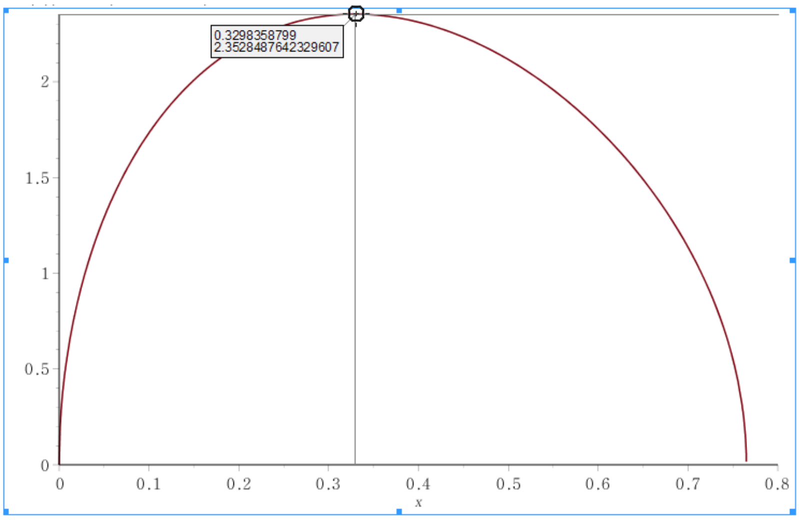
$x_1 = 0\,, \qquad \omega_1 = 0\,, \qquad h_2 = 0$

$1.2 \times 9.81 \times 0.4 \sin\theta = \frac{1}{2} \times 100 \times \left[0.6\sqrt{2}(\sqrt{1+\sin\theta} - 1)\right]^2 + \frac{1}{2} \times \frac{1}{3} \times 1.2 \times 0.8^2 \times \omega^2$

$4.7088 \cdot \sin\theta = 36 \cdot \left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right) + 0.128 \cdot \omega^2$

$0.128 \cdot \omega^2 = -31.2912 \cdot \sin\theta + 72\sqrt{1+\sin\theta} - 72$

$\therefore \omega = \sqrt{-244.4625 \cdot \sin\theta + 562.5 \cdot \sqrt{1+\sin\theta} - 562.5}$



If, $\omega = \sqrt{-244.4625 \cdot \sin\theta + 562.5 \cdot \sqrt{1+\sin\theta} - 562.5} = \sqrt{f}$

$$\frac{d\omega}{d\theta} = \frac{-244.4625 \cdot \cos\theta + 562.5 \cdot \dfrac{\cos\theta}{2\sqrt{1+\sin\theta}}}{2\sqrt{f}} = 0$$

$-244.4625 \cdot \cos\theta \cdot 2 \cdot \sqrt{1+\sin\theta} + 562.5 \cdot \cos\theta = 0$

$$\cos\theta \cdot \left(-488.925 \cdot \sqrt{1+\sin\theta} + 562.5\right) = 0$$

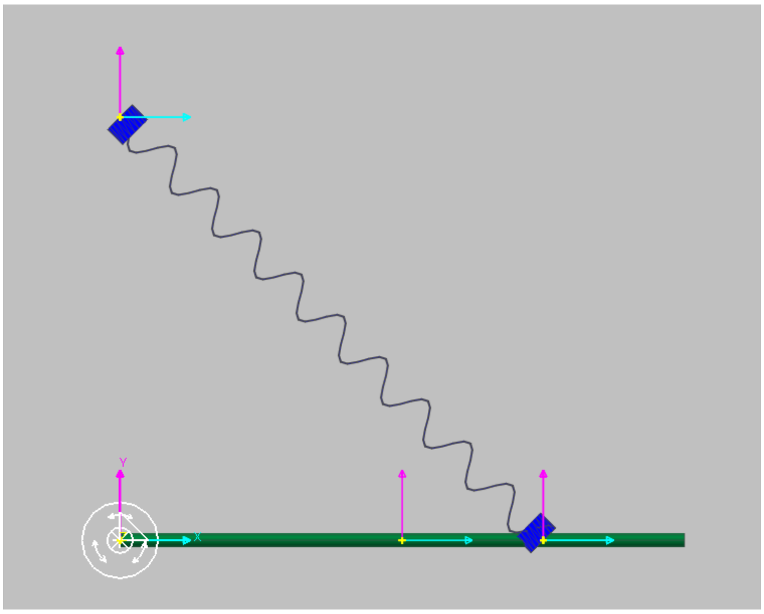
$$\cos\theta = 0 \quad or \quad \sqrt{1+\sin\theta} = 1.1505$$

$$\therefore \theta = 90° \quad or \quad \theta = 18.88°$$

$$\therefore \theta = 18.88° \; (0.3295\, rad) \;\; \rightarrow \;\; \omega_{max} = 2.353\, rad/s$$

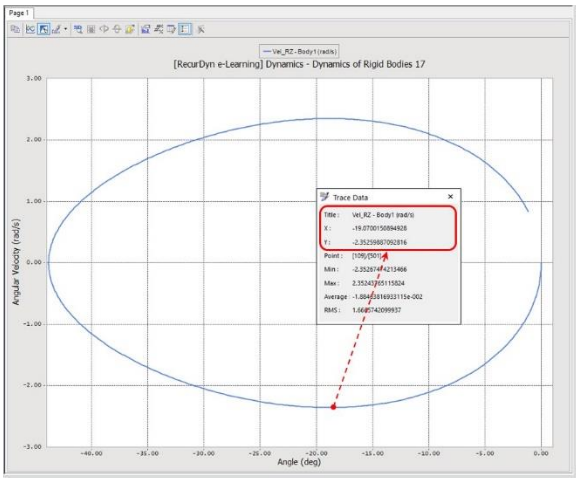
⭘ Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

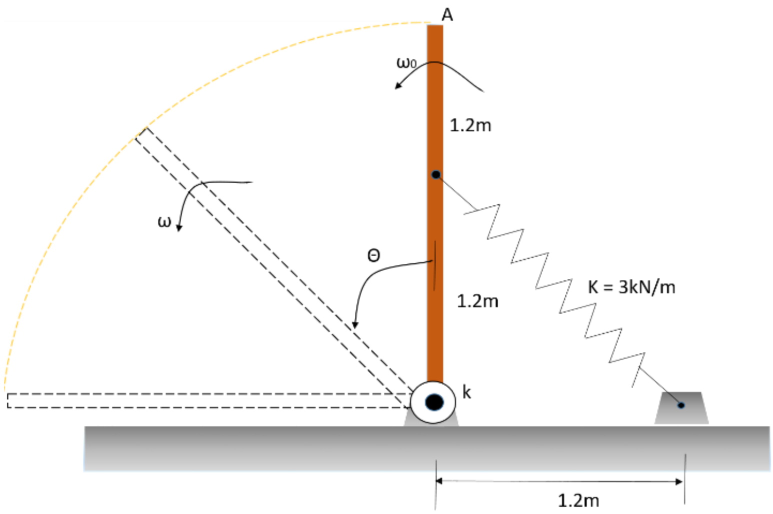
- The angular velocity on Z axis of the "AB" and "CA"



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\boldsymbol{\omega_{max}}$ $[\boldsymbol{rad/s}]$	2.353	2.353	0
$\boldsymbol{\theta}$ $[\boldsymbol{deg}]$	18.88	19.07	1.001

Dynamics of Rigid Bodies.28



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.511, P.6.229
Type of Analysis : Plane Kinematics of Rigid Bodies
Type of Element : Rigid Body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	30	kg
Angular velocity	ω_0	4	rad/s

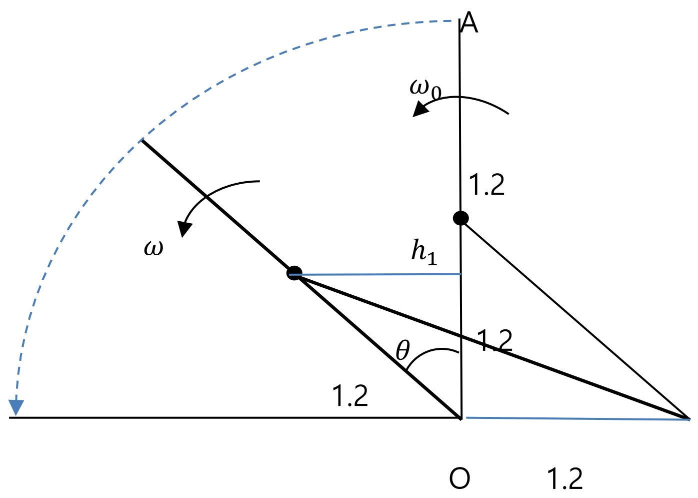
$\omega_0 = 4\,rad/s \;\rightarrow\; \delta_0 = 0$

$l = 2.4\text{ m}$

$l_2 = 0.2\text{ m}$

$k = 3\text{ kN/m}$

$$I_O = \frac{1}{3}ml^2 = \frac{1}{3} \times 30 \times 2.4^2 = 57.6\text{ m}$$



Work-energy Equation

$l_0 = 1.2\sqrt{2}\ \mathrm{m}$

The length of the spring with deformation when the angle is θ,

$l^2 = 1.2^2 + 1.2^2 - 2 \times 1.2 \times 1.2 \times \cos(90 + \theta) = 2 \times 1.2^2 \times (1 + \sin\theta)$

Thus, the equation of deformation of the spring is, $\delta = l - l_0 = \sqrt{2 \times 1.2^2 \times (1 + \sin\theta)} - 1.2\sqrt{2} = 1.2\sqrt{2}(\sqrt{1 + \sin\theta} - 1)$

$U_{1-2} = \Delta \mathrm{T} + \Delta V_g = 0$

$$mgh_1 + \frac{1}{2}k{x_1}^2 + \frac{1}{2}I_O{\omega_1}^2 = mgh_2 + \frac{1}{2}k{x_2}^2 + \frac{1}{2}I_O{\omega_2}^2$$

$x_1 = 0\,,\ \ h_2 = 0$

$$30 \times 9.81 \times 1.2 \times (1 - \cos\theta) + \frac{1}{2} \times 57.6 \times 4^2$$

$$= \frac{1}{2} \times 3000 \times \left[1.2\sqrt{2}(\sqrt{1 + \sin\theta} - 1)\right]^2 + \frac{1}{2} \times 57.6 \times \omega^2$$

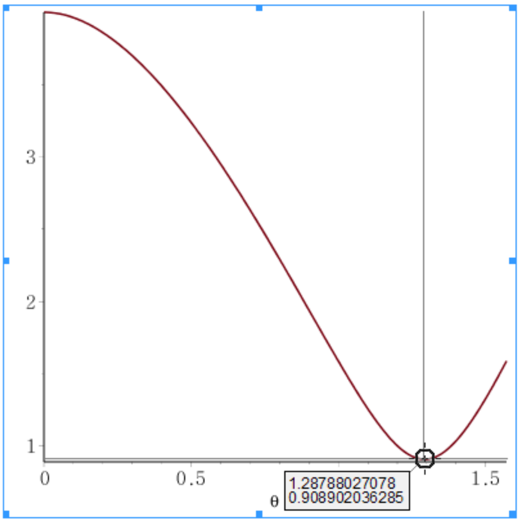
$$813.96 - 353.16 \cdot \cos\theta = 4320 \cdot (2 + \sin\theta - 2\sqrt{1 + \sin\theta}) + 28.8 \cdot \omega^2$$

$$28.8 \cdot \omega^2 = -4320 \cdot \sin\theta + 8640\sqrt{1 + \sin\theta} - 7826.04 - 353.16 \cdot \cos\theta$$

$$\therefore \omega = \sqrt{-150 \cdot \sin\theta + 300 \cdot \sqrt{1 + \sin\theta} - 271.74 - 12.263 \cdot \cos\theta}$$

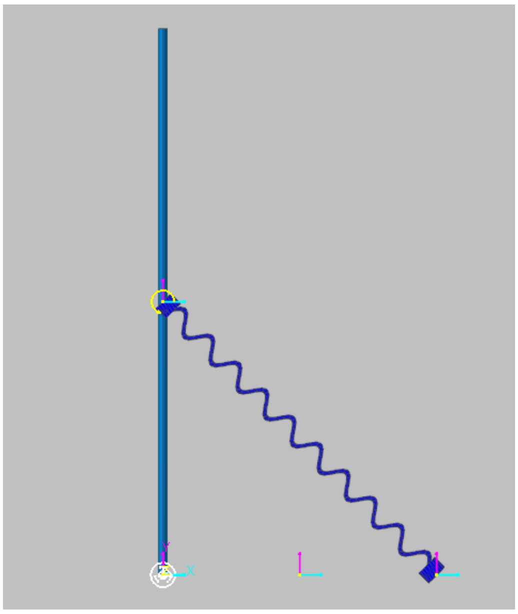
$\therefore \omega_{min} = 0.909\, rad/s \quad at \;\; \theta = 1.2879\, rad = 73.79°$

$\theta = 90° \;\; \rightarrow \;\; \omega = 1.589\, rad/s$



Numerical Solution - RecurDyn

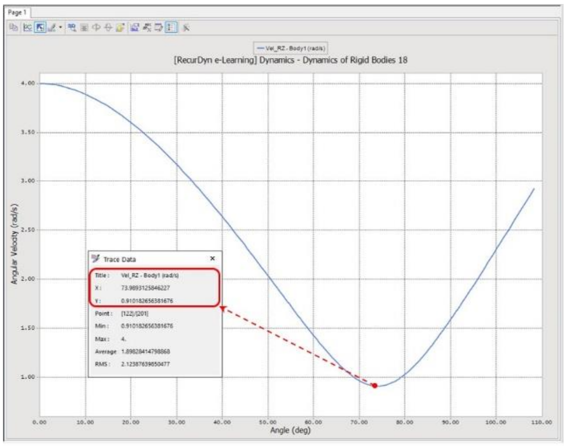
1. Modeling

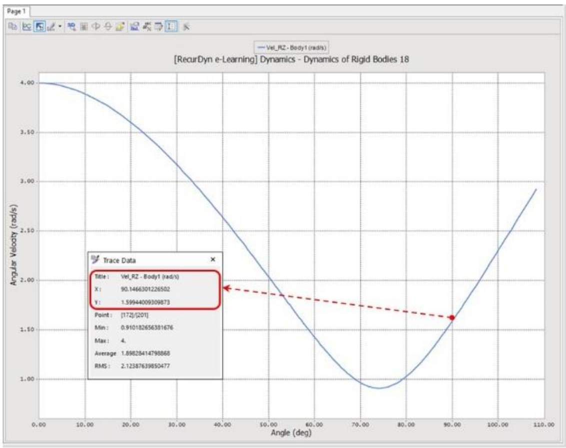


2. Plot the results

- The angular velocity on the Z axis of "A"

(X axis : the rotation angle of revolute joint “K”, Y axis : the angular velocity on the Z axis of “A”)

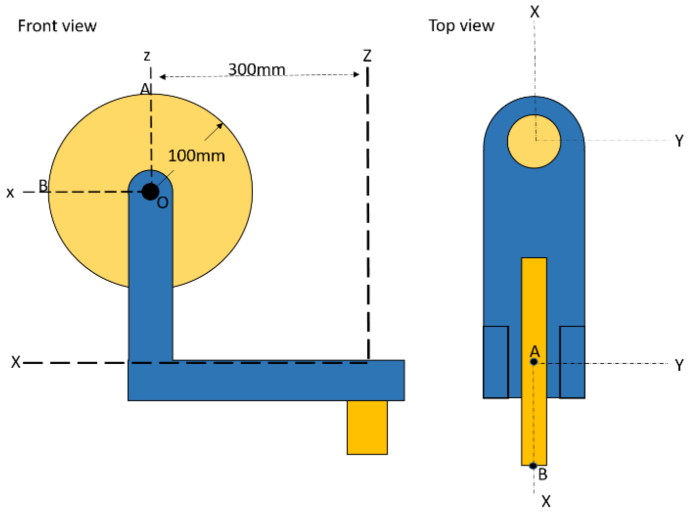




⭘ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
ω_{min} $[rad/s]$	0.909	0.901	0.88
θ $[deg]$	73.79	73.99	0.271
$\omega_{AB,h=0}$ $[rad/s]$	1.589	1.599	0.629

Dynamics of Rigid Bodies.29



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.536, P.7.46
Type of Analysis : 3-Dimensional Rigid Body Dynamics, Rotational Motion about a Fixed Point
Type of Element : Rigid body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Angular velocity of disk	$\vec{p}$	10π	rad/s
Angular velocity of frame	$\vec{\Omega}$	4π	rad/s

$$\vec{p} = 10\pi\vec{j}\ [rad/s] = p\vec{j}\ [rad/s] = \text{const}$$

$$\vec{\Omega} = 4\pi\vec{k}\ [rad/s] = \Omega\vec{k}\ [rad/s] = \text{const}$$

$$\vec{R}_O = 0.3\vec{i} + 0.1\vec{k}$$

$$\vec{r}_{A/O} = 0.1\vec{k}$$

Interaction Formula of Acceleration

$$\vec{\alpha} = \dot{\vec{p}} = \dot{p}\vec{j} + p\dot{\vec{j}} = \vec{\Omega} \times \vec{p} = \Omega\vec{k} \times p\vec{j} = -p\Omega\vec{i} = -40\pi^2\vec{i}$$

$$\vec{a}_A = \vec{a}_o + \dot{\vec{\Omega}} \times \vec{r}_{A/O} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}_{A/O}\right) + 2\vec{\Omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_o = \vec{\Omega} \times \left(\vec{\Omega} \times \vec{R}_O\right) = 4\pi\vec{k} \times \left(4\pi\vec{k} \times (0.3\vec{i} + 0.1\vec{k})\right) = 4\pi\vec{k} \times 1.2\pi\vec{j} = -4.8\pi^2\vec{i}$$

$$\dot{\vec{\Omega}} = \vec{0}$$

$$\vec{\Omega} \times \vec{r}_{A/O} = 4\pi\vec{k} \times 0.1\vec{k} = \vec{0}$$

$$\vec{v}_{rel} = \vec{p} \times \vec{r}_{A/O} = 10\pi\vec{j} \times 0.1\vec{k} = \pi\vec{i}$$

$$\vec{a}_{rel} = \vec{p} \times \left(\vec{p} \times \vec{r}_{A/O}\right) = 10\pi\vec{j} \times \left(10\pi\vec{j} \times 0.1\vec{k}\right) = 10\pi\vec{j} \times \pi\vec{i} = -10\pi^2\vec{k}$$

$$\therefore \vec{a}_A = -4.8\pi^2\vec{i} + \vec{0} + \vec{0} + 8\pi\vec{k} \times \pi\vec{i} - 10\pi^2\vec{k} = -4.8\pi^2\vec{i} + 8\pi^2\vec{j} - 10\pi^2\vec{k}$$

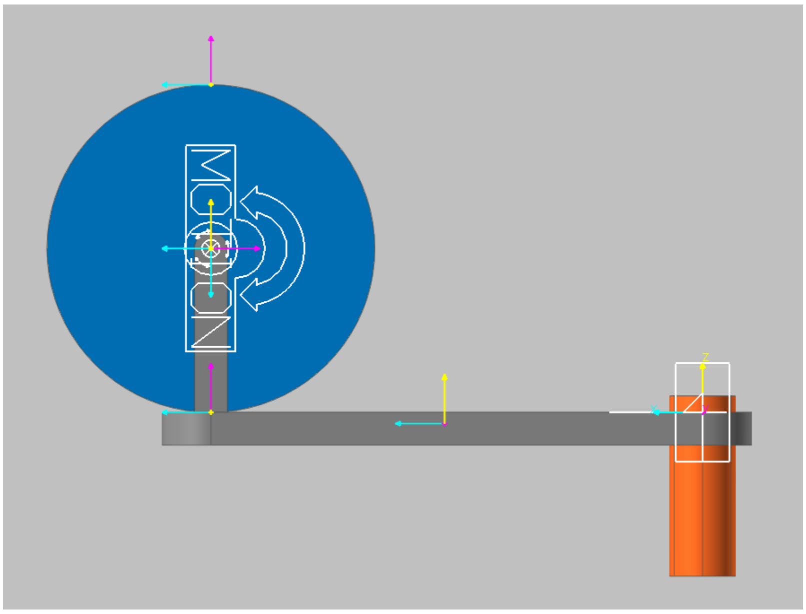
$$= 2\pi^2\left(-2.4\vec{i} + 4\vec{j} - 5\vec{k}\right)\ m/s^2$$

$$\therefore \vec{a}_A = -47.37\vec{i} + 78.96\vec{j} - 98.70\vec{k}\ \ m/s^2$$

$$\therefore |\vec{a}_A| = \sqrt{(47.37^2 + 78.96^2 + 98.70^2 +)} = 134.98\,m/s^2$$

Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

1-1) Create a scope of the point “A” which is on the disk

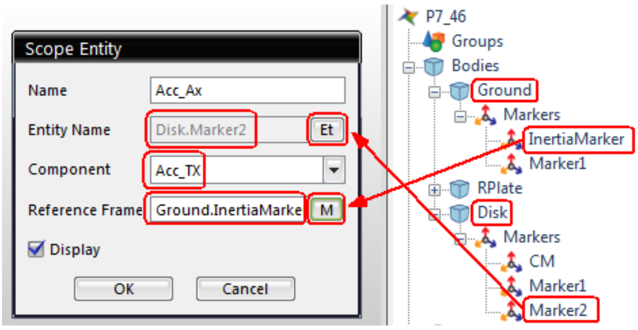
1. Enter the “Name” and click the “Et” button after select the “Entity” 

icon. And, drag and drop “Disk-Marker2” in the “Database” window

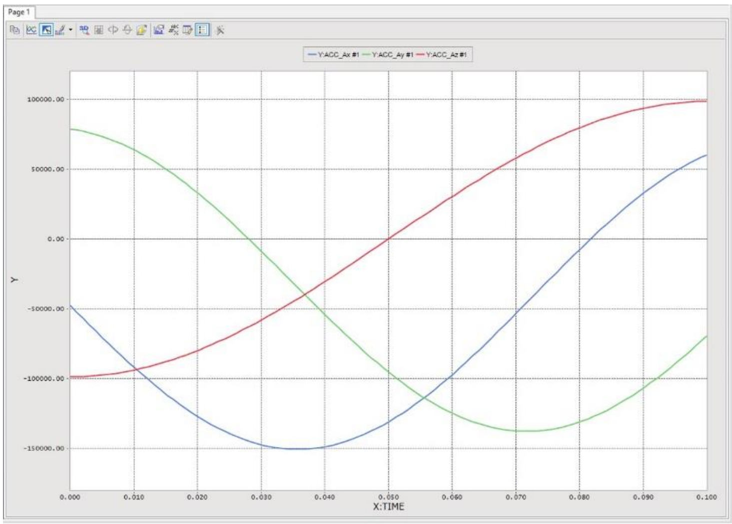
2. Change the “Component” to “Component – Acc_TX”

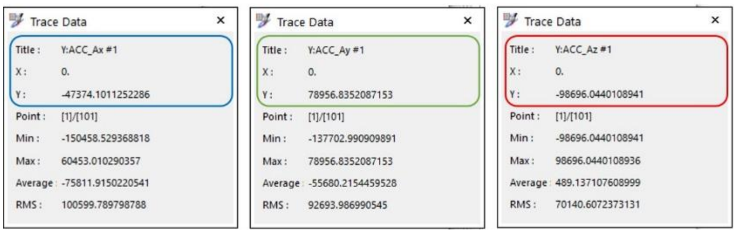
3. Change the “Reference Frame” to “Ground-InertiaMarker”

4. Click the “Add to Plot”  icon to add these scopes to the “Plot”

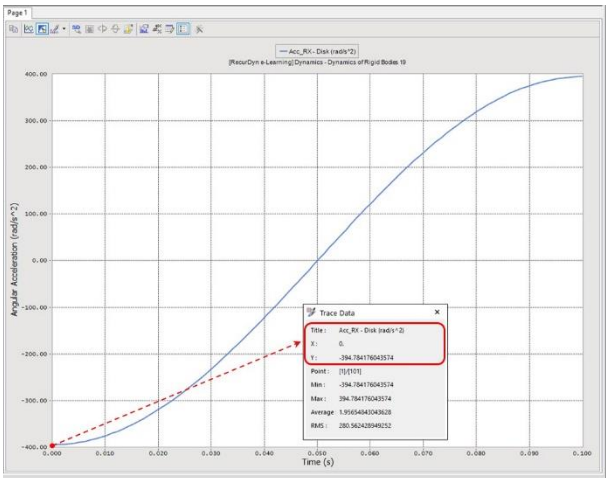


1-2) “Add to Plot” for the acceleration on the X,Y,Z axis of the “Disk”





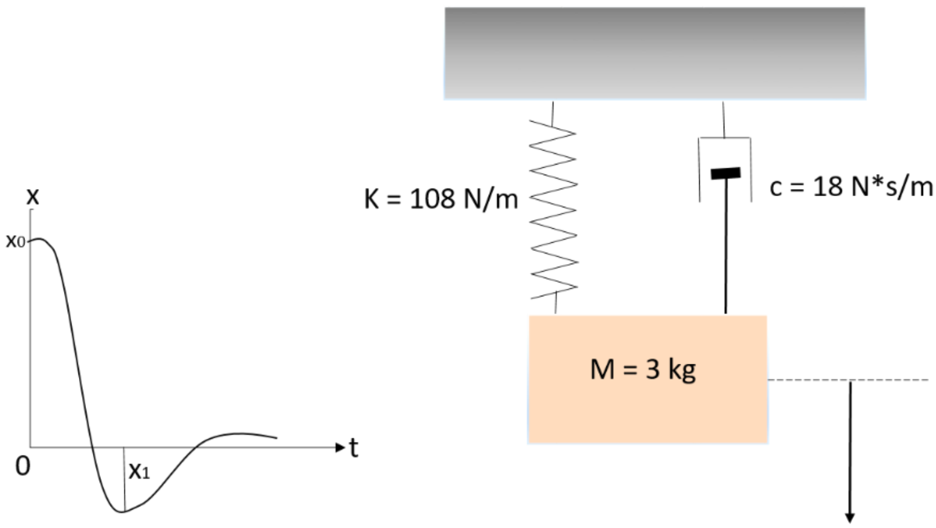
2) Draw a plot of an angular acceleration on the X axis of the “Disk”



Comparison of results

Object Value	Theory	RecurDyn	Error(%)
$\alpha\ [rad/s^2]$	-394.78$\vec{\imath}$	$-394.78\vec{\imath}$	0
$(a_A)_x\ [m/s^2]$	-47.37	-47.37	0
$(a_A)_y\ [m/s^2]$	78.96	78.96	0
$(a_A)_z\ [m/s^2]$	-98.70	-98.70	0

Dynamics of Rigid Bodies.30



References : Engineering Mechanics : Dynamics (J.L.Meriam, L.G.Kraige), 7th Edition, pp.598, P.8.36
Type of Analysis : Vibration and Time Response, Free Vibration of Particles
Type of Element : Rigid body (one part)

Theoretical Solution

Basic Conditions

Given	Symbol	Value	Unit
Mass	m	3	kg
Spring Coefficient	k	108	N/m
Damping Coefficient	c	18	$N \cdot s/m$

The Equation of Motion of Free Damped Oscillation

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{108}{3}} = 6\, rad/s$$

$$\zeta = \frac{c}{2\sqrt{mk}} = \frac{18}{2\sqrt{3 \cdot 108}} = 0.5$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6 \cdot \sqrt{1-0.5^2} = 5.196\, rad/s$$

The Solution of the Second Order Differential Equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = 0$$

$$x = (A_1 \cos\omega_d t + A_2 \sin\omega_d t)e^{-\zeta\omega_n t}$$

If, $t = 0 \;\rightarrow\; x = x_0$

$$\therefore A_1 = x_0$$

If, $t = 0 \;\rightarrow\; \dot{x} = 0$

$$\dot{x} = \omega_d(-A_1 \sin\omega_d t + A_2 \cos\omega_d t)e^{-\zeta\omega_n t} - \zeta\omega_n(A_1 \cos\omega_d t + A_2 \sin\omega_d t)e^{-\zeta\omega_n t}$$

$$0 = \omega_d A_2 - \zeta\omega_n A_1$$

$$\therefore A_2 = \frac{\zeta\omega_n}{\omega_d}A_1 = \frac{0.5 \times 6}{5.196}x_0 = 0.577x_0$$

$$\therefore x = x_0(\cos 5.196t + 0.577 \sin 5.196t)e^{-3t}$$

$$\omega_d = \frac{5.196}{2\pi}\; Hz = 0.827\; Hz$$

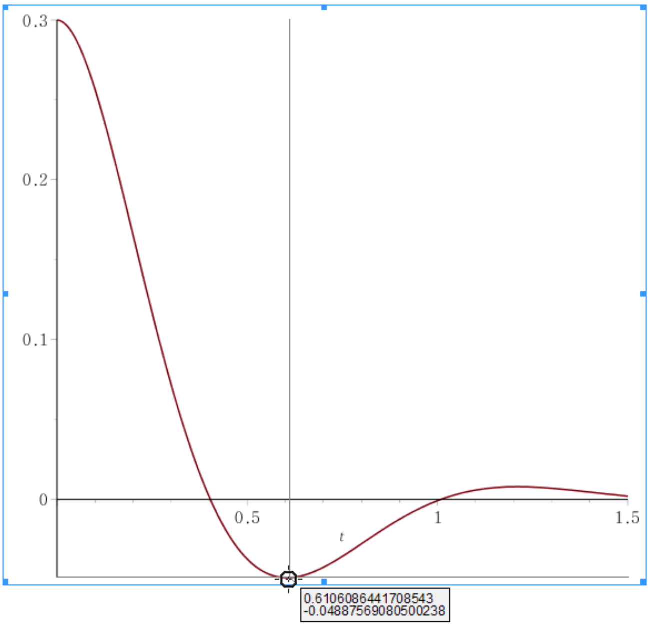
$$T = \frac{1}{\omega_d} = 1.209\; s/cycle$$

$$t = \frac{T}{2} = 0.605\; sec$$

$$\therefore x = x_1 = -0.163x_0$$

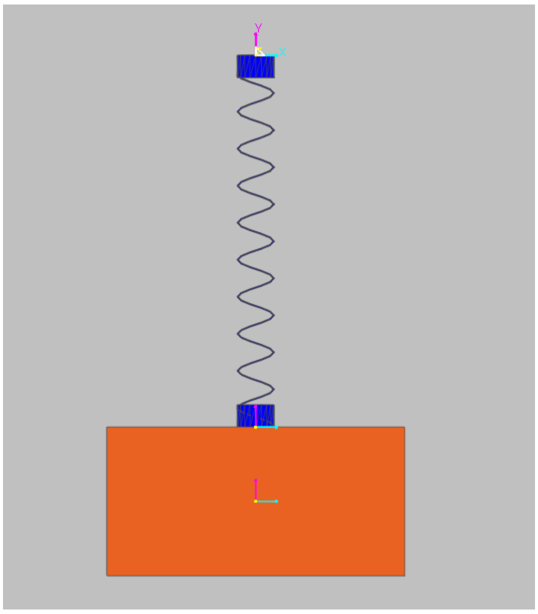
$$x_0 = 0.3\; m$$

$$x_1 = -0.0489$$



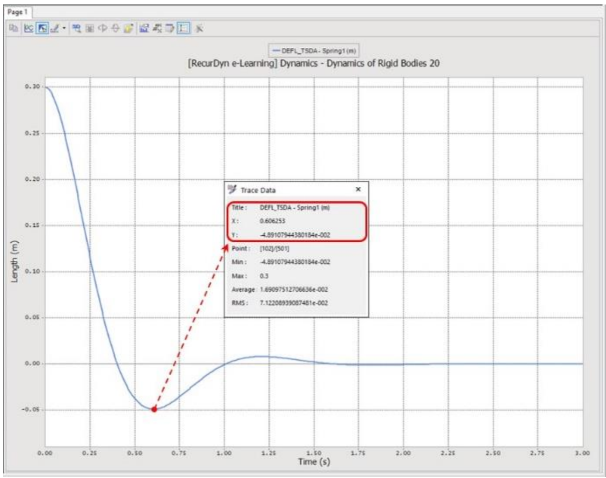
⭘ Numerical Solution - RecurDyn

1. Modeling



2. Plot the results

- Draw the “DEFL_TSDA” of Spring1 (the deformation of the spring) from the database



⚬ Comparison of results

Object Value	Theory	RecurDyn	Error(%)
x_1 $[m]$	-0.0489	-0.0489	0